Lecture 5 - Nodal analysis

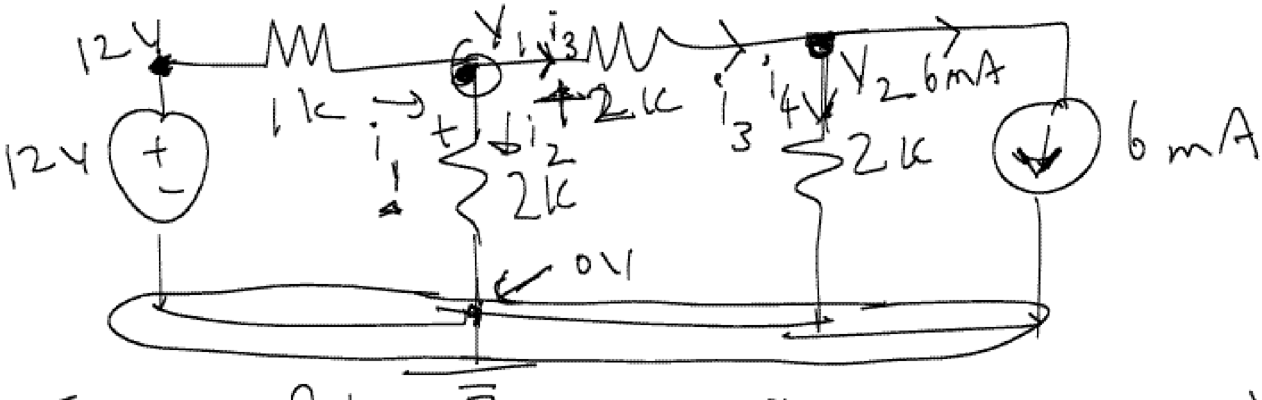
Administrivia
- → HW #2 change
- → HW #3 <u>big</u> change
- → Grades / HW #1 solutions up

<u>Today</u> → Chapter 4: Node voltage method ⟺ Nodal Analysis

Read section 4.1 → I won't be talking about essential nodes etc because you don't need it!

Goal of nodal analysis: Find unknown voltages in a circuit.

Consider the following circuit:



$$i_2 = \frac{V_1 - 0}{2k}$$

$$i_3 = \frac{V_1 - V_2}{2k}$$

(Q:) Solve circuit above using nodal analysis.

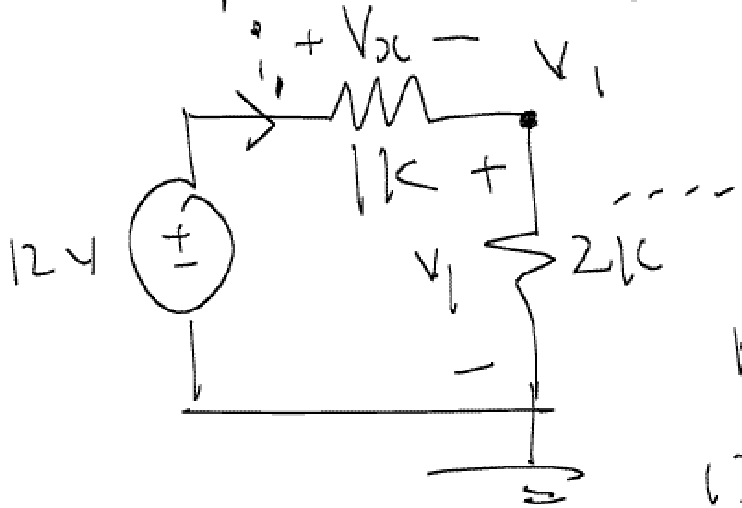
Sol: Step 1: Label/pick reference node!

Step 2: Label unknown node voltages

Step 3: Write KCL at each unknown node. : KCL @ V_1: $i_1 = i_2 + i_3$ —— (1)

KCL @ v_2: $i_3 = i_4 + 6\,mA$

Step 4: Rewrite KCL equation from Step 3 using unknown node voltages.



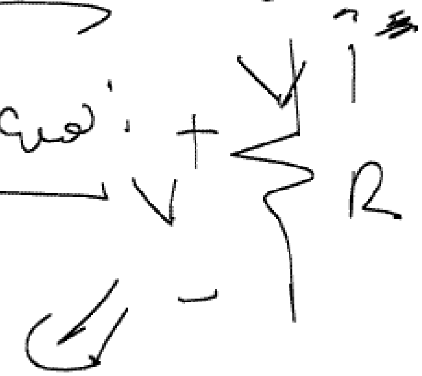
$$i_1 = \frac{V_x}{1K}$$

KVL:

$$12 - V_x - V_1 = 0$$

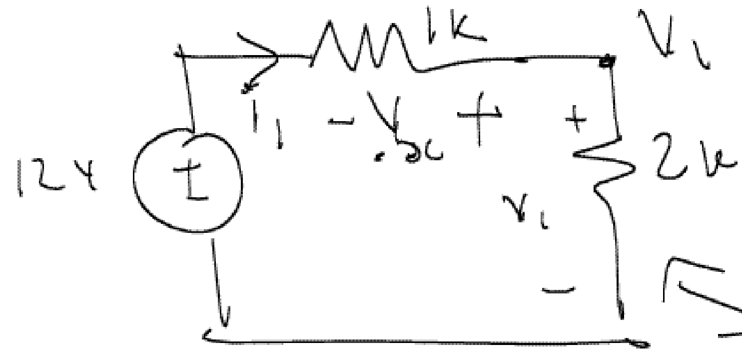
$$i_1 = \frac{12 - V_1}{1K} \Rightarrow V_x = 12 - V_1$$

Recall Ohm's law:



$$v = iR$$

Suppose:



$i_1 = \frac{-V_x}{1k}$

~~$i_1 = \frac{V_x}{1k}$~~

KVL: $12 + V_x - V_1 = 0$

$\Rightarrow V_x = V_1 - 12$

$\therefore i_1 = -\frac{(V_1 - 12)}{1k}$

$i_1 = \frac{12 - V_1}{1k}$

Similarly:

$i_1 = \frac{12 - V_1}{1k}$, $i_2 = \frac{V_1}{2k}$, $i_3 = \frac{V_1 - V_2}{2k}$, $i_4 = \frac{V_2}{2k}$

$$i_1 = i_2 + i_3 \Rightarrow \frac{12 - V_1}{1k} = \frac{V_1}{2k} + \frac{V_1 - V_2}{2k} \rightarrow (3)$$

$$i_3 = i_4 + 6mA \Rightarrow \frac{V_1 - V_2}{2k} = \frac{V_2}{2k} + 6mA \rightarrow (4)$$

<u>Note</u>: On the test, you will be responsible only for <u>2x2</u> systems

$$(3) \Rightarrow \frac{12}{1k} = \frac{V_1}{1k} + \frac{V_1}{2k} + \frac{V_1}{2k} - \frac{V_2}{2k}$$

(4) $\Rightarrow$ $6\,mA = \frac{V_1}{2k} + \frac{-V_2}{2k} - \frac{V_2}{2k}$

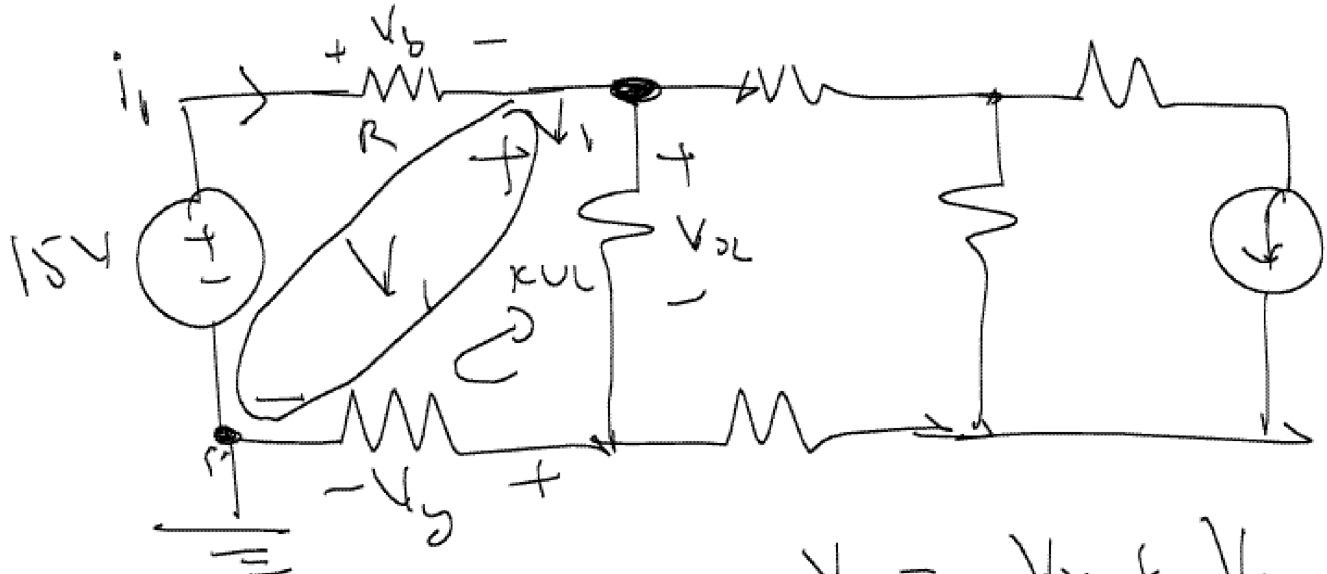
$$\Rightarrow \begin{bmatrix} \frac{1}{1k} + \frac{1}{2k} + \frac{1}{2k} & -\frac{1}{2k} \\ \frac{1}{2k} & -\frac{1}{2k} - \frac{1}{2k} \end{bmatrix}_{2\times2} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 12\,mA \\ 6\,mA \end{bmatrix}$$

↓ solved in SPICE

(Q:) what is the difference between

Voltage at a node & voltage across an element.

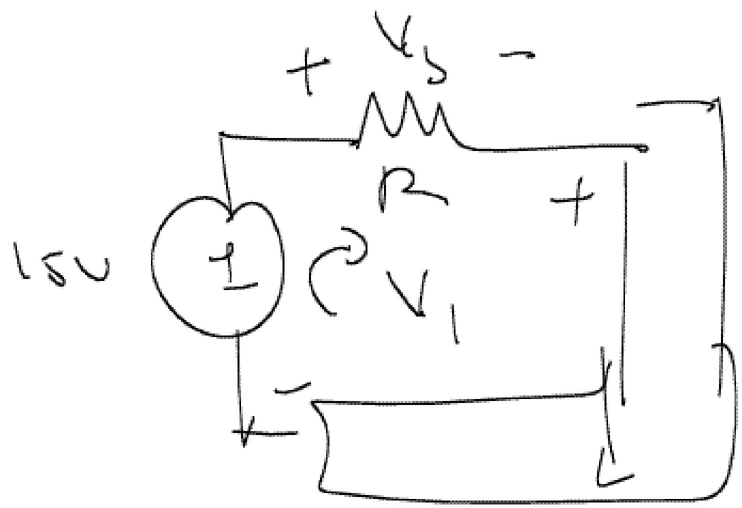
Answer with an example!



$V_1 = V_{3L} + V_y$

Hard to understand

$$i_1 = \frac{V_b}{R} = \frac{15 - V_1}{R}$$



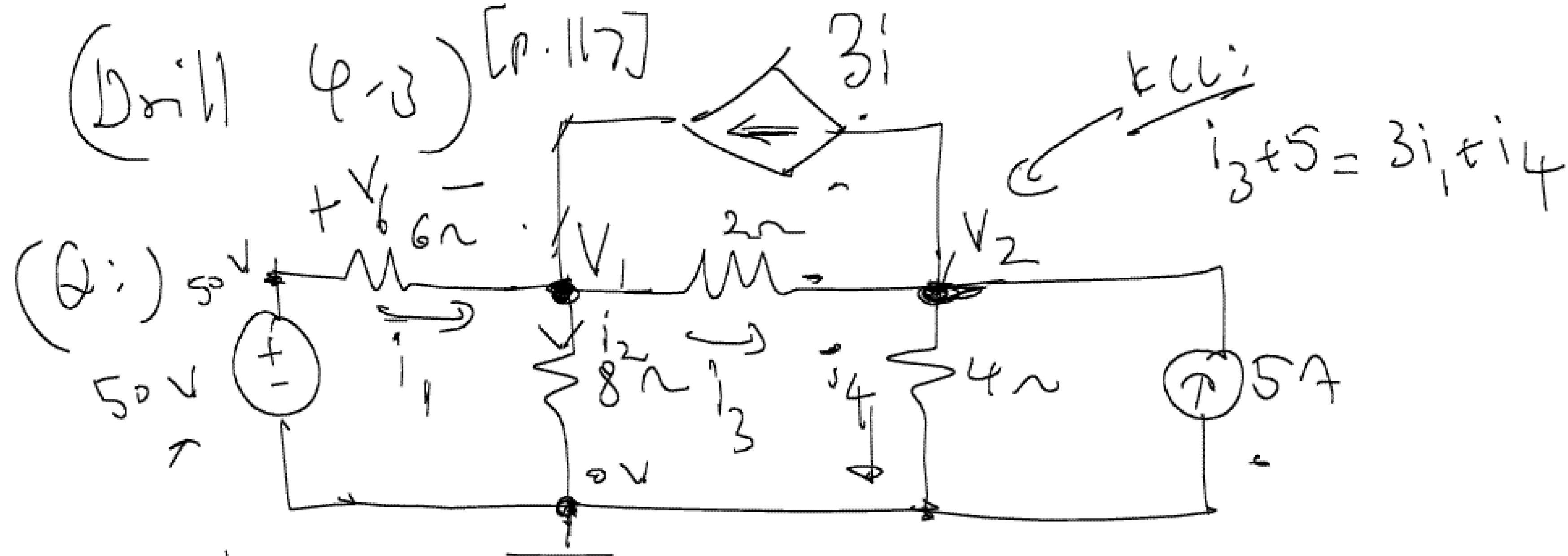
KVL: $15 - V_b - V_1 = 0$

$\Rightarrow V_b = 15 - V_1$

Section 4.3 - Node voltage method

& dependent sources

(Drill 4-3) [p. 117]



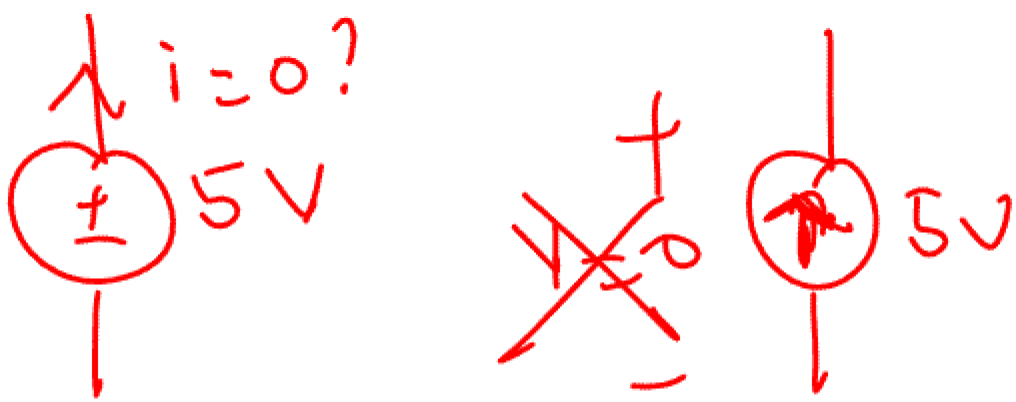
(a) Find power associated with each source

(b) State if each source is absorbing or delivering power.

KCL @ V_1: $i_1 + 3i_1 = i_2 + i_3$

Note: DO NOT ASSUME VOLTAGE ACROSS CURRENT SOURCE IS ZERO!

Analogy:



KCL @ V_2: $i_3 + 5 = 3i_1 + i_4$

$$i_1 = \frac{50 - v_1}{6}, \quad i_2 = \frac{v_1}{8}$$

$$i_3 = \frac{v_1 - v_2}{2} \quad i_4 = \frac{v_2}{4}$$

V_1:
$$4\left(\frac{50 - V_1}{6}\right) = \frac{V_1}{8} + \frac{V_1 - V_2}{2}$$

V_2:
$$\frac{V_1 - V_2}{2} + 5 = 3\left[\frac{50 - V_1}{6}\right] + \frac{V_2}{4}$$

$\underline{v_1:}$ $\frac{2}{3}(50 - v_1) = \frac{v_1}{8} + \frac{v_1 - v_2}{2}$

$\Rightarrow 16(50 - v_1) = 3v_1 + 12(v_1 - v_2)$

$\Rightarrow 800 - 16v_1 = 3v_1 + 12v_1 - 12v_2$

$\Rightarrow 800 = 31v_1 - 12v_2$

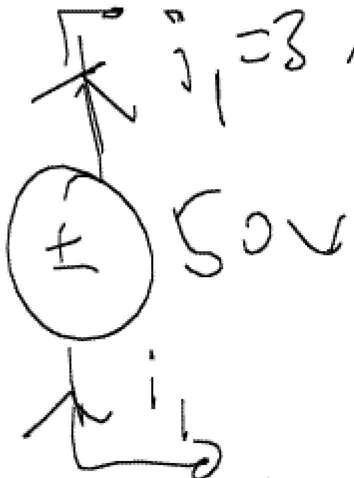
V_2:
$$\frac{V_1 - V_2}{2} + 5 = \frac{50 - V_1}{2} + \frac{V_2}{4}$$

$$\Rightarrow 2(V_1 - V_2) + 20 = 2(50 - V_1) + V_2$$

$$\Rightarrow 2V_1 - 2V_2 + 20 = 100 - 2V_1 + V_2$$

$$\Rightarrow 4V_1 - 3V_2 = 80$$

$$\boxed{V_1 = 32\ V, \quad V_2 = 16\ V}$$

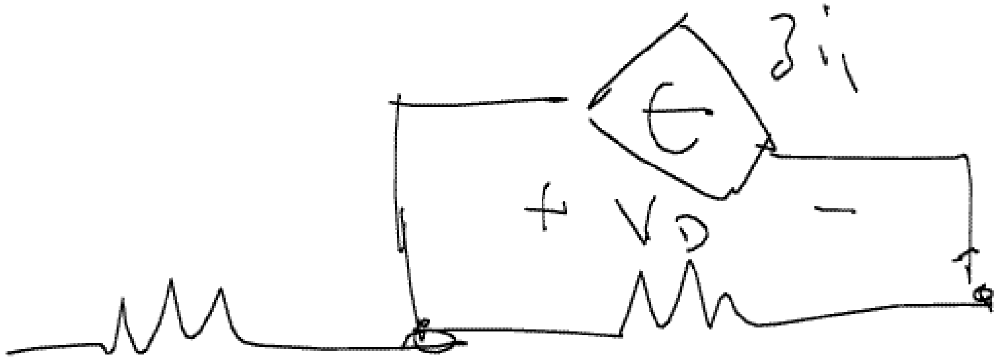


$$i_1 = \frac{50 - v_1}{6} = \frac{50 - 32}{6}$$

$$= 3 \text{ A}$$

$$P_{50v} = -vi = -150 \text{ W}$$

150 W delivered



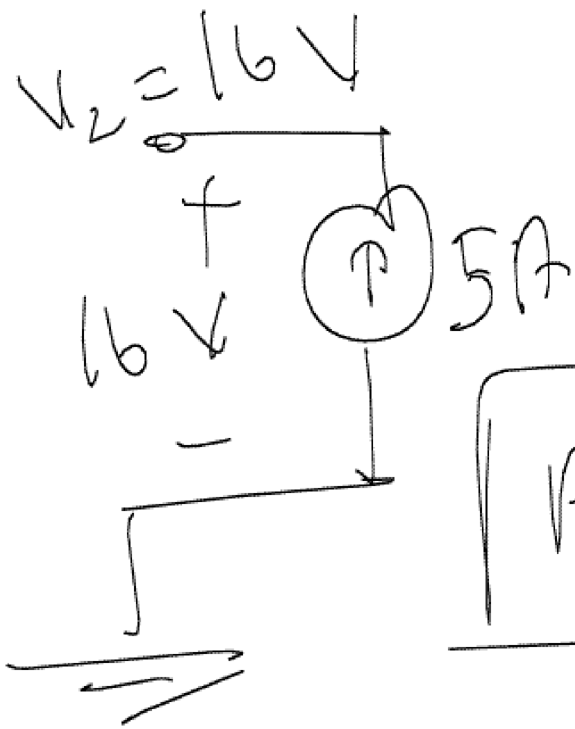
$v_2 = 16\,V$

$V_1 = 32\,V$

$v_D = 16\,V \quad \therefore P = -v_D(3i_1)$

$= -(16)(9)$

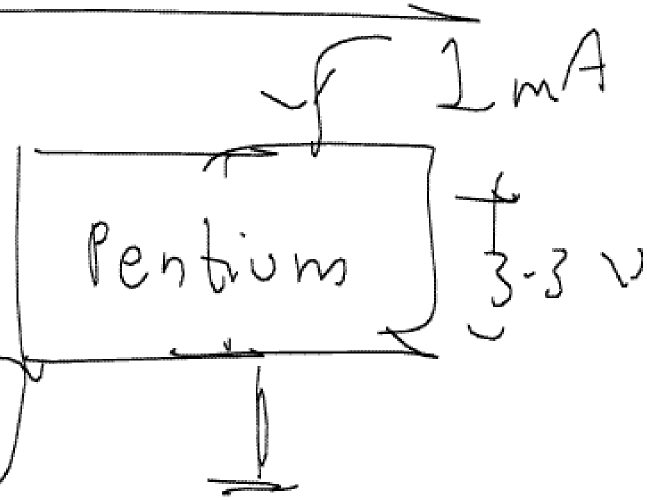
$$P_{3i_1} = -144\,W$$



$$P = -(16)(5)$$

$$P_{5A} = -80\text{ W}$$

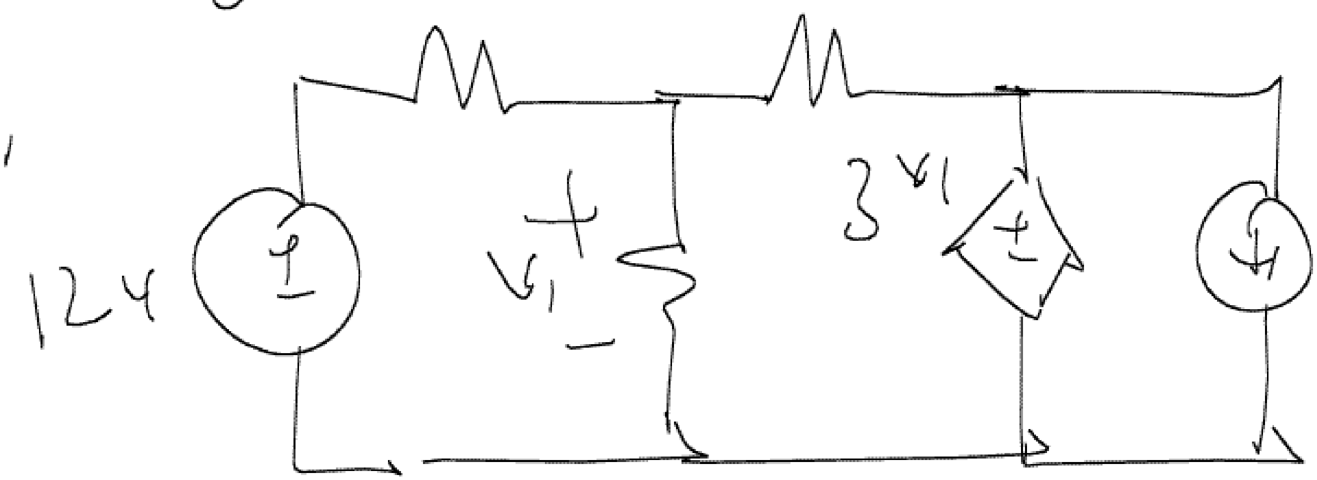
<u>Note:</u> $P = vi$



$$p = +3.3\text{ mW}$$

Note: Dependent sources in PSPICE ⇒ check "analog" library.

Eg:



(note: Check on webpage for schematic)

Next time;

- Justin will lecture
- More nodal analysis
- Overview of Mesh Analysis.