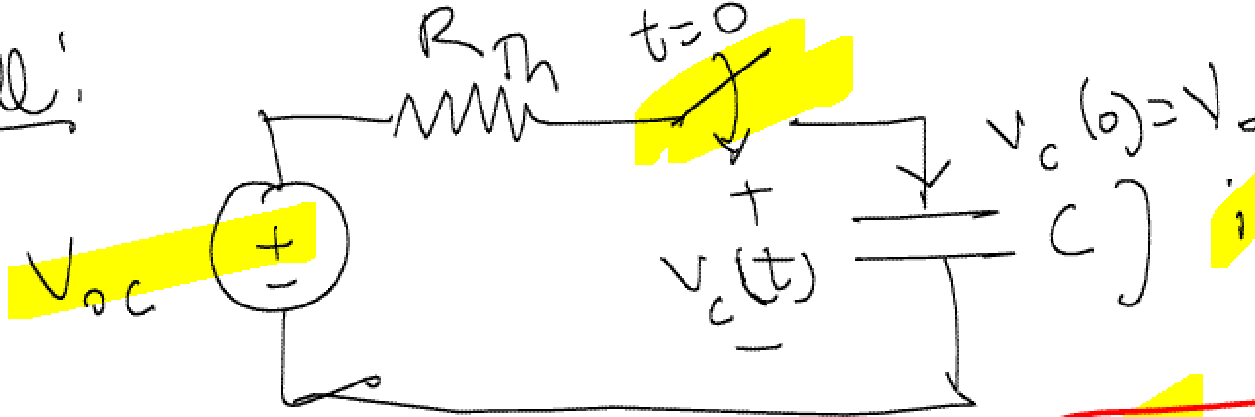
Lecture 11 - Chapter 7 Examples

Administrivia → HW policy ≡ If you don't write name / SID / lab section #/day, time → 5 POINTS OFF / HW, starting with HW #5

→ Midterm 1: 6:00pm - 9:00pm (??) ← Thursday, March 10th 2005

Today → RC examples (step / natural response)
→ PSPICE
→ RL examples ← maybe on Thursday } may not get to it today

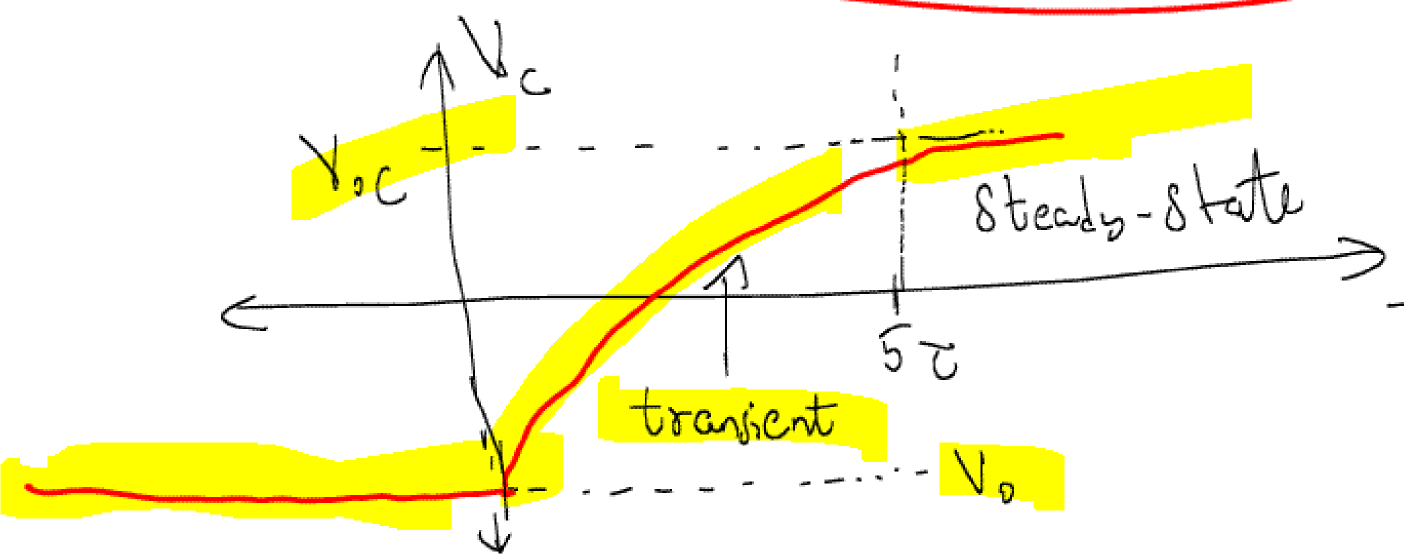
Recall:

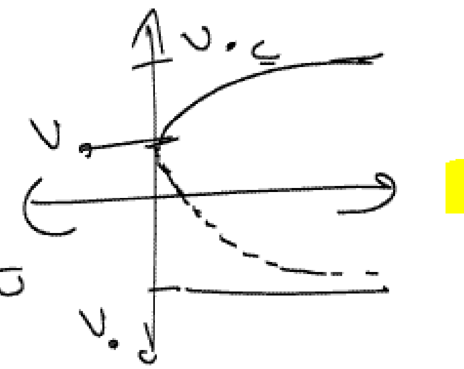


$$i = C\frac{dv_c}{dt}$$

$$\boxed{V_c(t) = V_{oc} + (V_0 - V_{oc})\, e^{-\frac{t}{R_{Th}C}}}$$

$R_{Th}C \rightarrow \tau$ (time constant)





Example 7.8 : (p. 289)

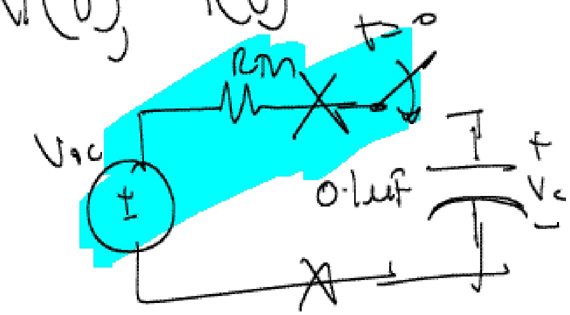
(Q!) Find $i(t)$, $v(t)$ [$t \geq 0$]

We will solve the circuit the difficult way

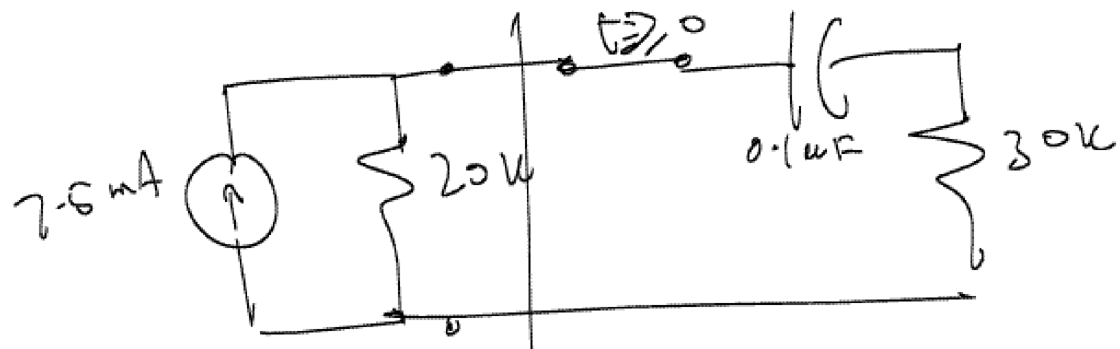
first, find Voltage across the capacitor

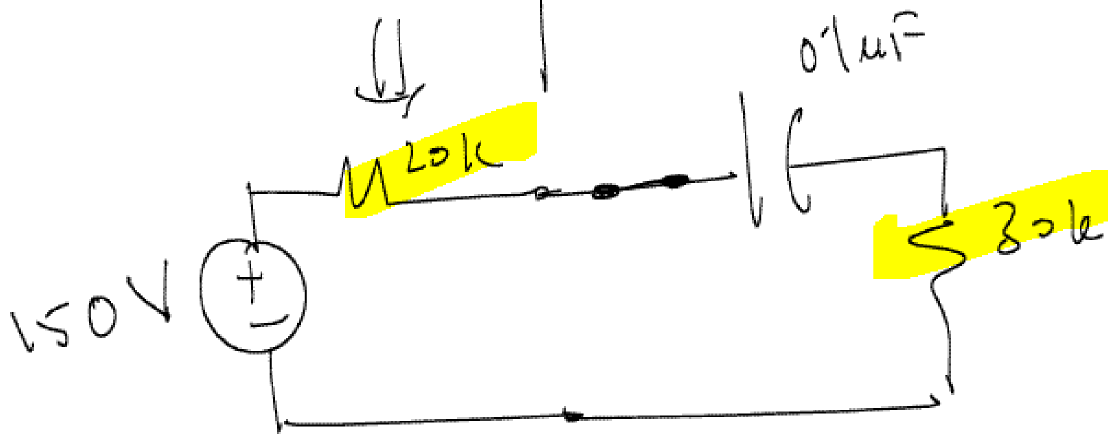
& then use it to find $V(t)$, $i(t)$

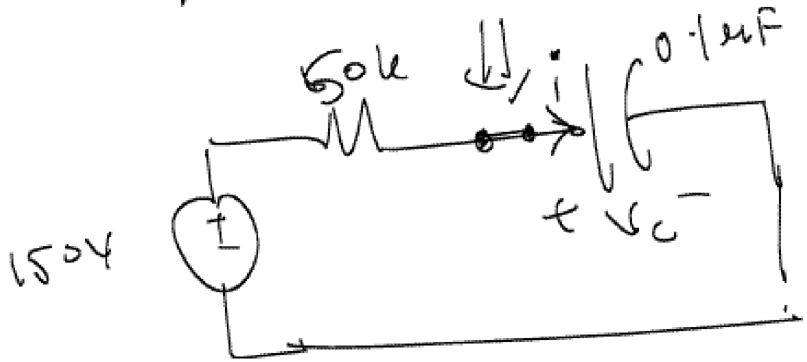
$$V_c(t) = V_{oc} + (V_0 - V_{oc})\, e^{-\frac{t}{R_{TH} C}}$$



$\boxed{V_0 = 0 \text{ V [initial voltage]}}$







$$v_c(t) = v_{oc} + (\overset{0}{\cancel{V_o}} - v_{oc})e^{-\frac{t}{R_{th}C}}$$

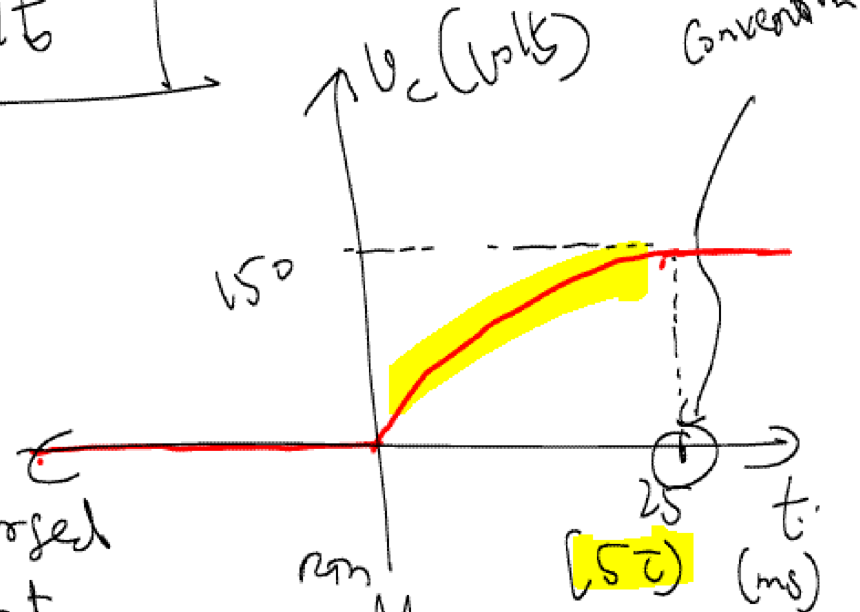
$$= v_{oc}\left(1 - e^{-\frac{t}{R_{th}C}}\right)$$

$$\boxed{V_{oc} = 150\ V}$$

$$\tau = R_{th}C = (50k)(0.1\mu)$$

$$\boxed{\tau = 5\ ms}$$

$$\boxed{v_c(t) = 150\left(1 - e^{-\frac{t}{5ms}}\right)\ \text{volt}}$$



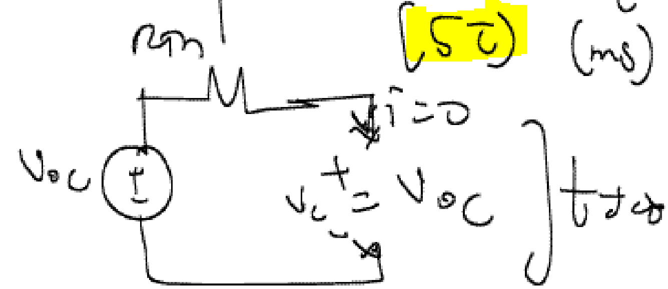
Note: Why V_{oc}?

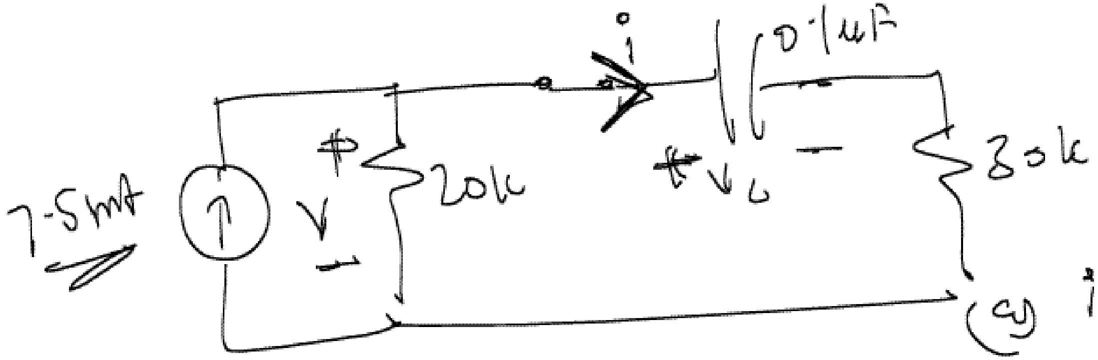
Because as $t \to \infty$

capacitor is fully charged

i.e. V_c is constant

$\therefore$ $i = C\dfrac{dv_c}{dt} = 0$





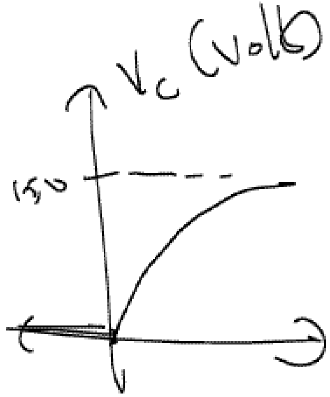
$V_c(t) = 150\left(1-e^{-\frac{t}{5ms}}\right)$ volts

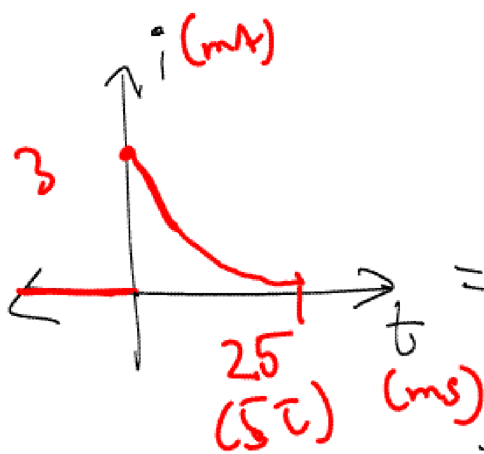
$i = C\frac{dv_c}{dt}$

$= (0.1\,\mu)\,150\left[-e^{-\frac{t}{5ms}}\cdot\frac{-1}{5ms}\right]$

$= \frac{15}{5}e^{-\frac{t}{5ms}}$ mA

$= 3e^{-t/5ms}$ mA ← τ



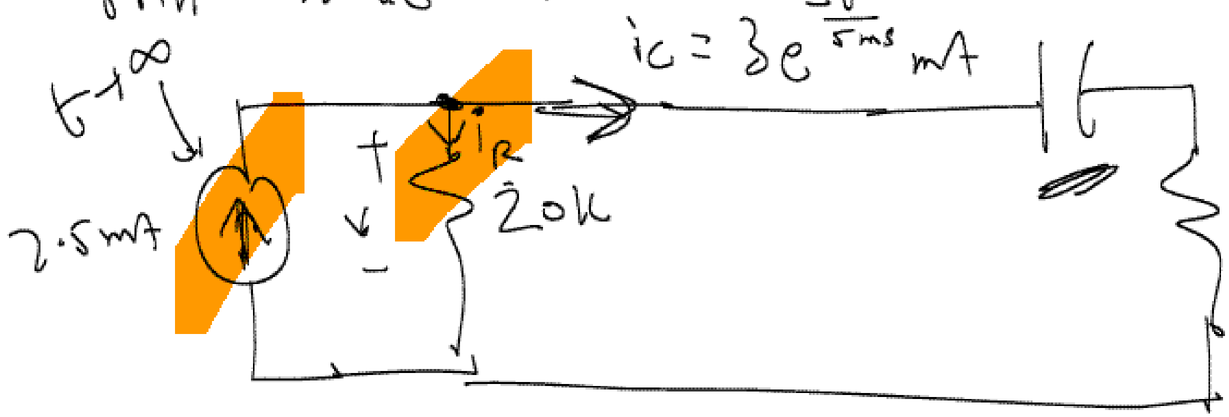


Notice:

(1) Voltage across a capacitor is continuous, ~~current usually isn't!~~ This is because $i = C\frac{dv}{dt}$ → cannot change instantaneously

(2) Notice $i(t)$ is also an exponential with the same time constant!

We still have to find v:



KCL: $7.5\text{mA} = i_R + i_c$

$$\Rightarrow i_R(t) = \left(7.5 - 3e^{-t/5ms}\right) \text{mA}$$

$$v(t) = (i_R)(20k) = \left(7.5 - 3e^{\frac{-t}{5ms}}\right)(20k)\ mA$$

$$\boxed{v(t) = 150 - 60e^{\frac{-t}{5ms}}\ V}$$

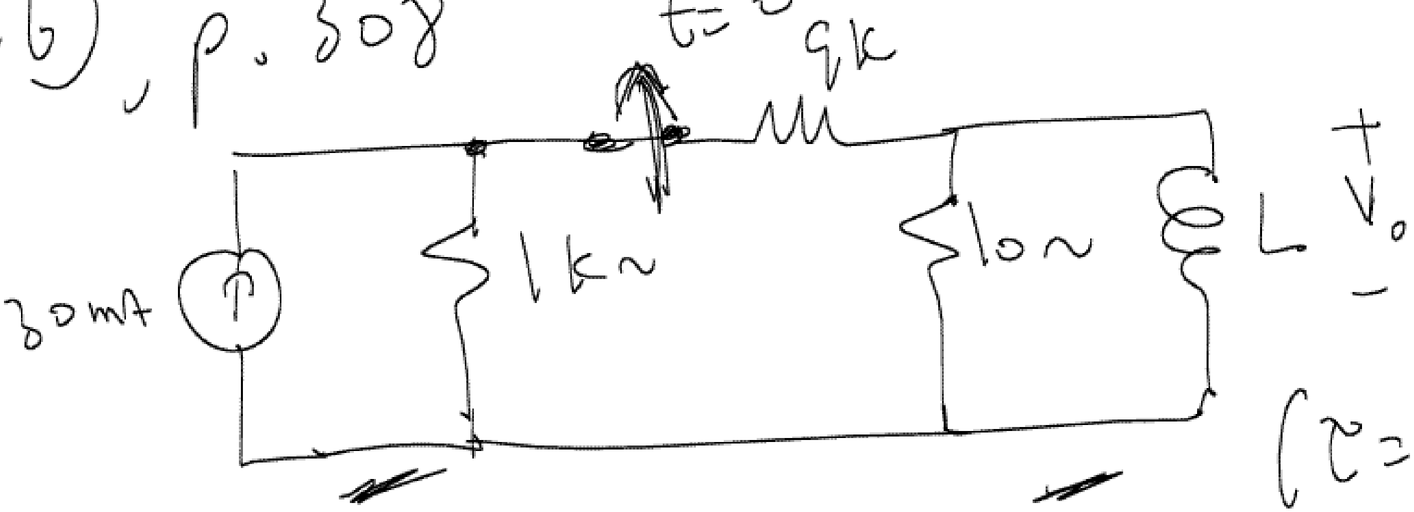
"Easier" way to solve transient problems

$$f(t) = f_{final} + (f_{initial} - f_{final})\, e^{\frac{-t}{\text{circuit time constant}}}$$

$R_{Th}C \qquad L/R_{Th}$

Example 7-8 is worked out using the method above; i.e. find final value(s), initial value (s) & time constant.

(P7.16), p. 308



$[\tau = L/R]$

(Q:1) (a) Find value of L so that $V_o(t)$ equals $0.5\,V_o(0^+)$ at $t = 1\,ms$

Note: (1) Inductors are "duals" of capacitor

i.e.. voltage is continuous in RC circuits,
current is continuous in RL circuit
$\left[V = L\frac{di}{dt}\right]$

(2) $t = 0^+$ means the "instant after" the switch is pushed.

i.e.. <u>RC:</u> $V_{capacitor}(t=0^-) = V_{capacitor}(t=0^+)$

<u>RL:</u> $i_{inductor}(t=0^-) = i_{inductor}(t=0^+)$

$v_o(0^+)$

<u>Sol:</u> $V_o(t) = V_{o\,final} + (V_{o\,initial} - V_{o\,final})\, e^{-t/\tau}$

$\tau = \frac{L}{R}$ ← unknown.

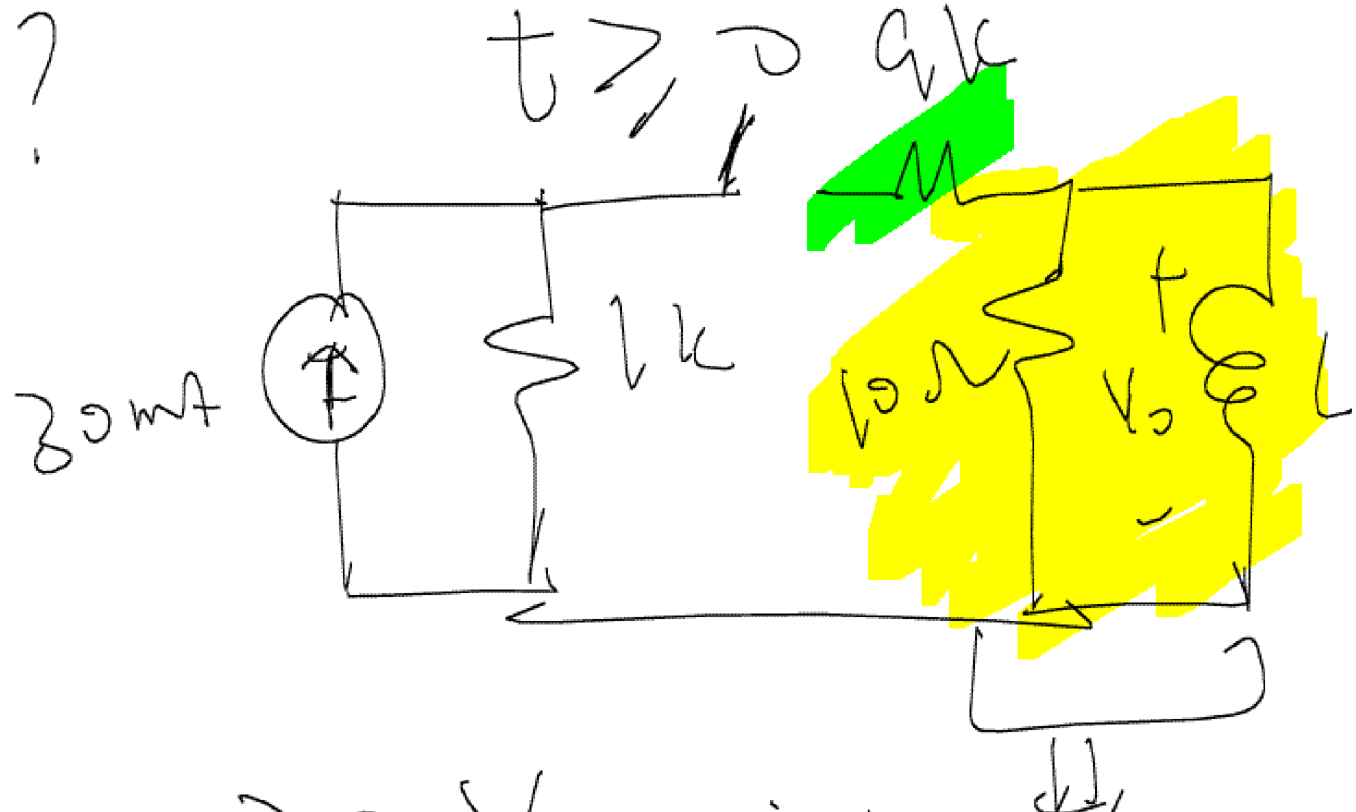
Given:

$$v_o(t = 1\,\text{ms}) = \frac{1}{2} V_o(0^+)$$

$$V_o(t = 1\,\text{ms}) = \frac{1}{2} V_{o\,\text{initial}}$$

$$\Rightarrow V_{o\,\text{final}} + \left(V_{o\,\text{initial}} - V_{o\,\text{final}}\right) e^{-t/\tau} = \frac{1}{2} V_{o\,\text{initial}}$$

$V_{o\,final} = ?$



$\therefore V_{o\,final} \rightarrow 0\,V$

$\therefore V_o(t) = 0 + (V_{o\,initial} - 0)e^{-t/\tau}$

inductor fully discharges

$$\therefore \cancel{V_{o\,initial}}\, e^{-\frac{t}{\tau}\big|_{t=1ms}} = \frac{1}{2}\underbrace{\cancel{V_{o\,initial}}}_{\parallel \atop V_o(0^+)}$$

$$\Rightarrow -t/\tau = \ln\left[\frac{1}{2}\right]$$

$$-\frac{t=1ms}{L/R_{Th}} = -0.693$$

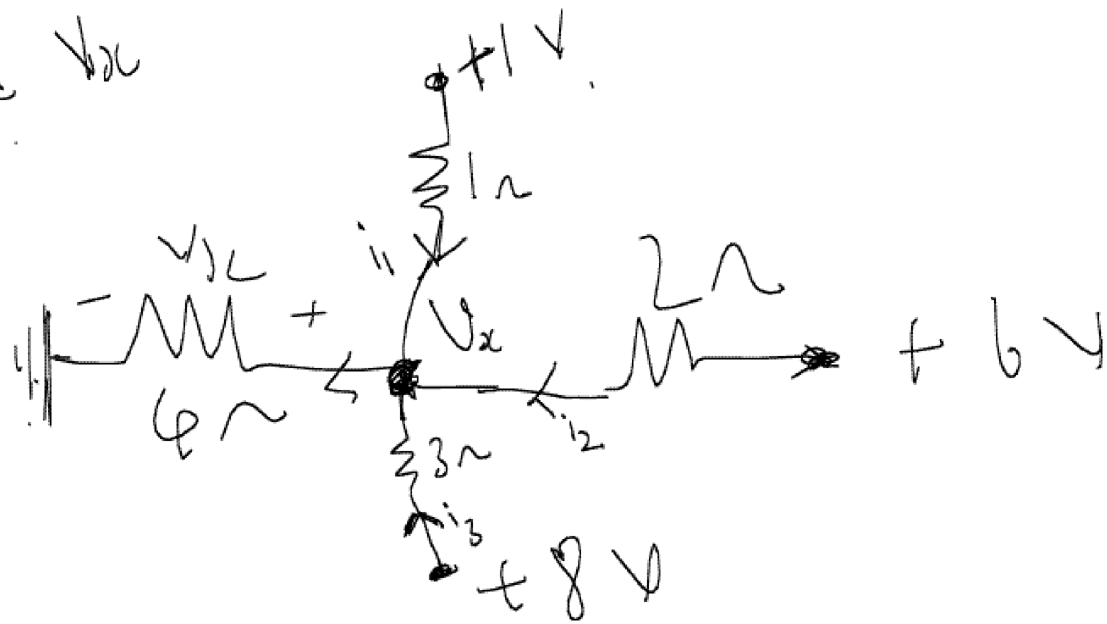
$$\Rightarrow \quad \frac{-1\,ms}{\frac{L}{10\,\Omega}} = -0.693$$

$$\Rightarrow \quad \frac{(-1\,ms)(10\,\Omega)}{L} = -0.693$$

$$\Rightarrow \boxed{L = \frac{10}{0.693}\ mH}$$

Midterm 1 spoiler (sorry TAs ☺)

(Q:) find V_x



KCL: $i_1 + i_2 + i_3 = i_4$

$$\Rightarrow \boxed{\frac{5 - V_{oc}}{1\,\Omega} + \frac{6 - V_{oc}}{2} + \frac{8 - V_{oc}}{3} = \frac{V_{oc}}{4}}$$

Next time: → More RC/RL examples