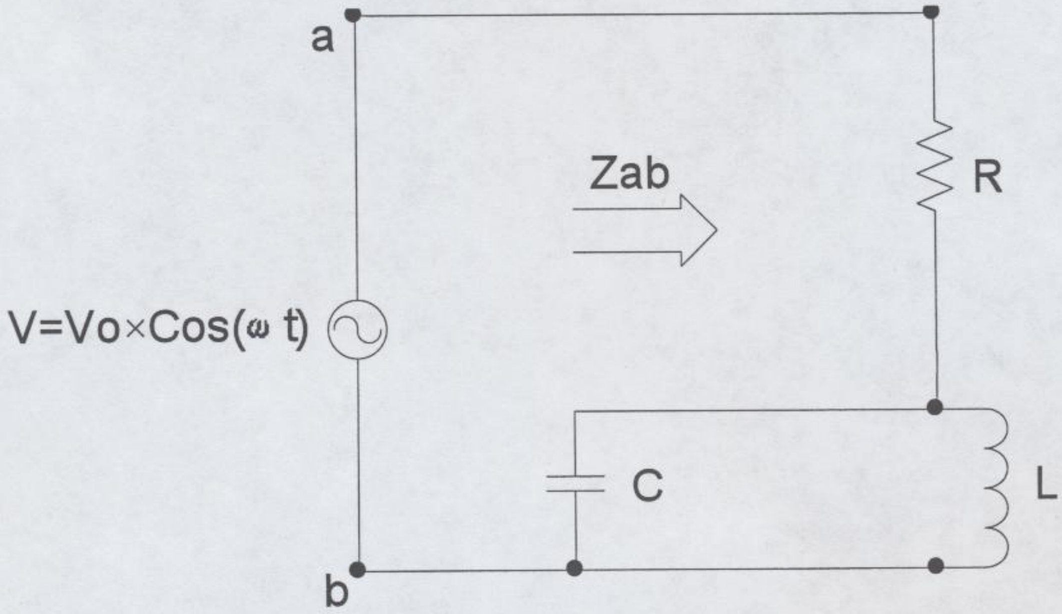
PROBLEM 1 [5 points]



Find an expression for ω (in terms of circuit variables L, C, *and/or* R) such that Zab (the impedance across nodes a and b) is purely resistive.

$$Z_{ab} = R + \left(\frac{1}{j\omega L} + j\omega C\right)^{-1} = R + \frac{j\omega L}{1-\omega^2 LC}$$

purely resistive. ⇒ imaginary part = 0

$$\frac{j\omega L}{1-\omega^2 LC} = 0 \quad \omega = 0$$

Expression for ω: $= 0$

#2 a) <u>Inductor</u> $\tau = \frac{L}{R} = 1 sec.$

$t < 0$: $i_L(t) = 0$

$v_L(t) = 0$

$0 < t < \infty$: $i_L(0^+) = 0$

$i_L(\infty) = 10A$

$i_L(t) = 10 + (0-10)e^{-t/\tau} = 10 - 10e^{-t}$

$v_L(t) = L\frac{di}{dt} = 1 \times 10e^{-t} = 10e^{-t}$

$v_L(0^+) = 10V$

$v_L(\infty) = 0$

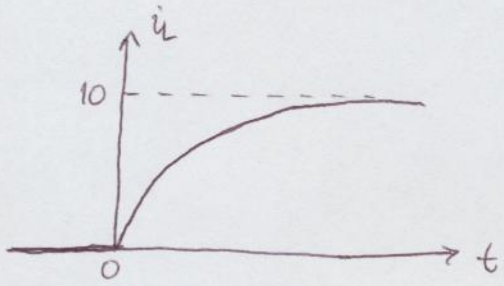
<u>Capacitor</u> $\tau = RC = 1 sec$

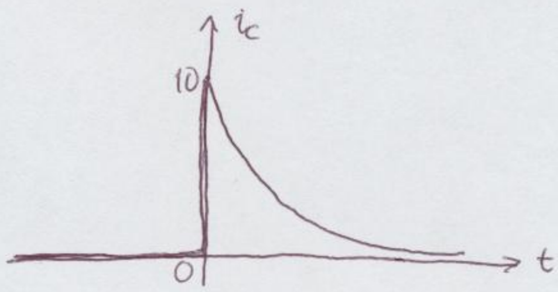
$t < 0$: $i_C(t) = 0$

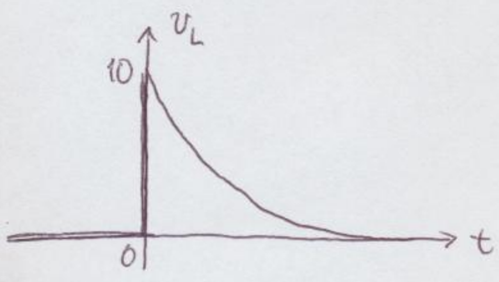
$v_C(t) = 0$

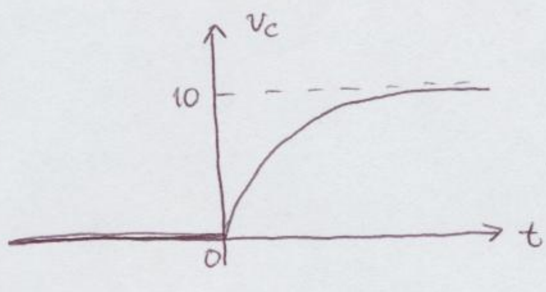
$0 < t < \infty$: $\left.\begin{array}{l} i_C(0^+) = 10A \\ i_C(\infty) = 0 \end{array}\right\} i_C(t) = 10e^{-t}$

$\left.\begin{array}{l} v_C(0^+) = 0 \\ v_C(\infty) = 10V \end{array}\right\} v_C(t) = 10 - 10e^{-t}$

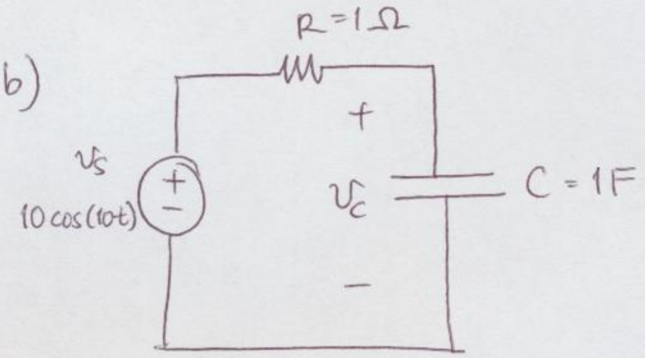








b)



$t \to \infty$: steady-state

Use voltage divider

$$v_C = V_s \left| \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \right| = V_s \left| \frac{1}{1 + j\omega RC} \right| = \frac{V_s}{\sqrt{1^2 + \omega^2 RC}}$$

$\omega = 10$ rad/s

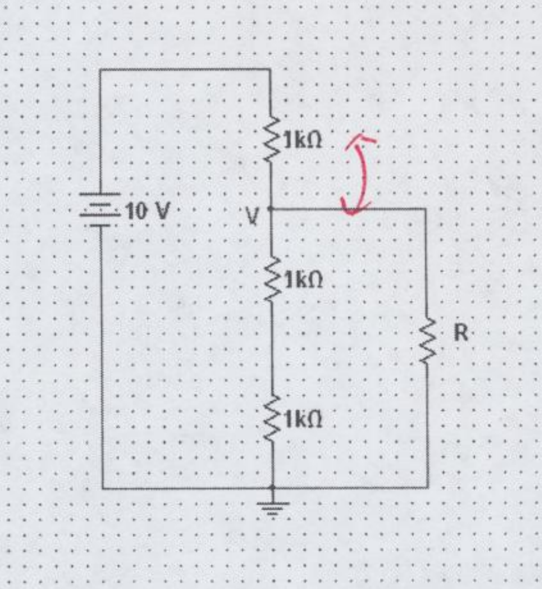
$R = 1\Omega$

$C = 1F$

$$\boxed{v_C = 0.995V}$$

PROBLEM 3 [8 points]

(a) In the circuit below, find the value of R such that the node V has a voltage of 5 volts.



$V = 5v.$

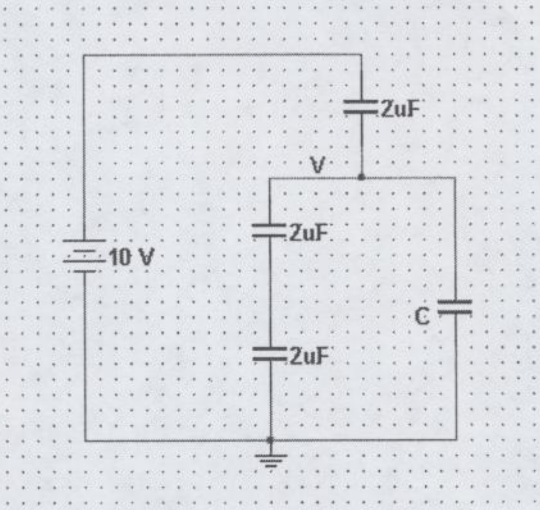
$R_{eq} = \frac{2kR}{2k+R}$

$5v = 10 \cdot \frac{\frac{2kR}{2k+R}}{\frac{2kR}{2k+R} + 1k}.$

$R = 2k\Omega.$

R: $2k\Omega$

(b) In the circuit below, find the value of C such that node V has a voltage of 5 volts.



$C_{eq} = \left(\frac{1}{2\mu F} + \frac{1}{2\mu F}\right)^{-1} + C.$

$V = 5v$

$2\mu F = C_{eq}$

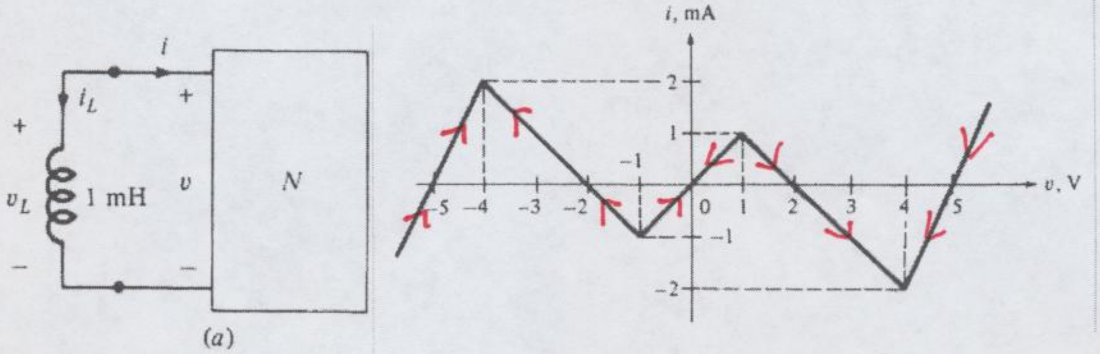
$2\mu F = \frac{4(\mu F)^2}{4\mu F} + C.$

$C = 1\mu F$

C: $1\mu F$

PROBLEM 4 [10 points]

Consider the circuit shown below along with its the *v*-*i* characteristic
(a) Sketch the dynamic route on the graph shown below.



(*a*)

$V = L\frac{di_L}{dt}$

$V_L = -L\frac{di}{dt}$

$V > 0 \Rightarrow i\downarrow$

$V < 0 \Rightarrow i\uparrow$

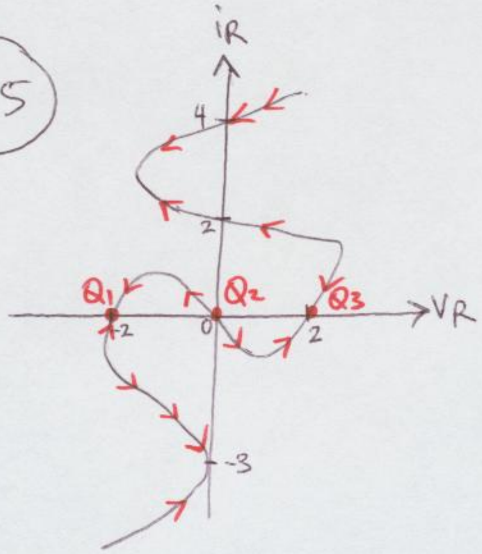
(b) For what range of initial condition $i_L(0)$ does this circuit exhibit oscillation ?

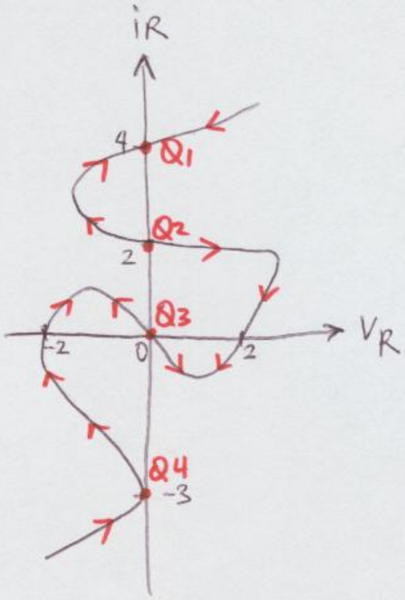
Conditions on $i_L(0)$: $V_L(0) < -1$ V

$V_L(0) > 1$ V

Solution

#5





Q_1 stable

Q_2 unstable

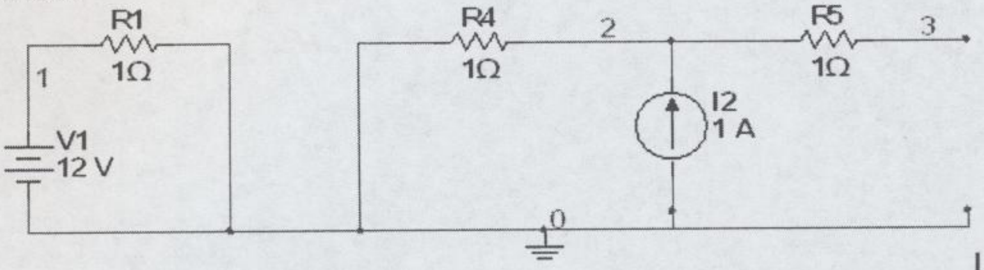
Q_3 stable

Q_1 stable

Q_2, Q_3, Q_4 } unstable

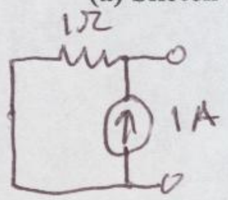
PROBLEM 6 [5 points]

Find the Thévenin and Norton equivalent circuit across terminals 3 and 0 for the circuit below.

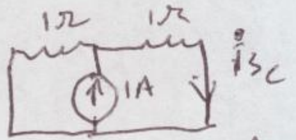


(a) Sketch the Thévenin equivalent circuit in the box:

V_{OC}



i_{SC}

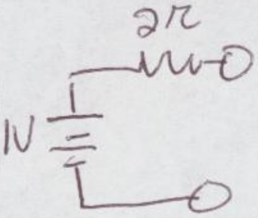


current divider

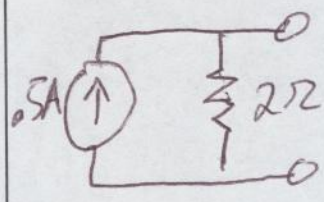
$$V_{oc} = (1\,A)(1\,\Omega) = 1\,V$$

$$i_{sc} = .5\,A$$

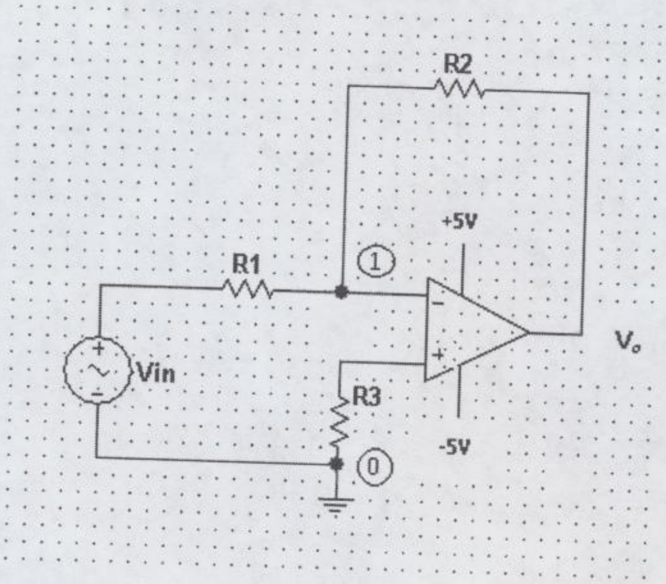
$$R_{Th} = \frac{V_{oc}}{i_{sc}} = 2\,\Omega$$



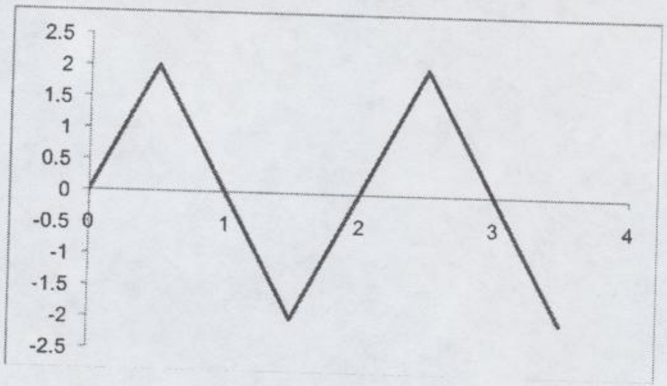
(b) Sketch the Norton equivalent circuit in the box:



PROBLEM 7 [10 points]



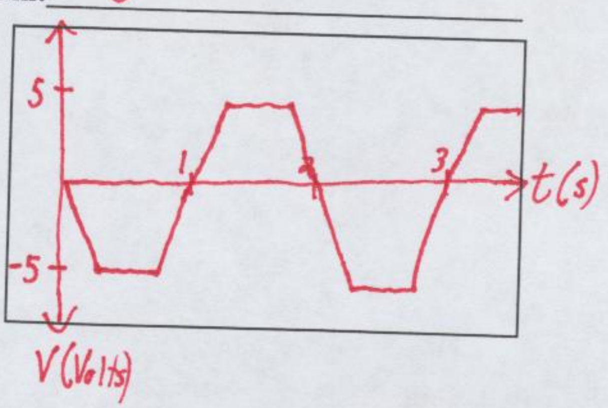
(a) Assuming the op-amp in the circuit above is ideal, derive V_o, in the linear regime, in terms of V_{in}, R_1, R_2, and/or R_3. Show your work on the next page.
(b) Find the gain (V_o/ V_{in}) in the *linear* regime for the circuit if R_1=1kΩ, R_2=5kΩ, and R_3=1kΩ.
(c) Sketch the output voltage versus time if it is given the following input voltage:



V_o: $-V_{in}\frac{R_2}{R_1}$

Gain: -5

Sketch the output voltage in the box:



EXTRA WORKSPACE FOR PROBLEM 7

a) $V^+ = V^- = \emptyset$

$$\frac{V_{in} - 0}{R_1} = \frac{0 - V_o}{R_2}$$

$$\frac{V_{in}}{R_1} = \frac{-V_o}{R_2}$$

$$V_o = -V_{in}\frac{R_2}{R_1}$$

b) $\text{Gain} = \frac{V_o}{V_{in}} = \frac{-R_2}{R_1} = \frac{-5000}{1000} = -5$

SOLUTIONS

PROBLEM 8 [15 points]

Non-Linear Region

$V_o = +5$ or -5

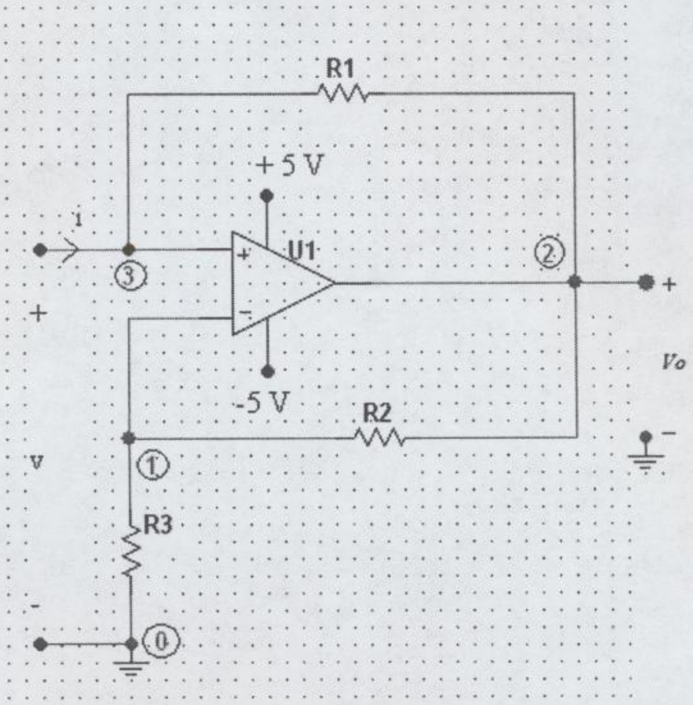
$\frac{V - V_o}{R_1} = i$

$\boxed{i = \frac{V-5}{R_1}}$ $(V_o = 5)$

$\boxed{i = \frac{V+5}{R_1}}$ $(V_o = -5)$

$i = \frac{V-5}{R_1}$ valid for $V \geq 2.5$

$i = \frac{V+5}{R_1}$ valid for $V \leq -2.5$



a) Node 3

(1) $\frac{V_3 - V_o}{R_1} = i$ $(V_3 = V)$

Node 1

(2) $\frac{V_1 - V_o}{R_2} + \frac{V_1 - \cancel{V_o}^{\,0}}{R_3}$ $(V_1 = V_3 = V)$

$V_1\left(\frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V_o}{R_2}$

$V_o = V_1 R_2 \left(\frac{R_2 + R_3}{R_2 R_3}\right)$

$V_o = V_1 \frac{(R_2 + R_3)}{R_3}$

(1) + (2) $\frac{V_3 - \frac{V_1 (R_2 + R_3)}{R_3}}{R_1} = i$

$\frac{V_3}{R_1} - \frac{V_1 (R_2 + R_3)}{R_1 R_3} = i$

$V_3 = V_1 = V$

$V\left(\frac{1}{R_1} - \frac{R_2 + R_3}{R_1 R_3}\right) = i$

$i = \frac{V}{R_1}\left(\frac{R_3 + R_2 - R_3}{R_3}\right)$

$\boxed{i = -\frac{V R_2}{R_1 R_3}}$

Linear Region

valid for $-2.5 \leq V \leq 2.5$

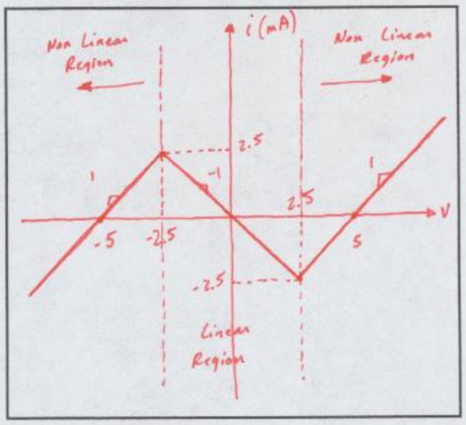
(a) Derive an expression for i in terms of v, R_1, R_2, and R_3 for all three regions of operation for the op-amp above. Show your work on the next page.

(b) If R_1=1kΩ, R_2=2kΩ, and R_3=2kΩ plot the IV characteristics below.

Expression for i =

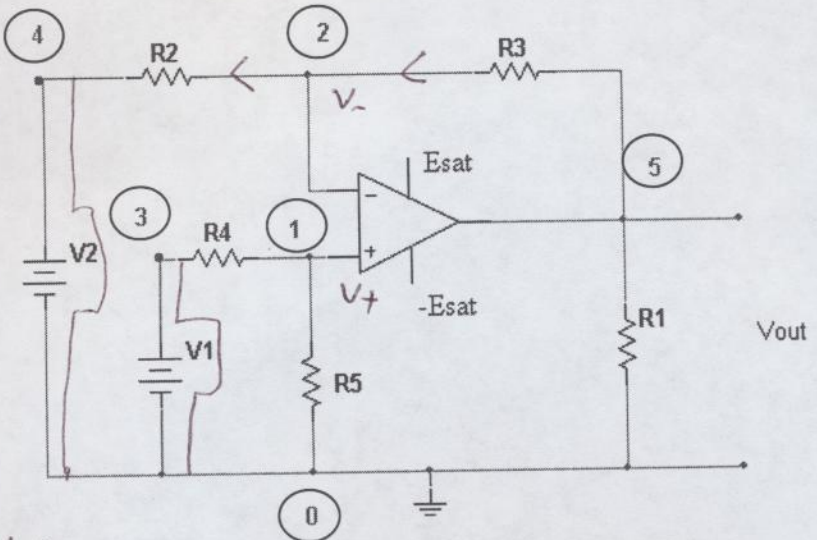
$i = \frac{-V R_2}{R_1 R_3}$	(linear
$i = (V-5)/R_1$	(+ve saturation)
$i = (V+5)/R_1$	(-ve saturation)

Sketch i-v graph in the box:



PROBLEM 9 [10 points]

Find an algebraic expression for the voltage Vout, the voltage across resistor 1, in the linear region. Hint: use superposition.



Shorting V_1:

$V_+ = 0$. By ideal opamp $V_+ = V_-$ → $\frac{V_{out} - 0}{R_3} = \frac{0 - V2}{R_2}$ → $V_{out} = -\frac{R_3}{R_2} V_2$

Shorting V_2:

$V_+ = \frac{R_5 V1}{R_4 + R_5}$ → $V_+ = V_-$ → $\frac{V_- - 0}{R_2} = \frac{V_{out} - V_-}{R_3}$ → $V_{out} = \frac{R_3 + R_2}{R_2} V_- = \frac{(R_3+R_2)R_5 V1}{R_2 (R_4+R_5)}$

voltage divider

By superposition $V_{out} = \frac{(R_3+R_2)R_5}{R_2(R_4+R_5)} V1 - \frac{R_3}{R_2} V_2$

Vout in the linear region (in terms of V1, V2, R1, R2, R3, R4 and/or R5):

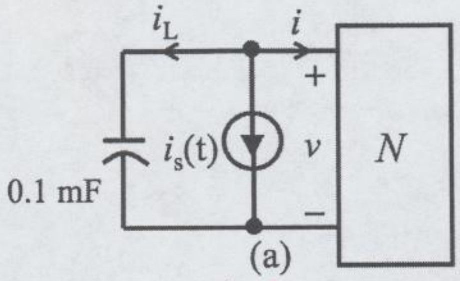
$V_{out} = \frac{(R_3+R_2)R_5}{R_2(R_4+R_5)} V1 - \frac{R_3}{R_2} V_2$

Find Vout if R1=R2=R3=R4=R5=1Ω, V1=V2=1V, Esat=5V, -Esat=-5V:

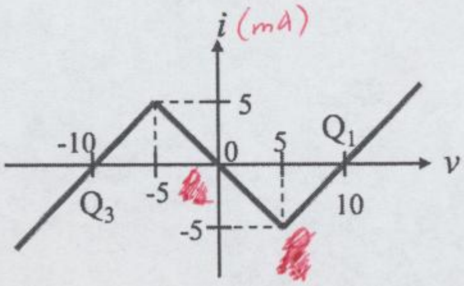
$V_{out} = \frac{(2)(1)}{1(2)} 1 - \frac{1}{1}(1) = 0$

PROBLEM 10 [15 points]

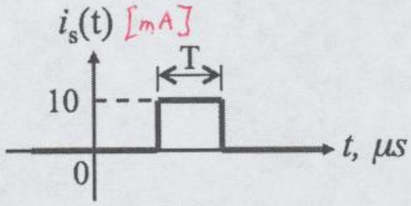
The circuit below is to be used as a flip-flop. In order to switch from Q_1 to Q_3 the triggering signal below is applied.



(a)



(b)



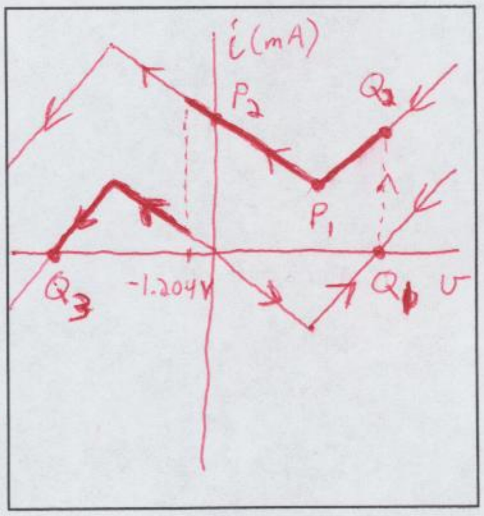
(c)

(a) Determine the minimum duration T of the pulse required for successful switching. *Show your work* on the next page.

(b) Sketch the relevant dynamic routes corresponding to a pulse width T = 0.15s.

(a) Minimum Duration of T:________________

(b) Sketch relevant dynamic routes in the box:



(a) $Q_2 \to P_1$: $\tau = 0.1mF(1k\Omega) = 0.1s$

$t_1 = \tau \ln\left[\frac{10-0}{5-0}\right] = 0.1\ln[2] = 0.0693s$

$P_1 \to P_2$: $\tau = -0.1s$

$t_2 = t_1 + \tau \ln\left[\frac{5-10}{0-10}\right] = 2(0.1)\ln(2)$

$t_2 = \boxed{T_{min} = 0.1386s}$

(b) $0.15 - 0.1\ln 2 = -0.1 \ln\left[\frac{5-10}{v(t=0.15s)-10}\right]$

$\to \boxed{v(t=0.15s) = -1.204V}$