Digraph and KVL

3 Consider the circuit shown in Fig. P1.3.

(*a*) Given $v_{2-3} = 10$ V, $v_{6-3} = 6$ V, and $v_{4-1} = 2$ V, find v_{6-1}, v_{4-6}, and v_{4-2}.

(*b*) Draw the digraph with terminal ③ chosen as the datum of the op amp and terminal ④ chosen as the datum of the transistor. Repeat (*a*) using this digraph.

(*c*) Repeat (*b*) but with terminal ⑤ chosen as the datum terminal for both the op amp and the transistor.

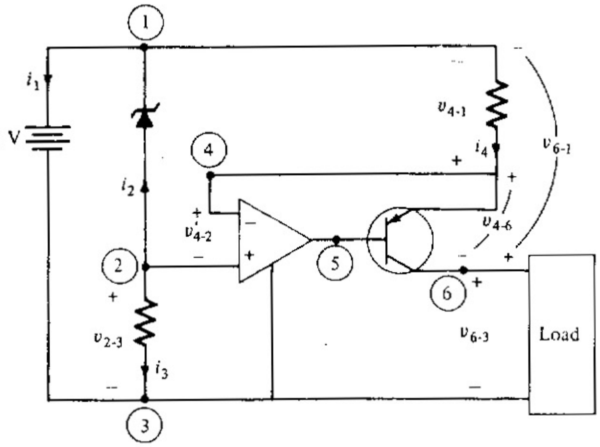

Figure P1.3

Digraph and closed node sequence

Incidence matrix and cut sets

11 For the digraph shown in Fig. P1.11:

(*a*) Write the incidence matrix $\mathbf{A}_a$.

(*b*) With node ③ as the datum node, write the KVL equations

$$\mathbf{v} = \mathbf{A}^T\mathbf{e}$$

(*c*) Give all the cut sets not already represented by $\mathbf{A}_a$.

(*d*) Select a maximum subset of the above which leads to linearly independent KCL equations.

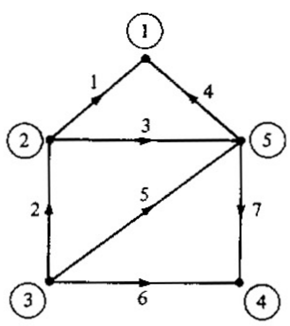

Figure P1.11

Degree of freedom, KVL, and KCL

Cut-set equations and linear independence

13 (*a*) Enumerate all cut sets for the digraph in Fig. P1.13.

(*b*) Write a KCL equation corresponding to each cut set from (*a*).

(*c*) Show that the equations from (*b*) are linearly dependent.

(*d*) Extract a subset of KCL equations from (*b*) containing a maximum number, ρ, of linearly independent equations. Verify that $\rho = n - p$, where n = number of nodes and p = number of connected components (or separate parts) in the digraph.

(*e*) Hinge nodes ⑤ and ⑦ and verify that the same KCL equations from (*b*) also hold for the resulting "connected" digraph.

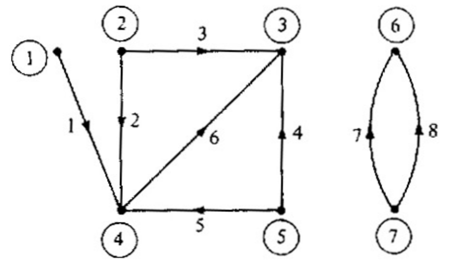

Figure P1.13

Incidence matrix, KVL, and KCL

14 (*a*) Write the incidence matrix $\mathbf{A}_a$ for the digraph in Fig. P1.14.

(*b*) Write the reduced incidence matrix $\mathbf{A}$ with node ⑤ as the datum node.

(*c*) Using $\mathbf{A}$, write a system of linearly independent KVL and KCL equations.

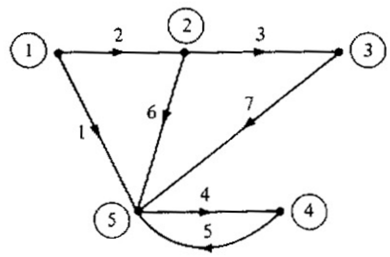

Figure P1.14

Degree of freedom, Tellegen's theorem

17 (*a*) Draw a digraph whose incidence matrix $\mathbf{A}_a$ is given by

$$\mathbf{A}_a = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix} \begin{matrix} \text{Node} \\ ① \\ ② \\ ③ \\ ④ \end{matrix}$$

$$\text{Branch} \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

(*b*) Given the following subgraphs of the digraph obtained in (*a*), identify the ones which form cut sets and the ones which are associated with gaussian surfaces.

$$\{1, 2, 3, 4\}, \{5, 6\}, \{2, 4, 5, 6\}, \{1, 3, 5, 6\}$$

(*c*) For the cut sets from (*b*), write down the corresponding KCL equations and also express these equations as the sum of appropriate node equations.

Cut sets, rank, and linear independence