

EX.

6.2

THE COMPLEX NUMBER $\frac{3+2j}{2-3j}$ CAN BE EXPANDED INTO ITS REAL AND $2-3j$ IMAGINARY PARTS :

$$\frac{3+2j}{2-3j} = \frac{(3+2j)(2+3j)}{(2-3j)(2+3j)} = \frac{6+4j+9j-6}{4+9} = j$$

$$\text{ALSO } 3e^{j(-50^\circ)} = 3 \cos 50^\circ - j3 \sin 50^\circ = 1.928 - j2.30$$

$$\begin{aligned} \therefore \frac{3+2j}{2-3j} + 3e^{j(-50^\circ)} &= j + 1.928 - j2.3 \\ &= 1.928 - j1.3 \\ &= 2.325 e^{j(-34^\circ)} \end{aligned}$$

THEREFORE $A = 2.325$; $\theta = -34^\circ$ ■

EX.

6.3

FIRST EXPRESS $(2-3j)$ AND $(-1+0.5j)$ IN THE FORM $Ae^{j\theta}$.

$$2-3j = \sqrt{2^2+3^2} e^{j \tan^{-1} \frac{3}{2}} = 3.6 e^{-j56.3^\circ}$$

$$-1+0.5j = \sqrt{1^2+0.5^2} e^{j \tan^{-1} \frac{0.5}{-1}} = 1.12 e^{j153.4^\circ}$$

$$\therefore X = \frac{[4e^{j(3/2)}][3.6e^{-j56.3^\circ}]}{1.12e^{j153.4^\circ}} = 12.8e^{-2.16}$$

$\therefore A = 2.16$; $\theta = -2.16$ radians ■

CHAPTER 6 - PROBLEMS

6.1 (1) PERIOD = $\frac{1}{f} = \frac{1}{60\text{Hz}} = \underline{16.7\text{msec}}$

ANGULAR FREQUENCY = $2\pi f = 2\pi(60) = \underline{377 \text{ radian/sec.}}$

(2) TIME DIFFERENCE = $\frac{\phi}{\omega}$
 $= \frac{18^\circ}{180^\circ} \times \pi \times \frac{1}{377} = \underline{0.83\text{msec}}$ ■

6.2 NO MAXIMA BETWEEN THE ONE AT $t_1 = 0.014$ SECOND AND THE ONE AT $t_2 = 0.018$ SECOND MEANS THAT

$$(\omega t_2 + \phi) - (\omega t_1 + \phi) = 2\pi$$

$$\Rightarrow \omega = \frac{2\pi}{t_2 - t_1} = \frac{2\pi}{0.004} = \underline{1571 \text{ rad/sec}}$$

A MAXIMUM OCCURS AT $t = t_1$; HENCE

$$\omega t_1 + \phi = 2n\pi \quad \text{WHERE } n \text{ IS}$$

$$\Rightarrow \phi = \underline{2n\pi - \omega t_1} \quad \text{ANY INTEGER}$$

THEREFORE ϕ IS NOT UNIQUE. ■

6.3 $f(t) = A \cos(\omega t + \phi)$

AT $t=0$, $f(0) = A = A \cos \phi$.

THUS $\phi = 2m\pi$, WHERE $m = \text{INTEGER}$

NOW WE WANT MINIMUM θ SUCH THAT $\cos(\theta + \phi) = 0$.i.e. $\theta + \phi = \frac{\pi}{2}(2n+1)$

WHERE $n = \text{INTEGER}$

HENCE $\theta = \frac{\pi}{2}(2n+1) - \phi$

$= \frac{\pi}{2}(2n+1) - 2m\pi = \frac{\pi}{2}(2n+1-4m)$

FOR $m, n = 0$, $\theta_{\min} = \frac{\pi}{2}$

6.4 $f(t) = 27 \sin(18t + 47^\circ)$
 $= 27 \cos(90^\circ - 18t - 47^\circ)$
 $= 27 \cos[-(18t - 43^\circ)]$
 $= 27 \cos(18t - 43^\circ)$
 $= A \cos(\omega t + \phi)$

FOR $A = 27$, $\omega = 18$, AND $\phi = -43^\circ$

6.5 $27V \angle 60^\circ = 27 \cos(\omega t + 60^\circ) V$.

WHERE $\omega = 2\pi f = 2\pi(1500) = 9425 \text{ rad/sec}$

6.6 WE MUST HAVE

$A(\cos \omega t + \sin \omega t) = B \cos(\omega t + \phi)$
 $= B[\cos \omega t \cos \phi + \sin \omega t \sin \phi]$

FROM EXAMPLE 4.4, $B = \sqrt{A^2 + A^2}$
 $= \sqrt{2} A$

ALSO $\tan \phi = -\frac{A}{A} = -1$

$\Rightarrow \phi = -45^\circ$

6.7 WE MUST HAVE

$A \sin \omega t + B \cos \omega t = C \sin(\omega t + \phi)$
 $= C[\sin \omega t \cos \phi + \cos \omega t \sin \phi]$

EQUATING COEFFICIENTS

$A = C \cos \phi$; $B = C \sin \phi$ — (*)

$\therefore A^2 + B^2 = C^2 [\cos^2 \phi + \sin^2 \phi] = C^2$

$\Rightarrow C = \sqrt{A^2 + B^2}$

FROM (*) $\tan \phi = \frac{B}{A}$ WHICH GIVES ϕ

6.8

$v(t) = 27 \cos(376t - 80^\circ)$
 $= 27[\cos 80^\circ \cos 376t + \sin 80^\circ \sin 376t]$
 $= 4.689 \cos 376t + 26.59 \sin 376t \text{ (volts)}$

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6.9 (1) $z^* = 12 - 17j$.

(2) THE EXPONENTIAL FORM IS $Ae^{j\theta}$

WHERE $A = \sqrt{12^2 + 17^2} = 20.81$

$\theta = \tan^{-1} \frac{-17}{12} = -54.8^\circ$

(3) IF $z = Ae^{j\theta}$, $z^* = Ae^{-j\theta}$

$\therefore z^* = \underline{20.81 e^{j54.8^\circ}}$

(4) $|z| = \underline{20.81}$

6.10 (1) $z_1^* = \underline{0.6 e^{0.8j}}$

(2) $z_1 = 0.6 [\cos 0.8 - j \sin 0.8]$
 $= \underline{0.418 - j0.43}$

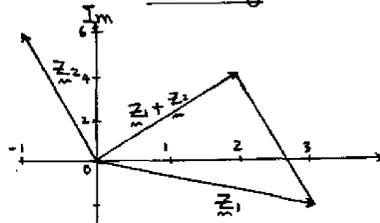
(3) $z_1^* = \underline{0.418 + j0.43}$

(4) $|z_1| = \underline{0.6}$

6.11 $j e^{j0.66} = j [\cos 0.66 + j \sin 0.66]$
 $= -j [-j \sin 0.66 - \cos 0.66]$
 $= \underline{-\sin 0.66 + j \cos 0.66}$

$= \cos(\frac{\pi}{2} + 0.66) + j \sin(\frac{\pi}{2} + 0.66)$
 $= \cos 2.23 + j \sin 2.23$
 $= e^{j2.23}$

6.12 $z_1 + z_2 = (3 - 2j) + (-1 + 6j)$
 $= \underline{2 + 4j}$



6.13 $z_1 = 2 + 3j$, $z_2 = 4 - 2j$

(1) $z_1 z_2 = (2)(4) + (3)(2) + 12j - 4j = \underline{14 + 8j}$

(2) $z_1 z_2 = \sqrt{14^2 + 8^2} e^{j \tan^{-1} \frac{8}{14}} = \underline{16.1 e^{j0.52}}$

(3) $z_1 = \sqrt{2^2 + 3^2} e^{j \tan^{-1} \frac{3}{2}} = \underline{3.61 e^{j0.98}}$
 $z_2 = \sqrt{4^2 + 2^2} e^{j \tan^{-1} \frac{-2}{4}} = \underline{4.47 e^{j(-0.46)}}$

(4) MULTIPLY z_1 & z_2 IN (3)
 $z_1 z_2 = (3.61)(4.47) e^{j(0.98 - 0.46)} = \underline{16.1 e^{j0.52}}$

PROBLEMS IN CALCULUS

$$(6.14) (1) \frac{z_4}{z_3} = \frac{6+5j}{0.2-0.3j} = \frac{(6+5j)(0.2+0.3j)}{0.2^2+0.3^2}$$

$$(2) \frac{z_4}{z_3} = \frac{-0.3+2.8j}{0.13} = -2.31 + 2.15j$$

$$(3) \frac{z_4}{z_3} = \sqrt{2.31^2+2.15^2} e^{j \tan^{-1} \frac{2.15}{-2.31}} = 2.16 e^{j 1.68}$$

$$(4) \frac{z_3}{z_4} = \frac{1}{2.16} e^{-j 1.68} = 0.463 e^{-j 1.68}$$

$$\frac{z_4}{z_3} = \sqrt{6^2+5^2} e^{j \tan^{-1} \frac{5}{6}} = 7.81 e^{j 0.695}$$

$$(5) \frac{z_4}{z_3} = \frac{7.81 e^{j 0.695}}{0.361 e^{j(-0.98)}} = 21.6 e^{j 1.68}$$

SAME AS IN (3)

$$(6.15) \underline{z}_1 = a+bj, \underline{z}_2 = c+dj$$

$$\begin{aligned} \left| \frac{\underline{z}_1}{\underline{z}_2} \right| &= \left| \frac{(a+bj)(c-dj)}{(c+dj)(c-dj)} \right| = \left| \frac{ac+bd+bcj-adj}{c^2+d^2} \right| \\ &= \left[\frac{(ac+bd)^2 + (bc-ad)^2}{c^2+d^2} \right]^{1/2} \\ &= \left[\frac{a^2+b^2}{c^2+d^2} \right]^{1/2} \end{aligned}$$

$$(6.16) e^{j\theta} = \sum_{k=0}^{\infty} \frac{(j\theta)^k}{k!}$$

102

$$\begin{aligned} &= \sum_{k=0}^{\infty} \frac{(j\theta)^{2k}}{(2k)!} + \sum_{k=0}^{\infty} \frac{(j\theta)^{2k+1}}{(2k+1)!} \\ &= \sum_{k=0}^{\infty} \frac{j^{2k} \theta^{2k}}{(2k)!} + \sum_{k=0}^{\infty} \frac{j^{2k+1} \theta^{2k+1}}{(2k+1)!} \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k \theta^{2k}}{(2k)!} + \sum_{k=0}^{\infty} \frac{j(-1)^k \theta^{2k+1}}{(2k+1)!} \\ &= \underline{\underline{\cos \theta + j \sin \theta}} \end{aligned}$$

$$(6.17) \text{ LET } \underline{z}_1 = A e^{j\theta}, \underline{z}_2 = B e^{j\alpha}$$

$$\begin{aligned} \text{THEN } \operatorname{Re}(\underline{z}_1 \underline{z}_2^*) &= \operatorname{Re}(AB e^{j(\theta-\alpha)}) \\ &= AB \cos(\theta-\alpha) \end{aligned}$$

$$\begin{aligned} \text{ALSO } \operatorname{Re}(\underline{z}_2 \underline{z}_1^*) &= \operatorname{Re}(AB e^{j(\alpha-\theta)}) \\ &= AB \cos(\alpha-\theta) \\ &= AB \cos(\theta-\alpha) \\ &= \underline{\underline{\operatorname{Re}(\underline{z}_1 \underline{z}_2^*)}} \end{aligned}$$

$$(6.18) (1) \underline{v}_1 = 17 e^{j10^\circ} \text{ V}$$

$$(2) \underline{v}_2(t) = 27.6 \sin(\omega t - 54^\circ) = 27.6 \cos(\omega t - 54^\circ - 90^\circ) = 27.6 e^{j(\omega t - 144^\circ)} \text{ V}$$

$$(3) \underline{i}(t) = 3.4 e^{-j0.42} \text{ mA}$$

103

(6.19) (1) $v_1(t) = 12 \cos(\omega t + 34^\circ) \text{ V}$
 (2) $v_2 = \sqrt{7^2 + 9^2} e^{\tan^{-1} \frac{9}{7}} = 11.4 e^{j52.1^\circ} \text{ V}$
 $v_2(t) = 11.4 \cos(\omega t + 52.1^\circ) \text{ V}$

(3) $i_1 = \sqrt{6^2 + 2^2} e^{\tan^{-1} \frac{2}{6}} = 6.32 e^{-j18.4^\circ} \text{ mA}$
 $\therefore i_1(t) = 6.32 \cos(\omega t - 18.4^\circ) \text{ mA}$

(6.20) (1) $z_1 = 3 + 2j = M e^{j\theta}$
 $M = \sqrt{3^2 + 2^2} = 3.61$
 $\theta = \tan^{-1} \frac{2}{3} = 33.7^\circ$

(2) AMPLITUDE = $M = 15$
 PHASE = -24°

(3) AMPLITUDE = $\frac{\sqrt{1.3^2 + (2.1)^2}}{\sqrt{0.3^2 + 0.8^2}} = 2.89$
 PHASE = $\tan^{-1} \frac{-2.1}{1.3} - \tan^{-1} \frac{-0.8}{0.3} = 127.7^\circ$

(6.21) (1) THE CORRESPONDING PHASOR IS
 $v_3 = v_1 + v_2 = 10 + 12j + (-7 - 9j)$
 $= 3 + 3j$

(2) MAGNITUDE OF $v_3 = \sqrt{3^2 + 3^2} = 4.24$

= AMPLITUDE OF $v_3(t)$
 ARGUMENT OF $v_3 = \phi = \tan^{-1} \frac{3}{3} = 45^\circ$
 = PHASE OF $v_3(t)$

THEREFORE $v_3(t) = 4.24 \cos(\omega t + 45^\circ)$

(6.22) WE CAN GET THE SUM BY SUMMING THE CORRESPONDING PHASORS

$v = 3e^{j62^\circ} + 4e^{j71^\circ}$
 $= 3 \cos 62^\circ - j3 \sin 62^\circ + 4 \cos 71^\circ - j4 \sin 71^\circ$
 $= 1.41 - j2.65 + 1.3 - j3.78$
 $= 2.71 - j6.43$
 $= 6.98 e^{-j67^\circ}$

$\therefore v(t) = 6.98 \cos(\omega t - 67^\circ)$

(6.23) WITHOUT ANY LOSS OF GENERALITY, WE CAN JUST DO THIS PROBLEM WITH ONLY TWO SINUSOIDS.

LET THE SINUSOIDS BE $A \cos(\omega t + \theta_1)$ AND $B \cos(\omega t + \theta_2)$.

THEN $A \cos(\omega t + \theta_1) + B \cos(\omega t + \theta_2) = 0$
 $\Rightarrow \text{Re}[(A \angle \theta_1 + B \angle \theta_2) e^{j\omega t}] = 0$

FROM RULE 2
 $\Rightarrow \text{Re}[v e^{j\omega t}] = 0$ WHERE
 $v = A \angle \theta_1 + B \angle \theta_2$

$\Rightarrow \text{Re}[X] \cos \omega t - \text{Im}[X] \sin \omega t = 0$
 SINCE $\cos \omega t$ AND $\sin \omega t$ ARE LINEARLY
 INDEPENDENT, $\text{Re}[X]$ AND $\text{Im}[X]$ MUST
 BE 0. HENCE THE SUM OF THE TWO
 PHASORS ARE ZERO.

6.24 (1) PHASE = $\tan^{-1} \frac{14}{12} = \underline{49.4^\circ}$

(2) PHASE = $\tan^{-1} \left(\frac{14}{15} / \frac{12}{15} \right) = \tan^{-1} \left(\frac{14}{12} \right) = \underline{49.4^\circ}$

(3) PHASE = $\tan^{-1} \left(\frac{12}{15} / -\frac{14}{15} \right) = \tan^{-1} \left(\frac{12}{-14} \right) = \underline{139.4^\circ}$

SINCE $\left(\frac{12}{15} j \right) + \left(\frac{14}{15} j \right) j = -\frac{14}{15} + \frac{12}{15} j$

6.25 LET $A = \frac{(6+3j)e^{j27^\circ}}{(2-6j)}$

$|A| = \frac{|6+3j| \cdot |e^{-j27^\circ}|}{|2-6j|} = \frac{\sqrt{6^2+3^2}}{\sqrt{2^2+6^2}} = \underline{0.94}$

6.26 $V_3(t) = V_2(t) - V_1(t)$
 $= 48 \sin(\omega t + 230^\circ) - 17 \cos(\omega t - 8^\circ)$
 $\therefore V_3 = 48 \angle (230^\circ - 90^\circ) - 17 \angle -8^\circ$

$V_3 = 48 \cos 140^\circ - 17 \cos 8^\circ + j(48 \sin 140^\circ + 17 \sin 8^\circ)$
 $= -53.6 + j33.2$

HENCE $|V_3| = \sqrt{53.6^2 + (33.2)^2} = \underline{63.1}$

ARG $V_3 = \tan^{-1} \frac{33.2}{-53.6} = \underline{148^\circ}$

6.27 LET $V(t) = \frac{19 e^{j(\omega t + 100^\circ)}}{2+3j}$

THEN $V(t) = \frac{19(2-3j)}{2^2+3^2} \cdot [\cos(\omega t + 100^\circ) + j \sin(\omega t + 100^\circ)]$

$\text{Re}[V(t)] = \frac{19}{13} [2 \cos(\omega t + 100^\circ) + 3 \sin(\omega t + 100^\circ)]$

$= \frac{19}{13} [\sqrt{2^2+3^2} \cos(\omega t + 100^\circ + \phi)]$

WHERE $\phi = \tan^{-1} \frac{-3}{2} = -56.3^\circ$

$\therefore \text{Re}[V(t)] = 19 \cos(\omega t + 100^\circ - 56.3^\circ)$
 $= \underline{19 \cos(\omega t + 43.7^\circ)}$

6.28 (1) $\text{Re}(V_1^*) = \text{Re}[(6-4j)(2-4j)]$
 $= (6)(2) + (4j)(4j) = \underline{-4}$

(2) $\text{Re}(jV^*) = \text{Re}[(2+4j)(6+4j)]$
 $= (2)(6) + (4j)(4j) = \underline{-4}$

NOTE THAT $\text{Re}[V_1^*] = \text{Re}[jV^*]$

6.29 $v = 16e^{j208^\circ}$; $i = 11e^{-j43^\circ}$

(1) $\text{Re}(v i^*) = \text{Re}[(16e^{j208^\circ})(11e^{j43^\circ})]$
 $= \text{Re}[16 \cdot 11 e^{j(208^\circ + 43^\circ)}]$
 $= 176 \cos(208^\circ + 43^\circ)$
 $= -170$

(2) $\text{Re}(i v^*) = \text{Re}[(11e^{-j43^\circ})(16e^{j208^\circ})]$
 $= \text{Re}[11 \cdot 16 e^{j(43^\circ - 208^\circ)}]$
 $= 176 \cos(43^\circ - 208^\circ)$
 $= -170$

6.30 LET $v = \frac{V_0 \omega RC}{1 + j\omega RC}$

THEN $v = \frac{V_0 \omega RC (1 - j\omega RC)}{1 + (\omega RC)^2}$

$\Rightarrow |v| = \frac{V_0 \omega RC}{1 + (\omega RC)^2} \cdot [1 + (\omega RC)^2]^{1/2}$

$= \frac{V_0 \omega RC}{\sqrt{1 + (\omega RC)^2}}$

ALSO PHASE = $\tan^{-1} \frac{-\omega RC}{1} = -\tan^{-1} \omega RC$

6.31 LET $v = \frac{V_0 (R + j\omega L)}{(1 - \omega^2 LC) + j\omega RC}$

$|v| = \frac{V_0 |R + j\omega L|}{|(1 - \omega^2 LC) + j\omega RC|}$
 $= \frac{V_0 \sqrt{R^2 + L^2 \omega^2}}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$

PHASE = $\phi = \tan^{-1} \frac{\omega L}{R} - \tan^{-1} \frac{\omega RC}{1 - \omega^2 LC}$
 $= \theta_1 - \theta_2$

SINCE $\tan(\theta_1 - \theta_2) = \frac{\tan \theta_1 - \tan \theta_2}{1 - \tan \theta_1 \tan \theta_2}$

$= \frac{\omega L/R - \frac{\omega RC}{1 - \omega^2 LC}}{1 - \frac{\omega L}{R} \cdot \frac{\omega RC}{1 - \omega^2 LC}}$
 $= \frac{\omega L(1 - \omega^2 LC) - R^2 \omega C}{R(1 - \omega^2 LC) + \omega L \omega RC}$

PHASE = $\theta_1 - \theta_2$
 $= \tan^{-1} \left[\frac{\omega L(1 - \omega^2 LC) - R^2 \omega C}{R(1 - \omega^2 LC) + \omega L \omega RC} \right]$

6.32 (1) PHASOR $v = 160e^{j0}$ VOLT = 160 VOLTS

(2) $i(t) = C \frac{dv(t)}{dt} \Rightarrow i = j\omega C v$ FROM RULE 4.
 $= j(377)(20\mu)(160e^{j0})$

$$\underline{i} = 1.21 \underline{j} = \underline{1.21 e^{j90^\circ}} \text{ A}$$

$$\begin{aligned} \text{(3) } i(t) &= \operatorname{Re}[i e^{j\omega t}] \\ &= \operatorname{Re}[1.21 e^{j90^\circ} e^{j\omega t}] \text{ mA} \\ &= 1.21 \cos(\omega t + 90^\circ) \text{ mA} \\ &= \underline{1.21 \cos(377t + \pi/2 \text{ radians})} \text{ A} \end{aligned}$$

(4) SINCE THE PHASE OF $i(t)$ IS 90° GREATER THAN THAT OF $v(t)$, $i(t)$ 'S MAXIMA COME EARLIER. I.E. IT LEADS THE VOLTAGE. ■

$$\begin{aligned} \text{(6.33) } \text{INSTANTANEOUS POWER} &= \frac{v^2(t)}{R} \\ &= \frac{15^2}{50} \cos^2(376t + 30^\circ) \\ &= \underline{4.5 \cos^2(376t + 30^\circ)} \text{ WATTS} \\ \text{(b) } P_{\text{AV}} &= \frac{V_0^2}{2R} \quad \text{FROM EQT. (4.9)} \\ &= \frac{15^2}{2(50)} = \underline{2.25 \text{ W}} \quad \blacksquare \end{aligned}$$

(6.34) THIS IS SIMILAR TO EXERCISE 4.2 EXCEPT THAT VOLTAGE IS GIVEN INSTEAD. SINCE $i(t) = \frac{v(t)}{R} = \frac{1}{R}[(10V)\cos 376t - (12V)\sin 376t]$

∴ FROM THE RESULT OF EXERCISE 4.4

$$\begin{aligned} P_{\text{AV}} &= \frac{R}{2} \left[\left(\frac{10}{R}\right)^2 + \left(\frac{-12}{R}\right)^2 \right] \\ &= \frac{1}{2} \left[\frac{100}{4700} + \frac{144}{4700} \right] = \underline{25.95 \text{ mW}} \quad \blacksquare \end{aligned}$$

(6.35) SINCE P_{AV} IS INDEPENDENT OF THE PHASE, THE RESULT OF EXERCISE 4.4 CAN STILL BE USED.
∴ $P_{\text{AV}} = \frac{1}{2}(10,000)[(15\text{m})^2 + (20\text{m})^2] = \underline{3.125 \text{ W}} \quad \blacksquare$

$$(6.36) (a) V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{24V}{\sqrt{2}} = \underline{16.97V}$$

$$(b) i(t) = \frac{v(t)}{R} = \frac{24A}{3000} \cos(376t - 20^\circ)$$

$$= 8 \cos(376t - 20^\circ) \text{ mA}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \underline{5.66 \text{ mA}}$$

$$(6.37) V_{rms}^2 = \frac{1}{T} \int_0^T v^2(t) dt$$

$$= \frac{1}{T} \int_0^T V_0^2 \cos^2 \omega t dt$$

$$= \frac{V_0^2}{T} \int_0^T \cos^2 \omega t dt$$

$$= \frac{V_0^2}{T} \left[\frac{T}{2} \right] = \frac{V_0^2}{2}$$

$\therefore V_{rms} = \frac{V_0}{\sqrt{2}}$, WHICH AGREES WITH
EQT 4.10

$$(6.38) V_{rms}^2 = \frac{1}{T} \int_0^T V_0^2 \cos^2(\omega t + \phi) dt$$

$$= \frac{V_0^2}{T} \int_0^T \cos^2(\omega t + \phi) dt$$

$$= \frac{V_0^2}{2T} \int_0^T [1 + \cos(2\omega t + \phi)] dt$$

$$= \frac{V_0^2}{2T} [T] = \frac{V_0^2}{2}$$

$$\Rightarrow V_{rms} = V_0 / \sqrt{2}$$

$$(6.39) \text{ FROM EQT 4.12}$$

$$V_{rms}^2 = \frac{1}{T} \int_0^T v^2(t) dt$$

$$= \frac{1}{4 \text{ msec}} \left[\int_0^1 0 dt + \int_1^3 3^2 dt + \int_3^4 0 dt \right]$$

$$= \frac{1}{4 \text{ msec}} [18 \text{ msec} \cdot V^2] = 4.5 V^2$$

$$\Rightarrow V_{rms} = \underline{2.12 V}$$

$$(6.40) V_{rms}^2 = \frac{1}{T} \int_0^T v^2(t) dt$$

$$= \frac{1}{2 \text{ msec}} \int_0^{2 \text{ msec}} \left(\frac{t}{0.001} \right)^2 dt$$

$$= \frac{1}{2 \text{ msec}} \frac{(2 \text{ msec})^3}{3(0.001)} = 1.33 V^2$$

$$\Rightarrow V_{rms} = \underline{1.155 V}$$

6.41 FROM RULE 5, THE AVERAGE POWER ENTERING THE ELEMENT IS $\frac{1}{2} \text{Re} [V_x i^*]$

$$\begin{aligned} P_{\text{AVG}} &= \frac{1}{2} \text{Re} [V_1 e^{j\phi} \cdot I_1 e^{j0}] \\ &= \frac{1}{2} \text{Re} [V_1 I_1 e^{j\phi}] \\ &= \frac{1}{2} V_1 I_1 \cos \phi \end{aligned}$$

NEGATIVE POWER MEANS ENERGY IS FLOWING OUT OF THE ELEMENT.

6.42 THE PHASOR OF THE CURRENT = $\frac{V}{R}$
 $P_{\text{AVG}} = \frac{1}{2} \text{Re} [V_x i^*] = \frac{1}{2} \text{Re} \left[\frac{V_1}{R} \cdot \frac{V_1^*}{R} \right] = \frac{1}{2} \text{Re} \left[\frac{|V_1|^2}{R} \right] = \frac{|V_1|^2}{2R}$

6.43 $i = V_1 j\omega C$

$$\begin{aligned} P_{\text{AVG}} &= \frac{1}{2} \text{Re} [V_1 V_1^* (-j)\omega C] \\ &= \frac{1}{2} |V_1|^2 \omega C \text{Re} [-j] = 0 \end{aligned}$$

THIS IS AS WE EXPECT PHYSICALLY. BEING UNABLE TO CONSUME POWER (AS A RESISTOR WOULD) THE CAPACITOR TAKES IN NO POWER, ON THE AVERAGE. (SOME ENERGY DOES SURGE IN AND BACK OUT DURING EACH CYCLE, THOUGH.)

CHAPTER 7 - EXERCISES

EX. 7.1
$$\begin{aligned} \underline{Z} &= \frac{R \left(\frac{1}{j\omega C} \right)}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC} \\ &= \frac{1000}{1 + j(2\pi)10^2 \cdot 10^{-3} \cdot 10^{-5}} = \frac{1000}{1 + 6.28j} \\ &= 24.7 - 155j \Omega \end{aligned}$$

(THE LAST STEP DONE BEST WITH A CALCULATOR)

EX. 7.2 THE PHASOR CURRENT THRU C_1 IS

$$\underline{i}_C = \underline{i}_1 - \underline{i}_2 = \underline{i}_1 \frac{C_1/C_2 + jR\omega C_1}{1 + C_1/C_2 + jR\omega C_1}$$

$$\begin{aligned} \underline{V}_{C1} &= \underline{i}_C \underline{Z}_{C1} = \frac{\underline{i}_C}{j\omega C_1} = \frac{\underline{i}_1}{j\omega} \frac{1/C_2 + j\omega R}{1 + C_1/C_2 + jR\omega C_1} \\ |\underline{V}_{C1}| &= \frac{|\underline{i}_1|}{\omega} \left| \frac{(3 \cdot 10^{-6})^{-1} + j2\pi(650)(100)}{1 + \frac{2}{3} + j2\pi(650)(100)(2 \cdot 10^{-6})} \right| \end{aligned}$$

$$= 0.69 \text{ V}$$