Lecture 10 - Chapter 6, 7 - RC/RL Circuits

Administrivia ≡ None ☺

Today → Chapter 6 ≡ I will cover section 6.1, 6.2; READ 6.3

That's all you need to know from chapter 6

→ skip 7.6, 7.7

→ Chapter 7 ≡ [Natural / Step response of RL/RC Circuits]

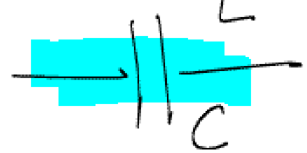
→ Section 7.4 today, do examples all next week from 7.1, 7.2, 7.3 & 7.5

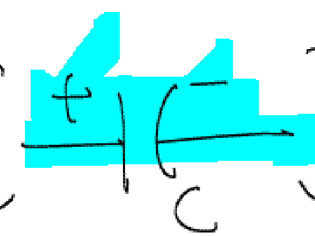
*Note: We will do chapter 5 in two weeks

Chapter 6 - Inductance & Capacitance

- Inductors (aka "choke" aka "coil") & Capacitors are elements that store energy in a magnetic field & electric field respectively.
- Inductance. Unit: Henry Typical: 1 mH

  Capacitance. Unit: Farad Typical: 1 μF
- Inductor symbol: 

  Capacitor symbol: 

  or [

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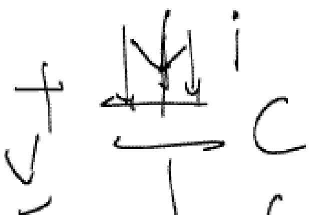
I-V RELATIONSHIP

$$V = iR$$

$$p = vi$$

$$= \frac{v^2}{R} = i^2R$$

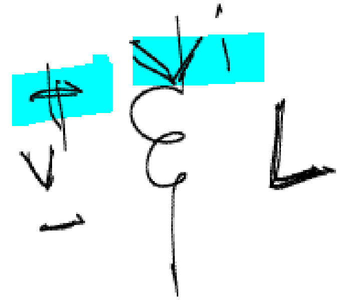


charge [C] (F) (volts)

$$q = Cv$$

$$\Rightarrow \frac{dq}{dt} = \frac{d}{dt}[Cv]$$

$$\Rightarrow \boxed{i = C\frac{dv}{dt}} + 0 \quad \cancel{v\frac{dc}{dt}}$$



$$V = L\frac{di}{dt}$$

$$p = vi$$

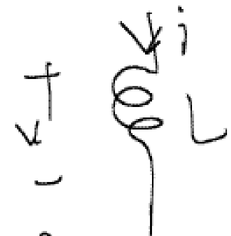
$$E = \int p = \frac{1}{2}Li^2 \quad \text{(Joules)}$$

more important

Note passive sign convention

Continuity Property for Inductors & capacitors

Inductor: $v = L\dfrac{di}{dt}$



(Q:) What happens to v if my i changes instantaneously?

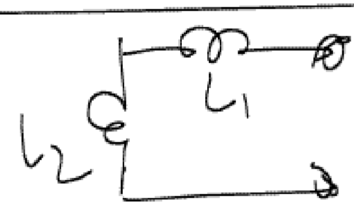
i.e.. $\left[\dfrac{di \Rightarrow \text{finite value}}{dt \rightarrow \text{zero}}\right] \Rightarrow v \rightarrow \infty$ [L is constant]

Current through an inductor does not change instanteously

Similarly, Voltage across a capacitor cannot change instantaneously [$i = C\frac{dv}{dt}$]

Mathematical details → look up Dirac delta function

→ You are not responsible for this!

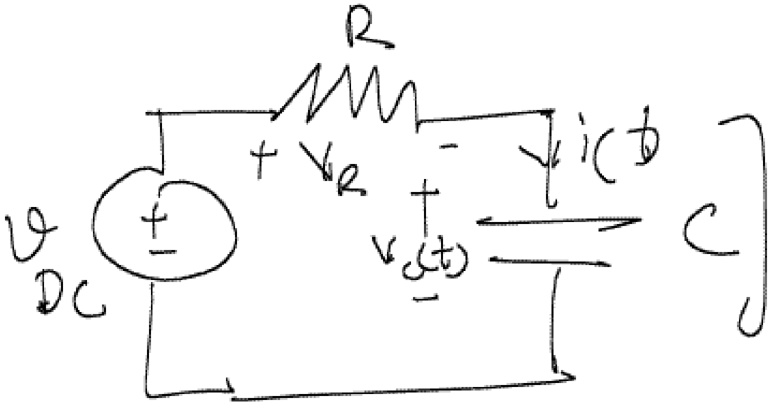
mini-H.W: Read 6.3 : 

$\Rightarrow$ $L_{eq} = L_1 + L_2$

Now, chapter 7!

Chapter 7 – RC / RL Circuits [Natural & Step response]

RC circuit :



$$i = C\frac{dv_c}{dt}$$

<u>Goal:</u> Find $V_c(t)$ & $i(t)$.

<u>Sol:</u> $V_{DC} = V_R + V_c$ [KVL]

First, lets find $V_c(t)$

Note: For capacitors, we find v_c first, instead of i. Because $i = C\frac{dv_c}{dt}$

$\Rightarrow \quad V_{DC} = iR + V_c$

$\Rightarrow \quad V_{DC} = \left(C\frac{dv_c}{dt}\right)R + V_c$

$\Rightarrow \quad V_{DC} = RC\frac{dv_c}{dt} + V_c \quad \text{—— } (1)$

units → seconds

(1) is an example of a first-order → no $\frac{d^2 v}{dt^2}$ etc

ordinary linear differential equation with

→ no partial derivatives

→ nothing like $\left(\frac{dv_c}{dt}\right)^2$, $\sin(v_c)$ etc

constant coefficients

→ because R, C & V_{DC} are constant

Q) What makes differential equations hard to solve?
That is, why is $x + 3 = 5$ easier?

Sol: (1) Solutions are not usually unique

eg: $x+3 = 5$, unique: $x = 2$

(2) Solution should satisfy ~~as~~ differential equation for all time ← hard to do!

Best way to solve diff. eqs. ⇒ Guess at a solution.

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(1) ⇒ $V_{DC} = RC \frac{dv_c}{dt} + v_c$ ⇒ looks like $K = A \frac{dx}{dt} + x$

"looks like"

$$\frac{dx}{dt} + x = 0 \longrightarrow \frac{dx}{dt} = -x$$

$$x(t) \simeq e^{-t}$$ (Forgetting boundary conditions)

Based on our intuition above, let us guess $v_c(t) = A + Be^{-t}$

<u>Note</u>: $$\beta \frac{dx}{dt} + x = 0 \Rightarrow \frac{dx}{dt} = \frac{-x}{\beta} \Rightarrow \boxed{x(t) = e^{-t/\beta}}$$

Therefore, our "final" guess is:

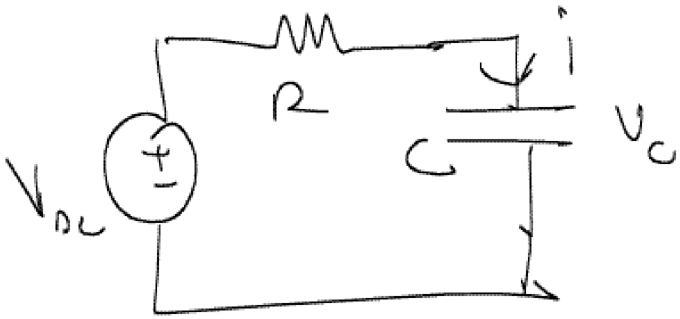
$$v_c(t) = A + Be^{-t/\tau}$$

<u>Goal:</u> Find A, B, τ

<u>eqn:</u> $V_{oc} = RC\dfrac{dv_c}{dt} + v_c$

<u>(1) Find A:</u> $\lim_{t\to\infty} v_c(t) = \lim_{t\to\infty}\left(A + Be^{-t/\tau}\right)$

$= A \quad \therefore A = v_c(t\to\infty)$

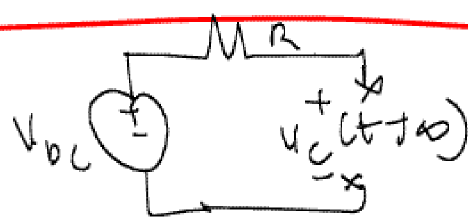


$t \to \infty$ means the circuit has been in operation for a very long time

i.e.. $v_c = A$ is a constant

$\Rightarrow \quad i = C\frac{dv_c}{dt} = 0 \text{ A}$

$\Rightarrow$ (1) After a very long time, capacitors are modelled as open circuit because they are fully charged

∴ 

∴ $\boxed{v_c(t\to\infty) = V_{DC} = A}$

$\therefore \quad V_c(t) = V_{DC} + B e^{-t/\tau}$

(2) Find B: Suppose $\underbrace{V_c(t=0) = V_0}_{\text{initial voltage across the capacitor.}}$

$\therefore \quad B \Rightarrow V_c(t=0) = V_{DC} + B \Rightarrow V_0 = V_{DC} + B$

$\Rightarrow \boxed{B = V_0 - V_{DC}}$

$$\therefore \quad V_c(t) = V_{DC} + (V_0 - V_{DC}) e^{-t/\tau} \qquad (2)$$

(3) Find τ : Notice (2) satisfies:

$x + 3 = 7$

$\hookrightarrow$ satisfy

$$v_{oc} = RC \frac{dv_c}{dt} + v_c$$

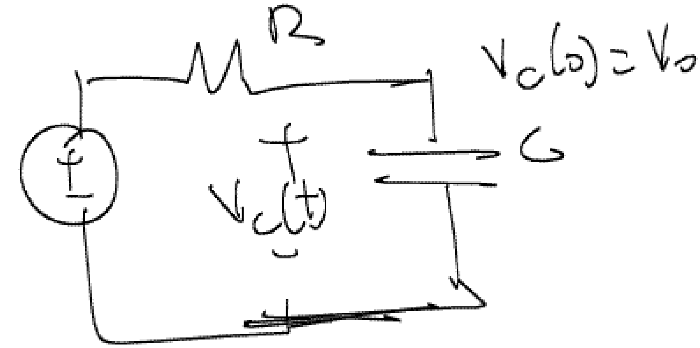
$$\Rightarrow v_{oc} = RC \frac{d}{dt}\left[v_{oc} + [v_o - v_{oc}] e^{-\frac{t}{\tau}}\right] + \left(v_{oc} + (v_o - v_{oc}) e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \cancel{v_{oc}} = RC\left[0 + \frac{[v_o - v_{oc}] e^{-t/\tau}}{-\tau}\right] + \cancel{v_{oc}} + [v_o - v_{oc}] e^{-\frac{t}{\tau}}$$

$$\Rightarrow \frac{RC\,\cancel{(V_0 - V_{DC})}\, e^{-t/\tau}}{\tau} = \cancel{(V_0 - V_{DC})}\, e^{-t/\tau}$$

$$\Rightarrow \boxed{\tau = RC} \checkmark$$

$$\boxed{V_c(t) = V_{DC} + (V_0 - V_{DC})\, e^{-t/RC}} \quad \text{— (3)}$$



Observations: I

(1) $V_c(0)$ in (3) $= V_0$ ✓

(2) $V_c(t \to \infty) = V_{DC}$ ✓

(3) Units! check RC ✓

$$[RC] = \left[\left(\frac{\cancel{V}}{I}\right)\left(\frac{q}{\cancel{V}}\right)\right] = \left[\frac{q}{\left(\frac{dq}{dt}\right)}\right] = \left[\frac{C}{\frac{C}{s}}\right] = \underline{\underline{\text{seconds}}}$$

Observations II: (1) Significance of $\tau = RC$ (TIME CONSTANT)

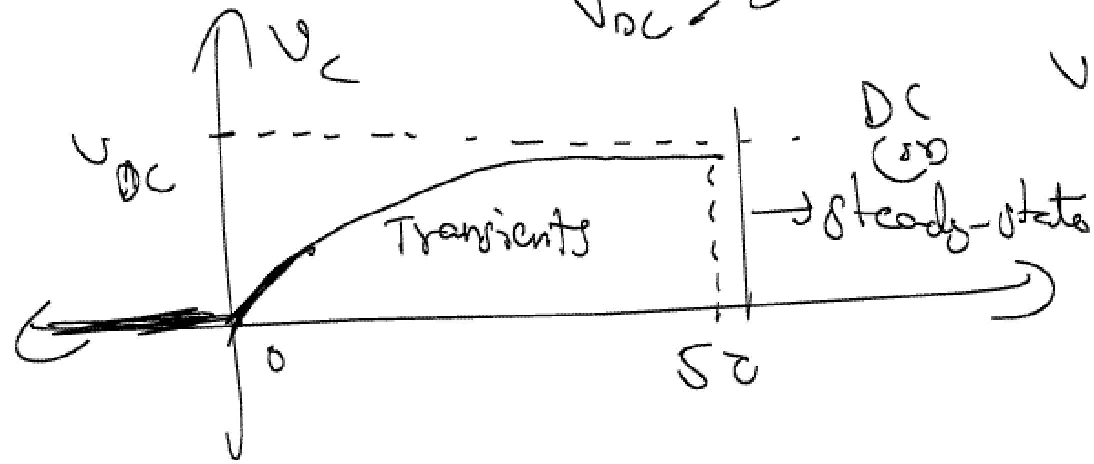
Now, $V_C\,[t = 5\tau] = V_{DC} + (V_0 - V_{DC})\,e^{-\frac{5\tau}{\tau}}$

$$= \left[V_{DC} + (V_0 - V_{DC})\,e^{-5}\right] \approx V_{DC}$$

That is, problem is "over" in 5 time constants

(2) Easier to understand from a plot of $V_C(t)$ [assume $V_0 = 0$ V]

$V_{DC} > 0$



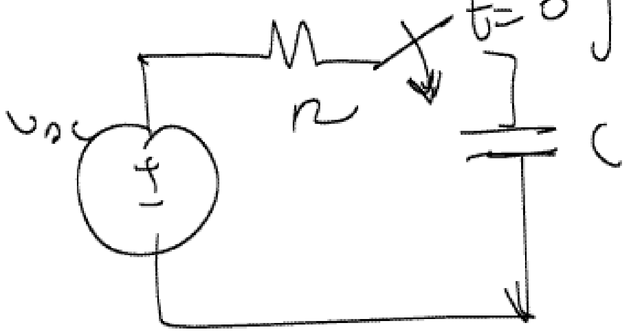
$$v_C = V_{DC} - V_{DC} e^{-\frac{t}{\tau}} = V_{DC}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Note: This $v_C(t)$ is called the step response

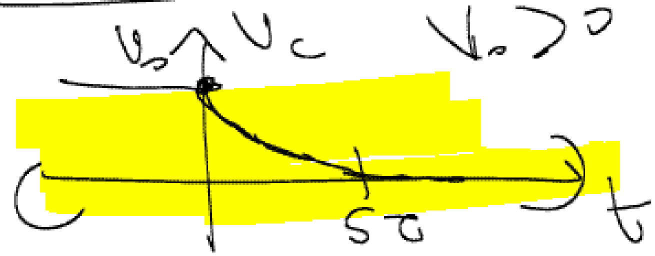
(or) a forced response as opposed to

natural response ← response to initial conditions only

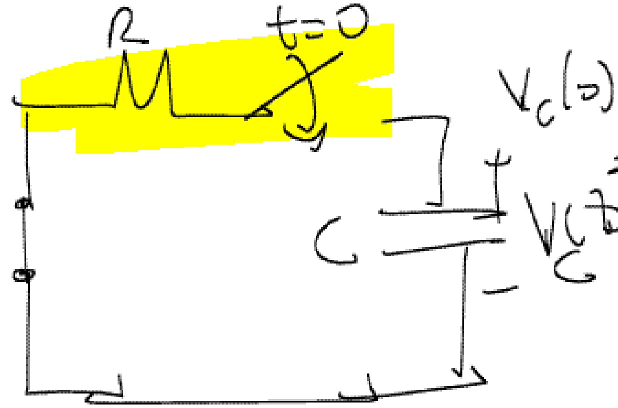
STEP RESPONSE



] switch closes at t=0



NATURAL RESPONSE

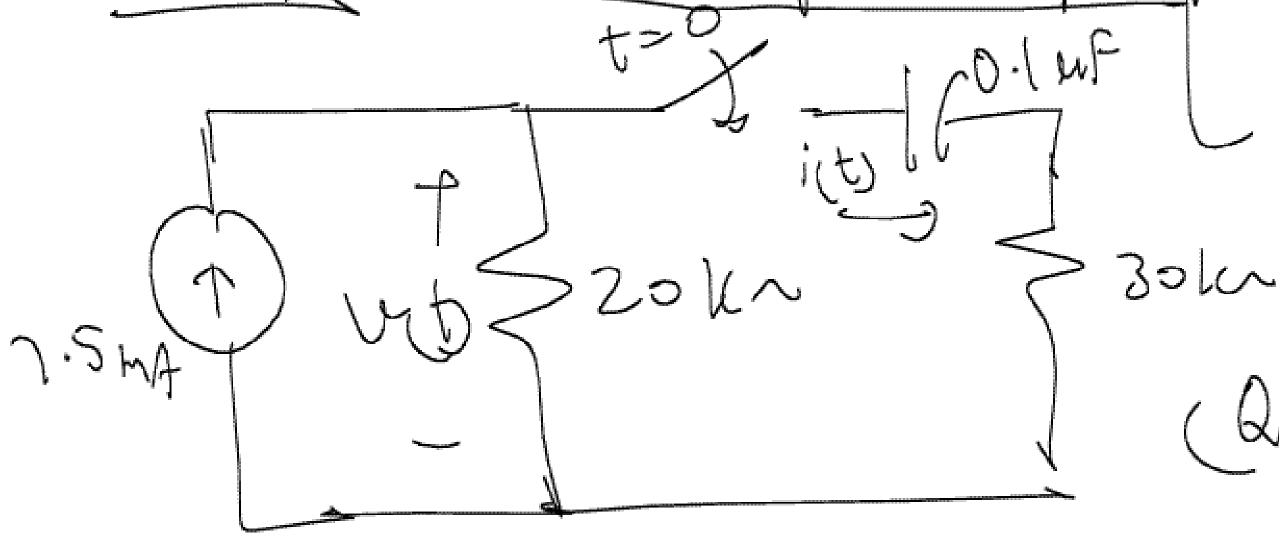


$V_c(0) = V_0$

$$V_c(t) = \underbrace{V_{DC}}_{0} + (V_0 - \underbrace{V_{DC}}_{0})e^{-\frac{t}{\tau}}$$

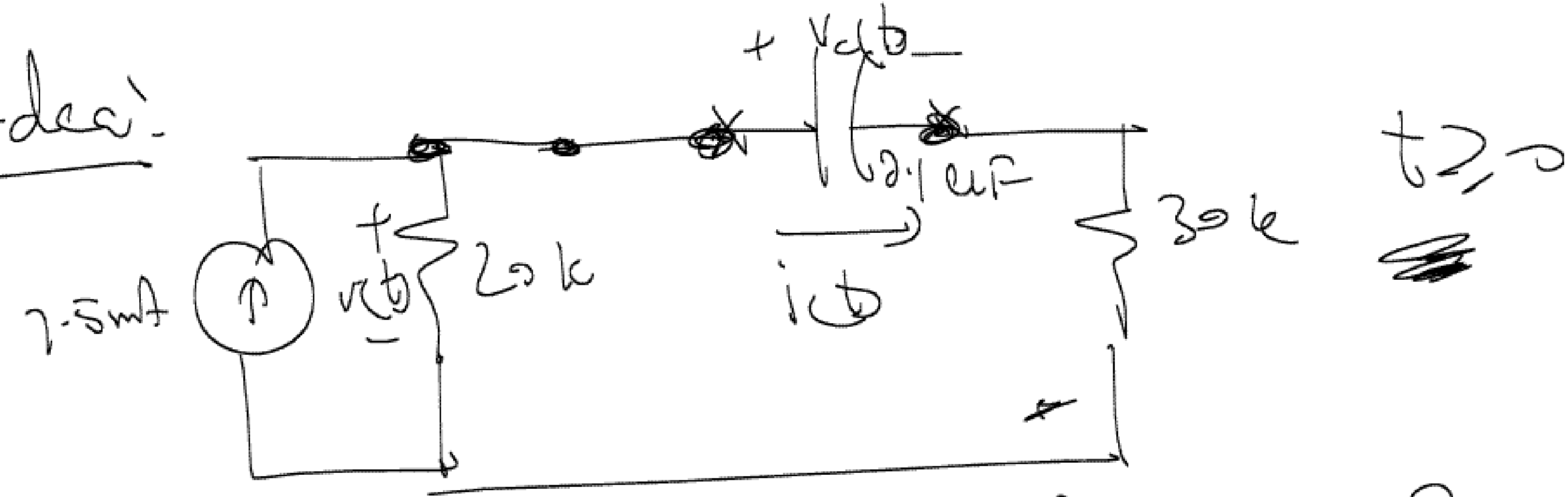
$$V_c(t) = V_0 e^{-t/\tau}$$

Example: Eg (7-8), p. 389 [Setup only, solve on Tuesday]

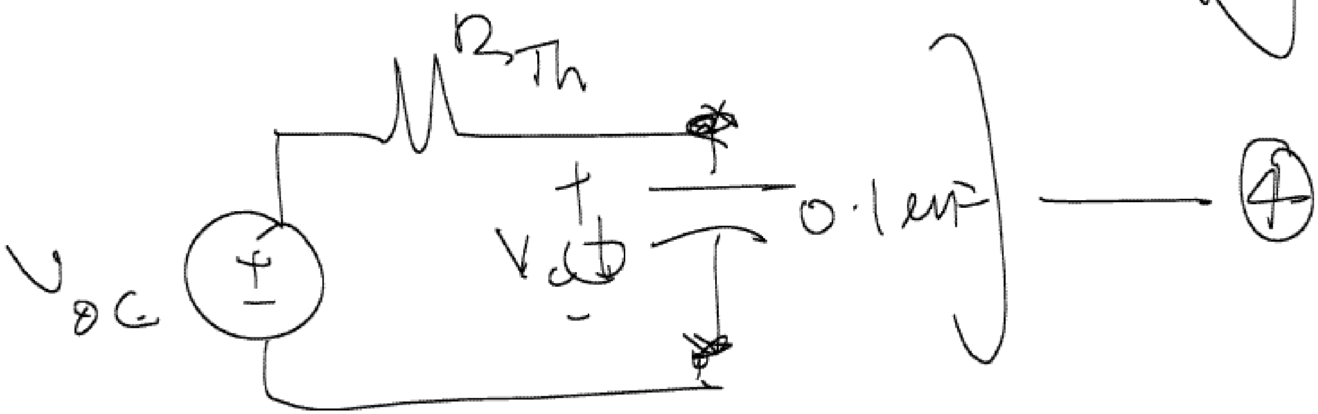


(Q:) Switch has been open for a very long time, closes at $t=0$. Find $i(t)$, $v(t)$, $t \geq 0$

Idea:



Find i by finding V_c $\left[i = C \frac{dV_c}{dt} \right]$



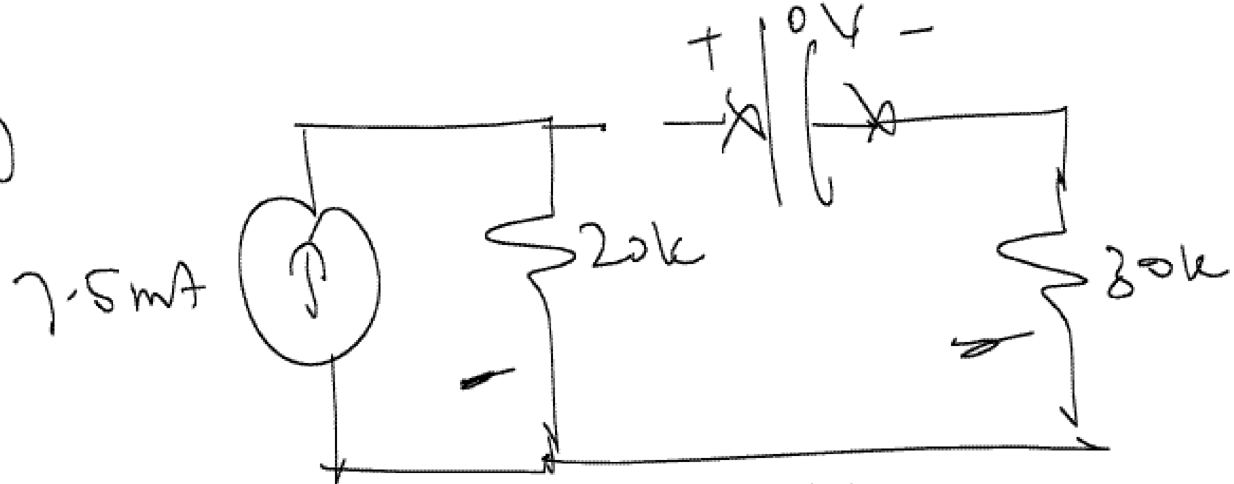
(4) can be solved !!!

$$V_C(t) = V_{oc} + (V_0 - V_{oc}) e^{-t/R_{Th}C}$$

Notice: To find V_0, you have to use circuit analysis.

That is, problem stated the switch had been open for a very long time.

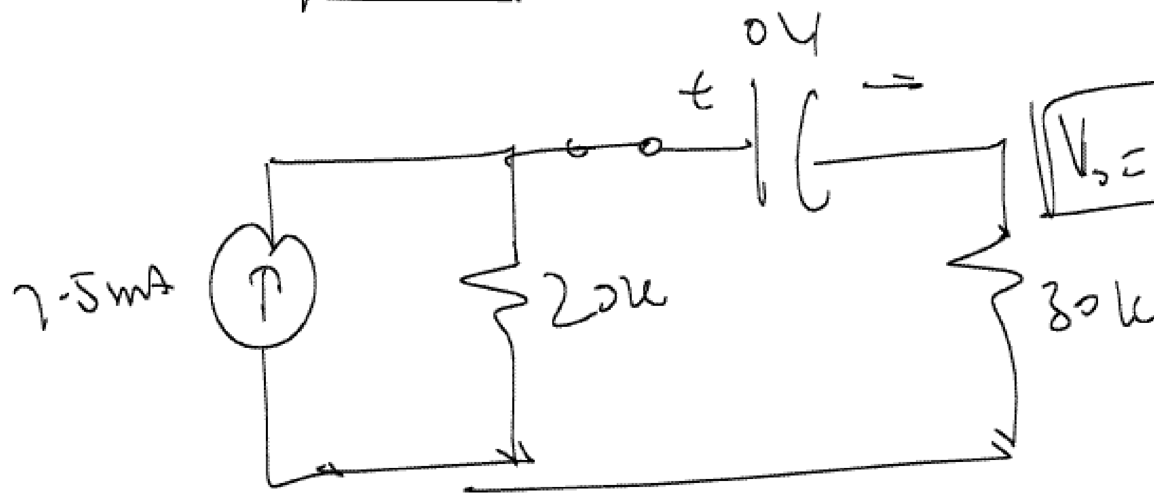
(1)



t<0

$V_o = 0$

(2)



$\boxed{V_o = 0\ V}$ $t = 0^+$

the instant after switch closes

Next time

- Finish example above
- More examples!!!!