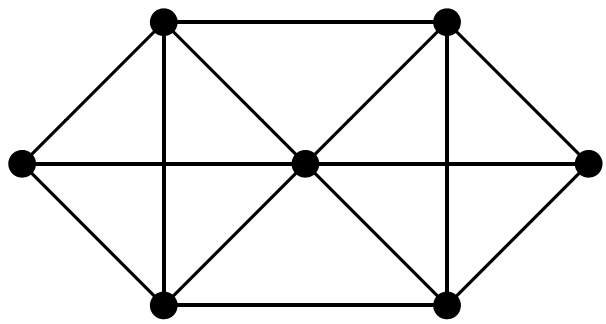
CS 70 Discrete Mathematics and Probability Theory
Summer 2015 Chung-Wei Lin Discussion 8A

1. Degree

Prove that for any graph of $n \geq 2$ vertices, there are at least two vertices which have the same degree.

2. Eulerian Tour and Eulerian Walk



(a) Is there an Eulerian tour in the graph above?

(b) Is there an Eulerian walk in the graph above?

(c) What is the condition that there is an Eulerian walk in an undirected graph?

3. Hypercubes

The vertex set of the n-dimensional hypercube $G = (V, E)$ is given by $V = \{0,1\}^n$, where recall $\{0,1\}^n$ denotes the set of all n-bit strings. There is an edge between two vertices x and y if and only if x and y differ in exactly one bit position. These problems will help you understand hypercubes.

(a) Depict 1-, 2-, and 3-dimensional hypercubes.

(b) Show that the edges of an n-dimensional hypercube can be colored using n colors so that no pair of edges sharing a common vertex have the same color.

(c) Show that the vertices of an n-dimensional hypercube can be colored using 2 colors so that no pair of adjacent vertices have the same color. (This is equivalent to showing that a hypercube is *bipartite*: the vertices can be partitioned into two groups (according to color) so that every edge goes between the two groups.)

4. Bipartite Graph

Consider an undirected bipartite graph with two disjoint sets L, R. Prove that a bipartite graph has no cycles of odd length.