EE-100 Lab: Basic CNN Cell Experiments - Theory

1. Objective
In this laboratory measurement you will learn some basics of cellular neural networks (CNN), a possible op amp realization of the basic CNN cell, and some potential cellular neural network applications.

2. Introduction: Cellular Neural Network – CNN
A *Cellular Neural Network* is any spatial arrangement of *locally-coupled cells*, where each cell is a *dynamical system* evolving according to some prescribed dynamical laws. Therefore, a CNN is defined by two mathematical constructs:
- the *dynamics* of the cell
- the *coupling law* relating one or more relevant variables of each cell to all neighbor cells within a prescribed sphere of influence.

A standard two-dimensional CNN architecture consist of an *MxN* rectangular array of cells ($C_{i,j}$) where every cell is connected locally to its neighbors only as shown in Figure 1.
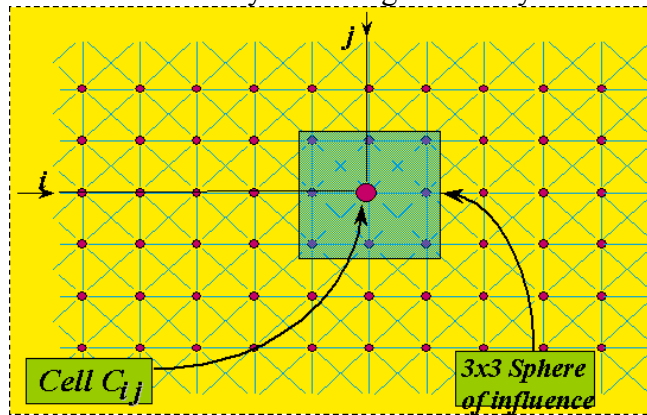

Figure 1 A CNN cell C_{ij} with a 3x3 sphere of influence

Each cell is a dynamical system which has an *input*, an *output*, and a *state* variables evolving according to the following dynamical laws:

$$\frac{dx_{ij}}{dt} = -x_{ij} + \sum_{kl \in S_{ij}} a_{kl} \cdot y_{kl} + \sum_{kl \in S_{ij}} b_{kl} \cdot u_{kl} + z_{ij},$$

where

$$y_{ij} = f(x_{ij}) = \frac{\left|x_{ij} + 1\right| - \left|x_{ij} - 1\right|}{2}$$

is depicted in Figure 2.
- where *u*, *x*, *y*, stand for *input*, *state*, and *output* variables of a cell, respectively;
- *f(x)* is the cell output function;
- a_{ij} and b_{ij} are feed-back and feed-forward weighting coefficients, respectively, describing the coupling law among cells;
- z_{ij} is the cell bias term (or offset level).

$$y_{ij} = f(x_{ij})$$
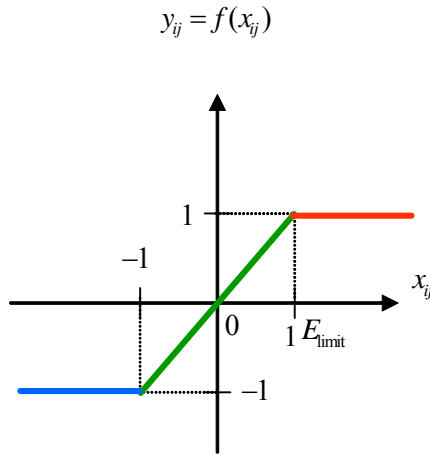
Figure 2 Output function of the standard CNN cell

5. Cellular Neural Network Applications

CNN has found numerous applications. Many artificial, physical, chemical, as well as living (biological) systems can be very conveniently modeled via CNN (chaos, self organization, dissipative structures, etc.). This paradigm allows not only the modeling but also the engineering of complex systems. For instance, as a high speed analog array signal processor, CNN can process two-dimensional images solve many time consuming image processing problems in real time. Some of these applications include high-speed target recognition and tracking, real-time visual inspection of manufacturing processes, intelligent vision capable of recognizing context sensitive and moving scenes. As an example, let us consider a relatively simple image processing task, i.e. edge detection. The following CNN program (coefficient weights) called a *CNN template* extracts edges from a gray-scale image and produces a binary output (black and white) image containing only the object boundaries, as shown in Figure 3.

A =

0	0	0
0	2	0
0	0	0

B =

-1	-1	-1
-1	8	-1
-1	-1	-1

z = | -0.5 |

A and *B* stand for feed-back and feed-forward coefficients, respectively.


a)   b)
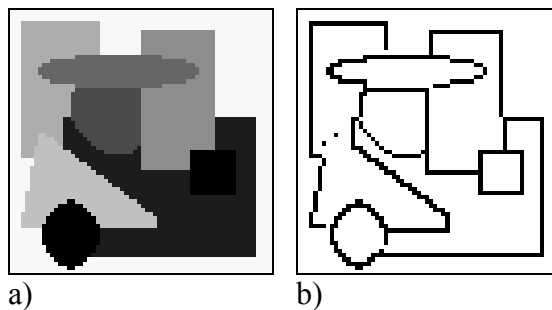
Figure 3 a) Input gray-scale image, b) Binary output image containing edges of objects n the left

You can get a CNN template and algorithm simulator running on Windows from the following link: http://lab.analogic.sztaki.hu/Candy/ - CANDY CNN Simulator.

4. Op Amp Realization of the Basic CNN Cell

The circuit shown in Figure 4 is a simplified CNN cell circuit of cellular neural networks. It consists of the basic circuit elements, namely, a linear capacitor C_x, a linear resistor R_x, a linear voltage-controlled current source, and an output sub-circuit with the piecewise-linear output function - $f(x_{ij})$.
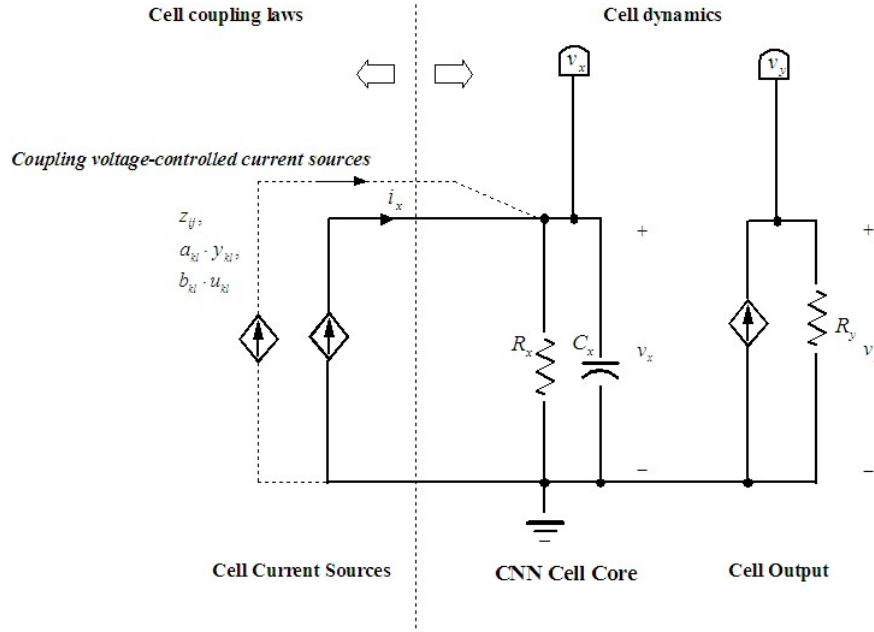
Figure 4 Simplified CNN cell circuit

The dynamics of the CNN cell core is the following.

$$C_x \cdot \frac{dv_x}{dt} = -\frac{v_x}{R_x} + i_x,$$

and the cell output is

$$v_y = f(v_x)$$

One possible op amp implementation of the above circuit is shown in Figure 5. The voltage-controlled current source is realized by op amp A_1. It can be shown (good exercise for you) that $i_x = -\dfrac{R_4}{R_3 \cdot R_7} \cdot v_2$ under the condition that $\dfrac{R_4}{R_3} = \dfrac{R_6 + R_7}{R_5}$. In this case, the output current (i_x) of the VCCS is independent of the value of the load resistor (R_x), i.e. it works as an ideal VCCS.

The piecewise-linear output function is realized by op amp A_2 with the constraint that $\dfrac{R_8 + R_9}{R_8} = \dfrac{R_{10} + R_{11}}{R_{11}} = E_{sat} / E_{limit}$, where usually $E_{limit} = 1V$.
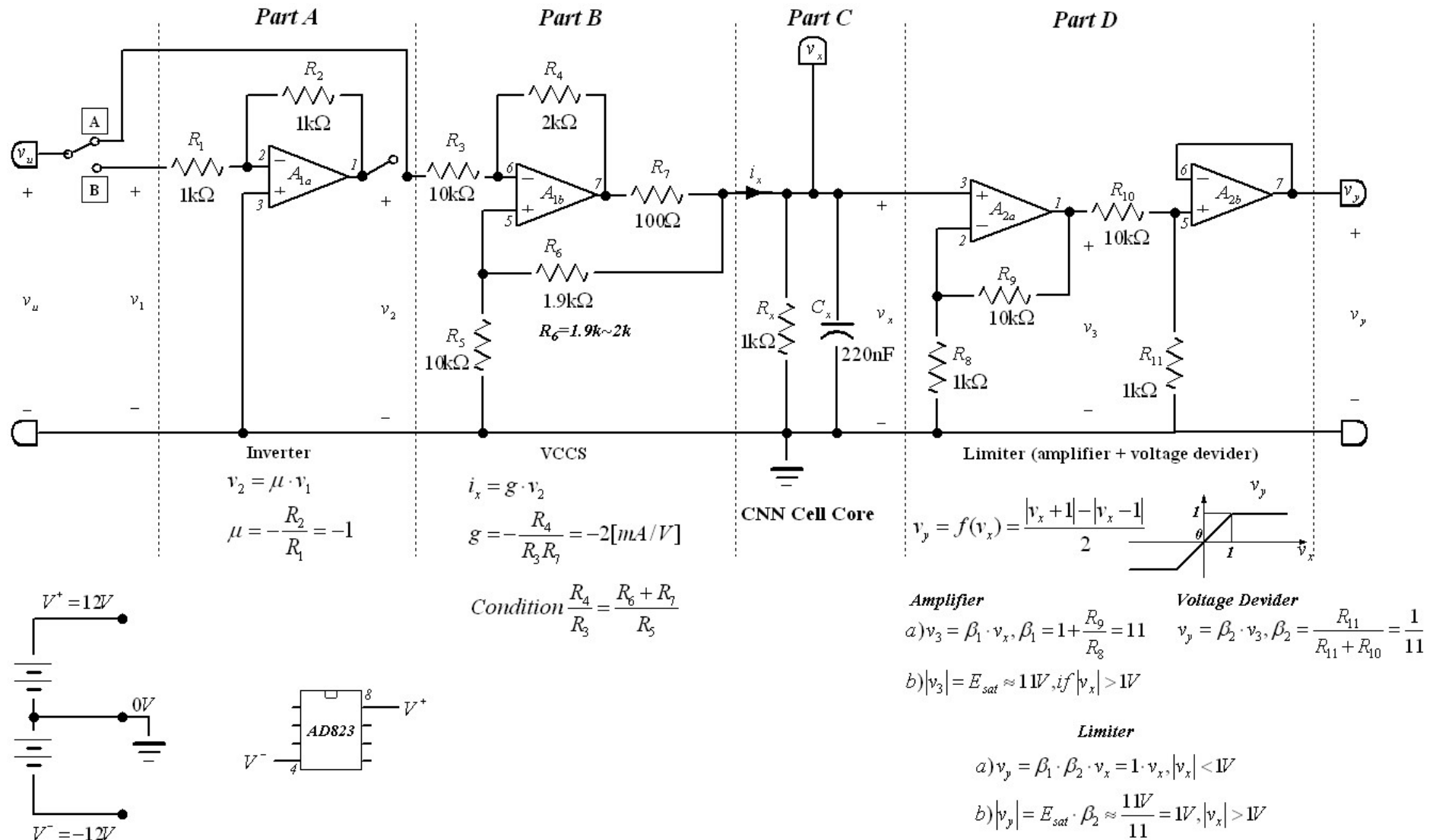
3

Part A

Part B

Part C

Part D

R_2

1kΩ

R_4

2kΩ

v_x

A

v_u

R_1

1kΩ

A_{1a}

R_3

10kΩ

A_{1b}

R_7

100Ω

i_x

A_{2a}

R_{10}

10kΩ

A_{2b}

v_y

B

R_6

1.9kΩ

$R_6 = 1.9k \sim 2k$

R_9

10kΩ

v_3

v_u

v_1

v_2

R_5

10kΩ

R_x

1kΩ

C_x

220nF

v_x

R_8

1kΩ

R_{11}

1kΩ

v_y

Inverter

VCCS

CNN Cell Core

Limiter (amplifier + voltage devider)

$$v_2 = \mu \cdot v_1$$

$$\mu = -\frac{R_2}{R_1} = -1$$

$$i_x = g \cdot v_2$$

$$g = -\frac{R_4}{R_3 R_7} = -2[mA/V]$$

$$v_y = f(v_x) = \frac{|v_x + 1| - |v_x - 1|}{2}$$

$V^+ = 12V$

0V

$V^- = -12V$

AD823

V^+

V^-

$$Condition \; \frac{R_4}{R_3} = \frac{R_6 + R_7}{R_5}$$

Amplifier

$$a) v_3 = \beta_1 \cdot v_x, \beta_1 = 1 + \frac{R_9}{R_8} = 11$$

$$b) |v_3| = E_{sat} \approx 11V, if \; |v_x| > 1V$$

Voltage Devider

$$v_y = \beta_2 \cdot v_3, \beta_2 = \frac{R_{11}}{R_{11} + R_{10}} = \frac{1}{11}$$

Limiter

$$a) v_y = \beta_1 \cdot \beta_2 \cdot v_x = 1 \cdot v_x, |v_x| < 1V$$

$$b) |v_y| = E_{sat} \cdot \beta_2 \approx \frac{11V}{11} = 1V, |v_x| > 1V$$

Figure 5 Op amp realization of the Basic CNN Cell

EE-100 Lab: Basic CNN Cell Experiments – Experiment Guide

You will be provided a special box designed for this laboratory measurement (Figure 8).

Exp1 – Measurement of the CNN cell core and the nonlinear output function
Set N_1, …, N_4 switches to their UP position (inverter is ON). Set N_5, …, N_8 switches to their LEFT position (VCCS output is not connected to the core cell). Set N_9 switch to its RIGHT position (no *Initial state*). Set V_{DD}=12V and V_{SS}= -12V.
   a) Set the voltage level of the Initial state around zero. Switch N_9 to the Initial state (LEFT position) and measure both V_x and V_y. Determine the levels of E_{sat} and E_{limit} as well. A LED should be red if the V_x is above $+E_{limit}$ and green if it is below $-E_{limit}$.
   Checking the nonlinear output function
   b) Set N_9 switch again to its RIGHT position (no *Initial state*). Set the output of the function generator to a 200mV$_{pp}$, 1kHz sine wave with 0 DC offset and connect it to the CNN cell (label V_x).Using the scope, observe the waveform of the input signal (V_x) and the output of the limiter (V_y). Enlarge the voltage level up to 2V. Draw both input and output signals and determine the saturation level. *Hint: to determine the saturation level more exactly you can change to a more appropriate waveform (e.g. a triangle waveform).* Switch to XY mode and determine the transfer characteristic.
   c) Now, set the scope to display the transfer characteristic (X/Y mode). Set the voltage scale to 500mV/div. Draw the transfer characteristic and determine the slope of the linear region and the saturation levels as well.

Exp2 – Measurement of the voltage-controlled current source (VCCS)
Connect the output of the first current generator to the core of the CNN cell (N_5 is set to RIGHT). **Do not connect the cell capacitor (C_x) yet.** The load resistor will only be R_x.
   a) Using the 6V output of the power supply set the input voltage of the first VCCS to 1V, 2V, and 3V respectively (N_1 should be UP). Measure the output current (i_x) and determine the current amplification (g). Repeat the measurement using different CNN cell resistance (let R_x be 470Ω and 100Ω, respectively). Does the current depend on the load resistor strongly? Explain possible reasons of this behavior.
   b) Set the output of the function generator to a 1kHz sine wave with V$_{pp}$ amplitude 2V and apply it as the input voltage of the VCCS. Let R_x again be 1k. Measure and display both the input and the output voltages (V_u and V_x). What is the phase between these two signals and explain this behavior. *Hint: if you change to X/Y mode you can display the transfer characteristic of the VCCS and this should reflect the same behavior.*
   c) *Saturation measurement* (with load resistor R_x = 1k): Knowing the saturation level (E_{sat}) of the op amp and the current amplification (g), derive the maximum input voltage when there is no clipping yet. Check your calculation by increasing the amplitude of the input signal and measuring the lowest voltage when clipping occurs. *Hint: Use the previous sine wave (or triangle wave) as an input signal.*

d) Now, connect C_x as well and let it be 0.22 µF. Set the input signal to a 100Hz sine wave with a 3V V_{pp} amplitude. Display both the input and the output voltages (V_2 and V_x) in the X/Y mode of the scope. Increase the frequency (up to 1kHz) and explain the reasons of the observed behavior.

e) Repeat the previous measurement but display the output voltage of the limiter (V_y) instead of V_x. Use the X/Y mode of the scope and explain the observed behavior.

Exp3 –Cellular cell measurement
   a) Behavior of self-feedback

Feed back the output voltage (V_y) of the CNN cell to its input (V_u) either through the inverter (switch is UP) or connected directly to the input of the VCCS (switch is DOWN). Measure the output voltage (V_y) in both cases and explain the reason of the observed behavior.

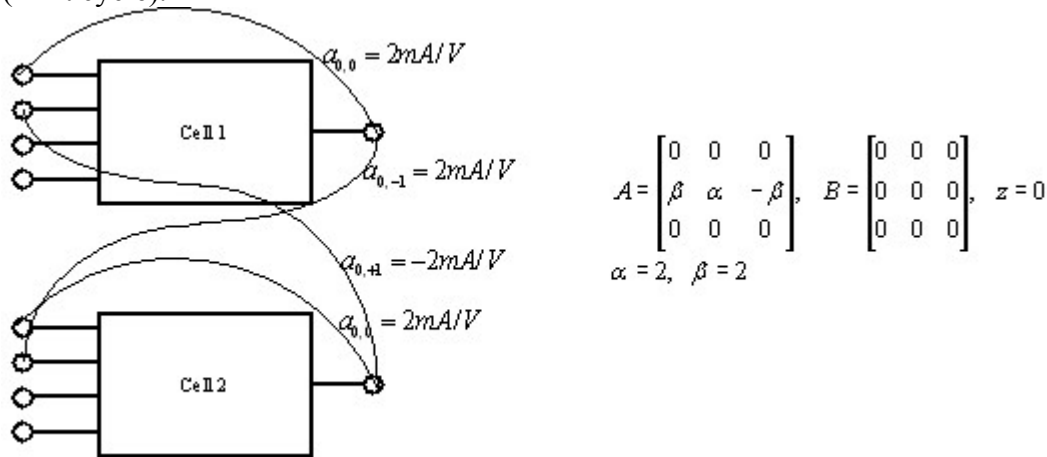Now, you will connect two CNN boxes together to generate more complex behavior.
   b) An oscillatory CNN with only two cells.

Consider a two-cell CNN characterized by the following template shown in Figure 6. The state equation for this CNN is given by

$$\dot{x}_1 = -x_1 + \alpha y_1 - \beta y_2$$
$$\dot{x}_2 = -x_2 + \alpha y_2 + \beta y_1$$

Note that the solution waveforms instead of converging to a dc equilibrium point as in all of our previous measurement, the state variables will converge to a *periodic* waveform (limit cycle).

$$A = \begin{bmatrix} 0 & 0 & 0 \\ \beta & \alpha & -\beta \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad z = 0$$

$$\alpha = 2, \quad \beta = 2$$

Figure 6   Two-cell CNN oscillatory network

Condition for oscillation is

$$\left. \begin{array}{l} \alpha > 1 \\ \beta > \alpha - 1 \end{array} \right\}$$

Observe both waveforms of y_1 and y_2. Switch to X/Y mode and record the trajectory as well.

c) A chaotic CNN with only two cells and one sinusoidal input

Suppose we apply a sinusoidal input to the first cell and choose $\alpha = 2$ and $\beta = 2$ as its parameters. In this case, we will have the following non-autonomous system of two nonlinear differential equations

$$\dot{x}_1 = -x_1 + 2y_1 - 1.2y_2 + 4.04\sin\left(\frac{\pi}{2}t\right)$$

$$\dot{x}_2 = -x_2 + 2y_2 + 1.2y_1$$

The solution waveforms do *not* converge to a periodic waveform. The behavior of this circuit is called *chaotic*, and the associated trajectory is called *strange atractor*.
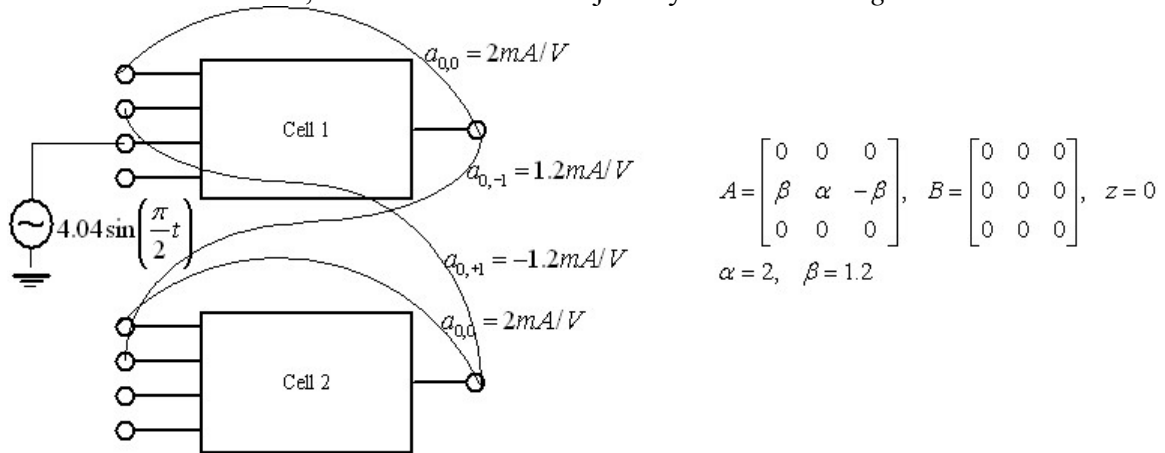
Figure 7   **Two-cell CNN chaotic network**

Observe both waveforms of y_1 and y_2. Switch to X/Y mode and record the trajectory as well.
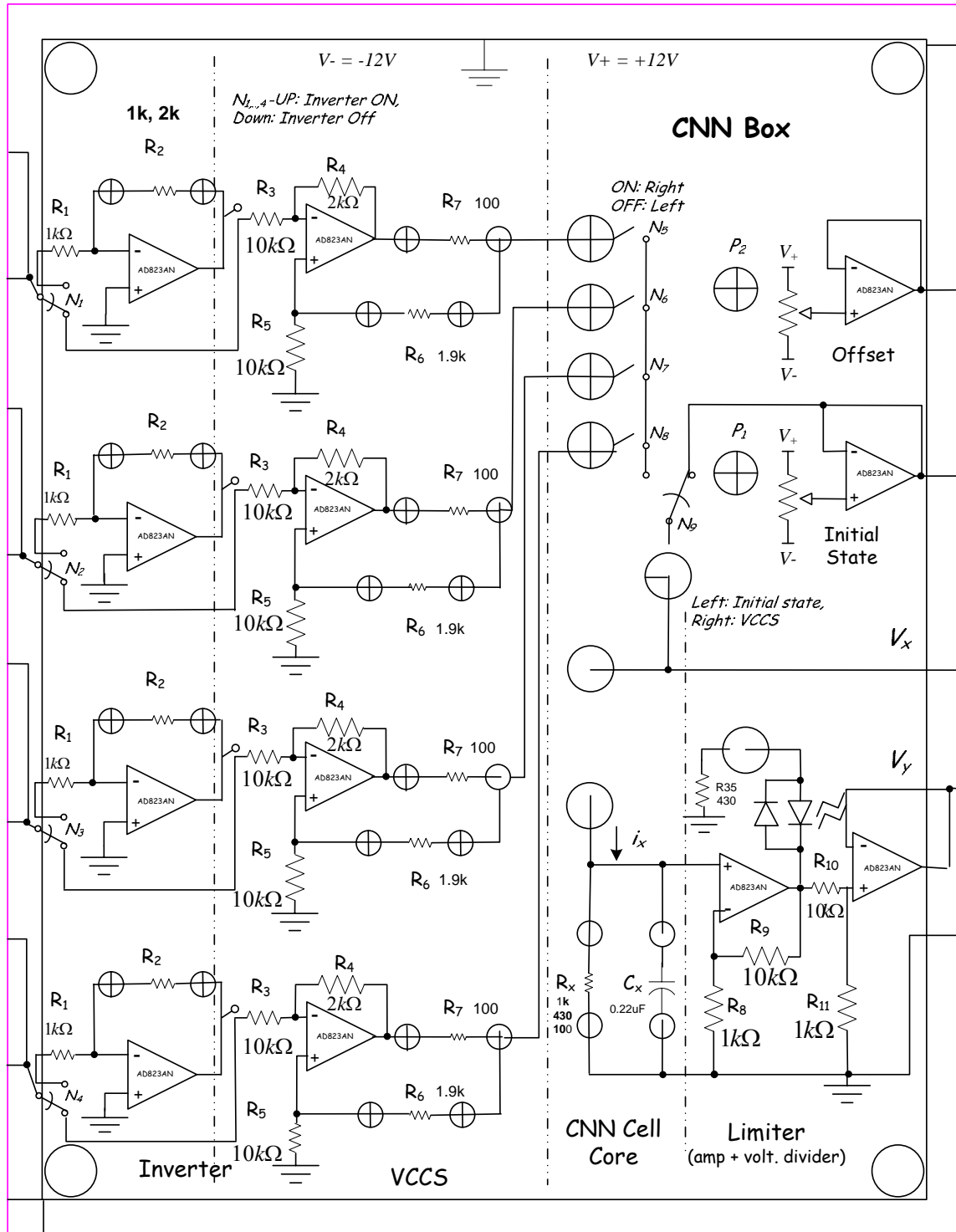
Appendix

Figure 8   CNN Box – designed for laboratory measurement

The arrangement of the components of the CNN Box matches the corresponding schematic (Figure 5). You can see four voltage controlled current sources (VCCS) with

their corresponding inverters. There are also two constant DC signal sources (*Offset* and *Initial state*). The *Initial state* can be connected directly to the CNN core cell (N_9).

The box requires power supply refer to the basic lab guide what the proper connection is. You can see labels showing which banana socket belongs to the positive power supply (*V+= +12V*, and red color), the common terminal (always in the middle), and the negative power supply (*V- = -12V*). Do not mix them. If you fill unsure ask your TA to help you. Passive components (resistors, capacitors, etc) are prepared in advance you need only to put them to the proper small banana sockets. Large banana sockets on the left side of the box are used for input, while sockets on the right for the output. If you need to set DC input voltage you can use the 6V output of the power supply. To set periodic input voltage using the function generator you can use BNC-banana converters. Also, there are prepared banana plugs with small pins so you can connect the alligator clips of the scope probe and measure any desired signal.

$N_1, ..., N_4$: Switch inverter either ON or OFF. Position UP – connects the inverter before the VCCS, thus providing positive signal amplification (VCCS is negative). Position DOWN – no inverter before the VCCS thus gives a negative signal amplification.

$N_5, ..., N_8$: Switches current sources to the core cell. ON – Right, OFF – Left position.

N_9: Switches either the *Initial state* or currents to the core cell. Left: *Initial state*, Right: VCCS.