Lecture 9 - Finish chapter 4

Administrivia - (1) No labs this week (02/14) & next week (02/21), but TAs will be in lab to answer questions*

(2) Grades online now arranged by increasing SID nos.

Don't do part (b) in 4-58 in HW#4

Today: Thevenin / Norton; Maximum power transfer — Finish chapter 4 & Superposition

Not responsible for mesh analysis

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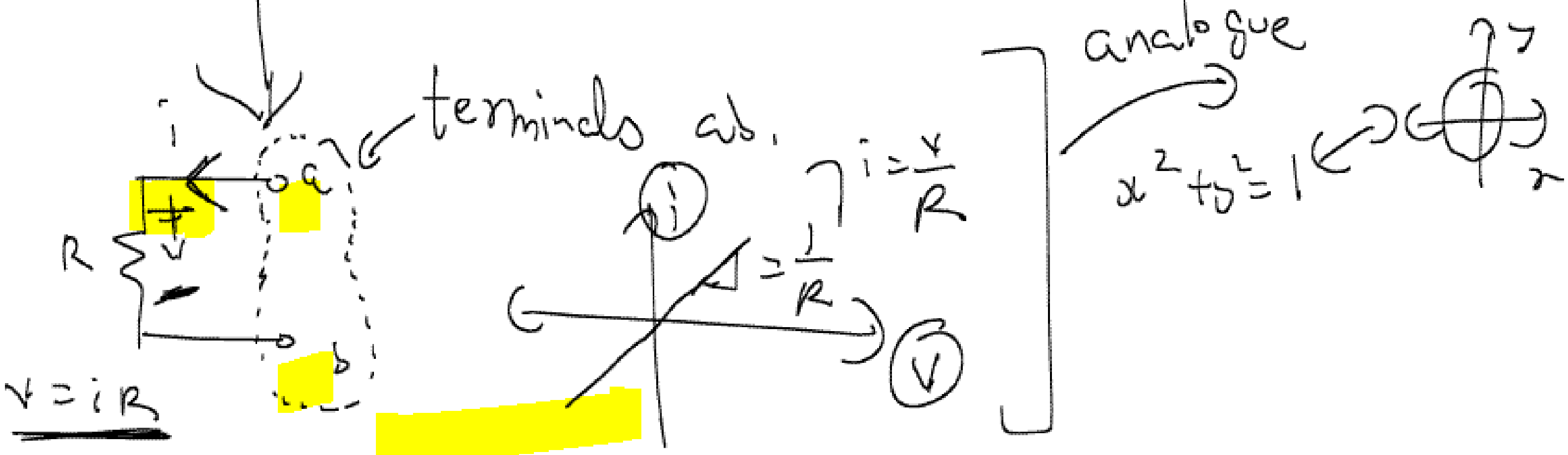
* Jaleh does not have lab tonight

Thevenin's theorem: Statement unimportant,

French dude

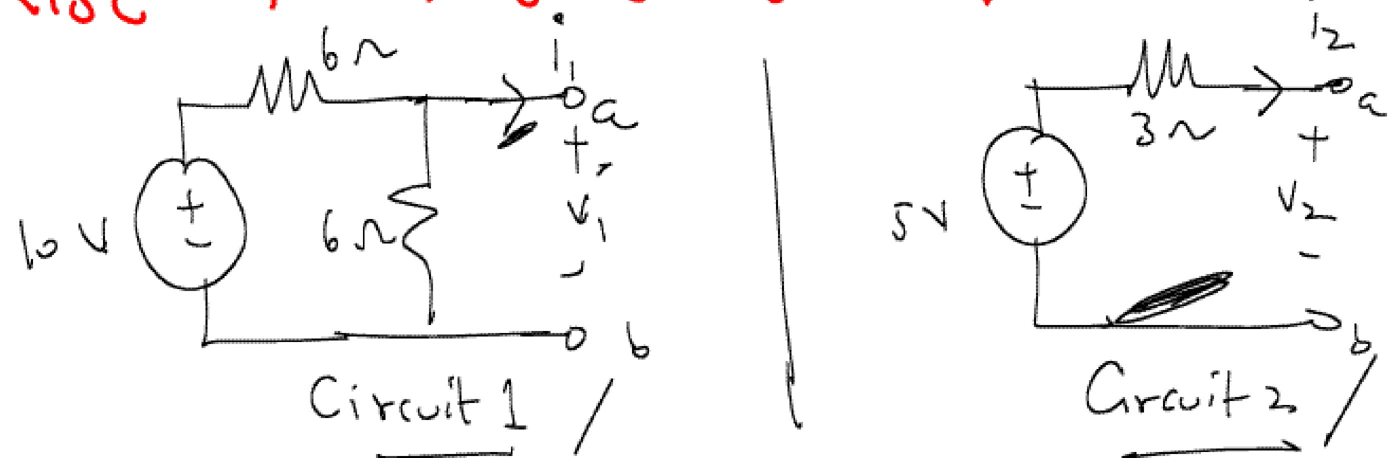
Concept: EQUIVALENT CIRCUIT



Thevenin noticed something very interesting: If you have a linear circuit, then the i-v graph will be a straight line. Now, you need two point to plot a straight line. **KEY CONCEPT:**

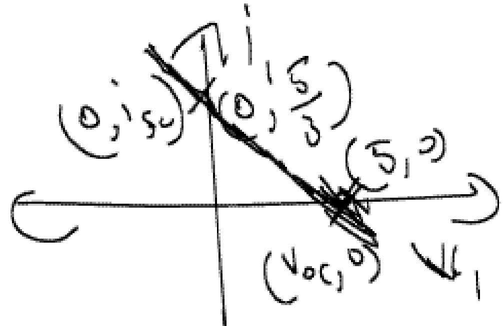
TWO DIFFERENT LINEAR CIRCUITS CAN GIVE RISE TO THE SAME I-V GRAPH!

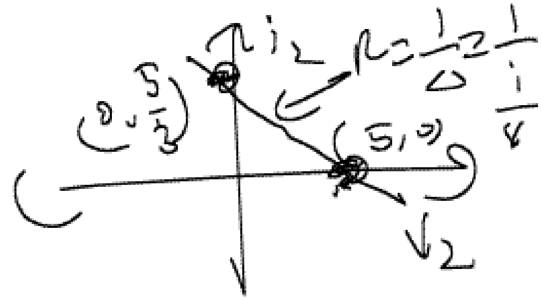
Eg 1:



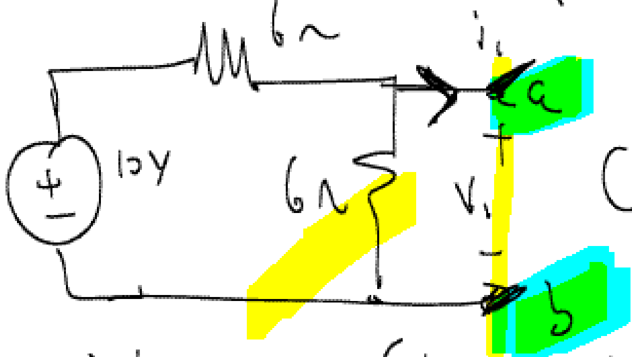
Turns out i vs v graph for both circuits at terminals ab are the same!

Mathematically:

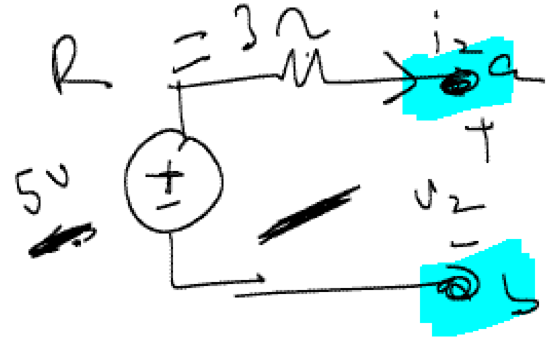




intercepts help plot straight line(s) easily!



Circuit 1

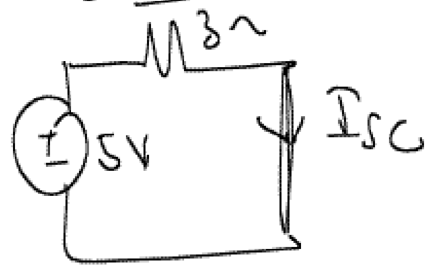


Circuit 2

x-intercept: $(V_{oc}, 0) \rightarrow$ open circuit

y-intercept: $(0, i_{sc}) \rightarrow$ short circuit

$V_{oc} = 5\ V$



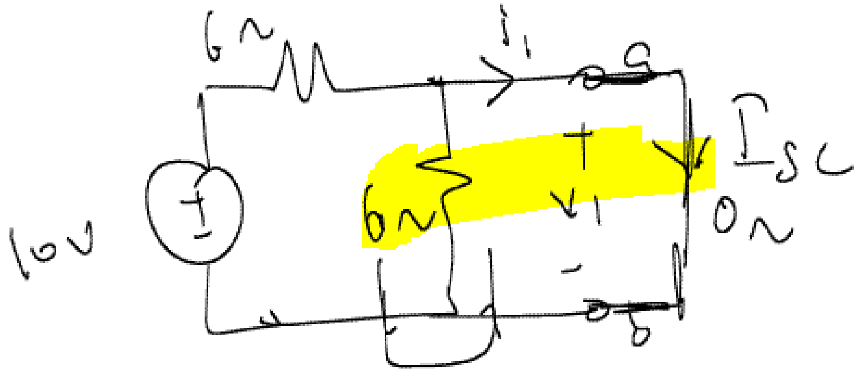
$I_{sc} = \frac{5}{3} A$

V_{oc}:

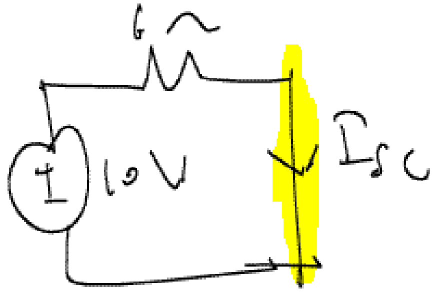
$$V_{oc} = \left(\frac{6}{6+6}\right) 10 \quad \text{[voltage divider]}$$

$$= 5\,V$$

I_{sc}:



ignore $(0\Omega \,\|\, 6\Omega = 0\Omega)$



$$I_{sc} = \frac{10}{6} = \frac{5}{3}\ A$$

Note: $V_{R_2} = V_{oc}$

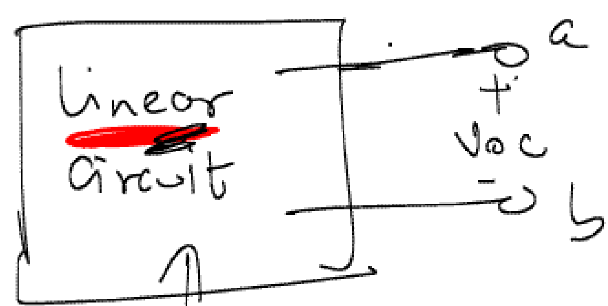
How is R related to V_{oc} & I_{sc}?

$$R = \frac{V_{oc}}{I_{sc}} = \frac{5}{\frac{5}{3}} = 3\Omega$$

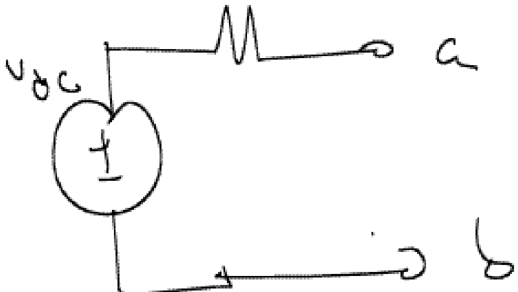
$$\frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{0}$$

$$\Rightarrow \frac{1}{R_{eq}} = \infty \Rightarrow R_{eq} = 0\Omega$$

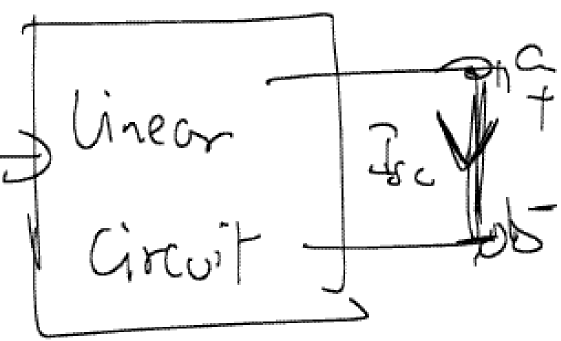
Point of Thevenin:



$$R_{Th} = \frac{V_{oc}}{I_{sc}}$$

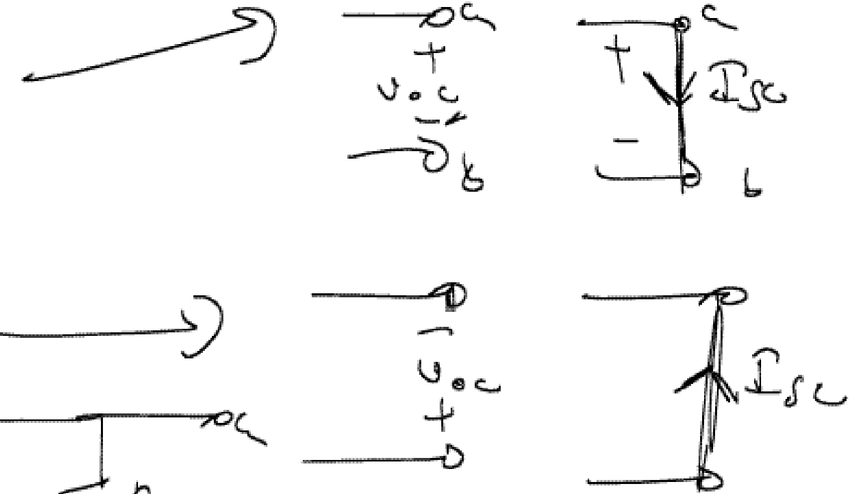


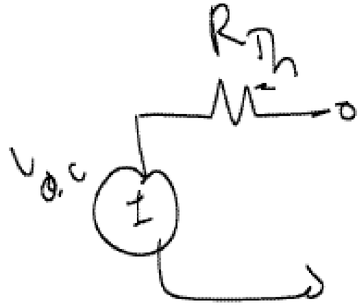
Thevenin same

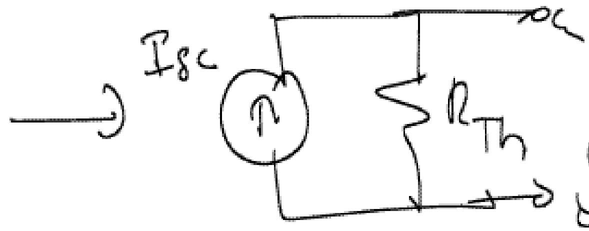


Thevenin equivalent

Note: $R_{Th} > 0$

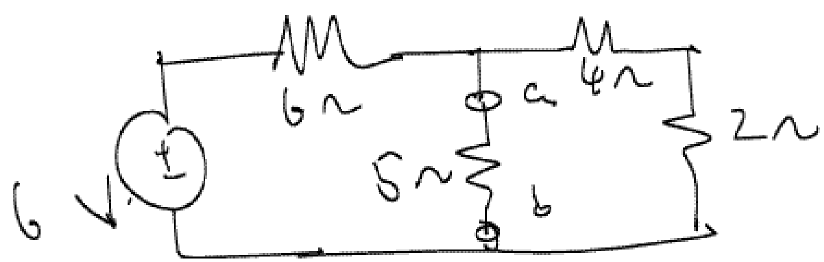




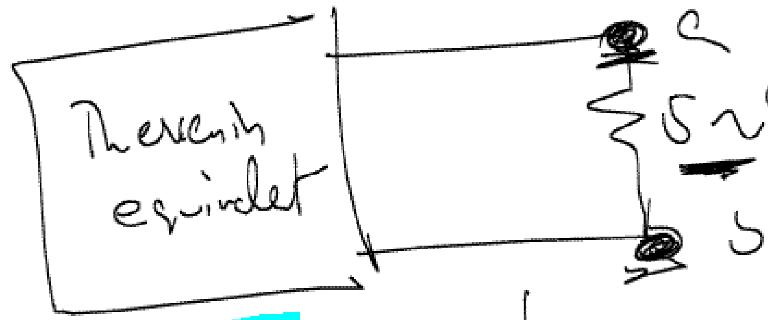


← Norton equivalent

Eg: Find thevenin & norton equivalent at ab

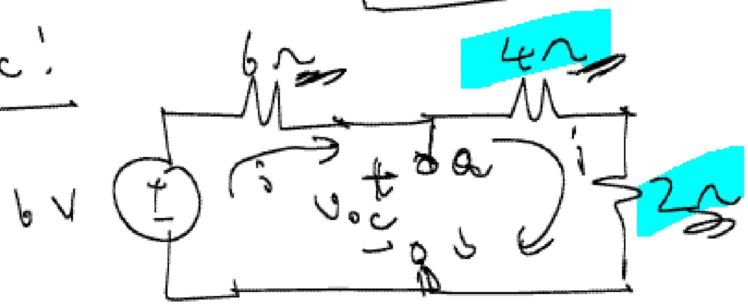


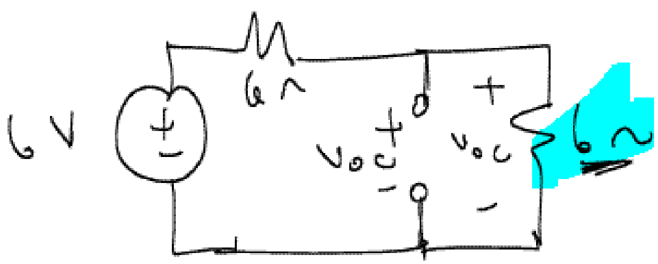
Solution:



Don't include resistence in calculation

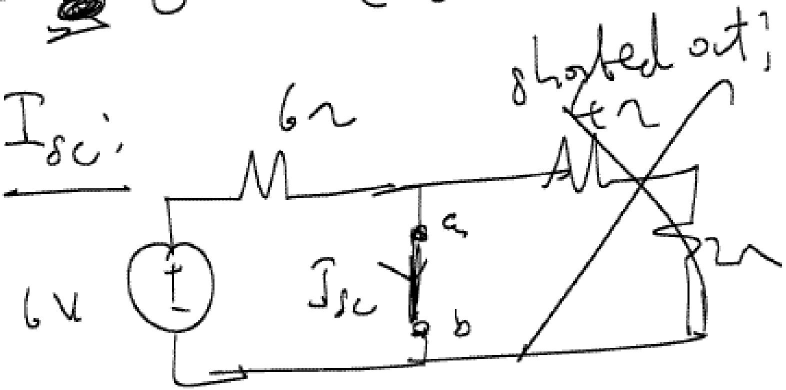
Voc:





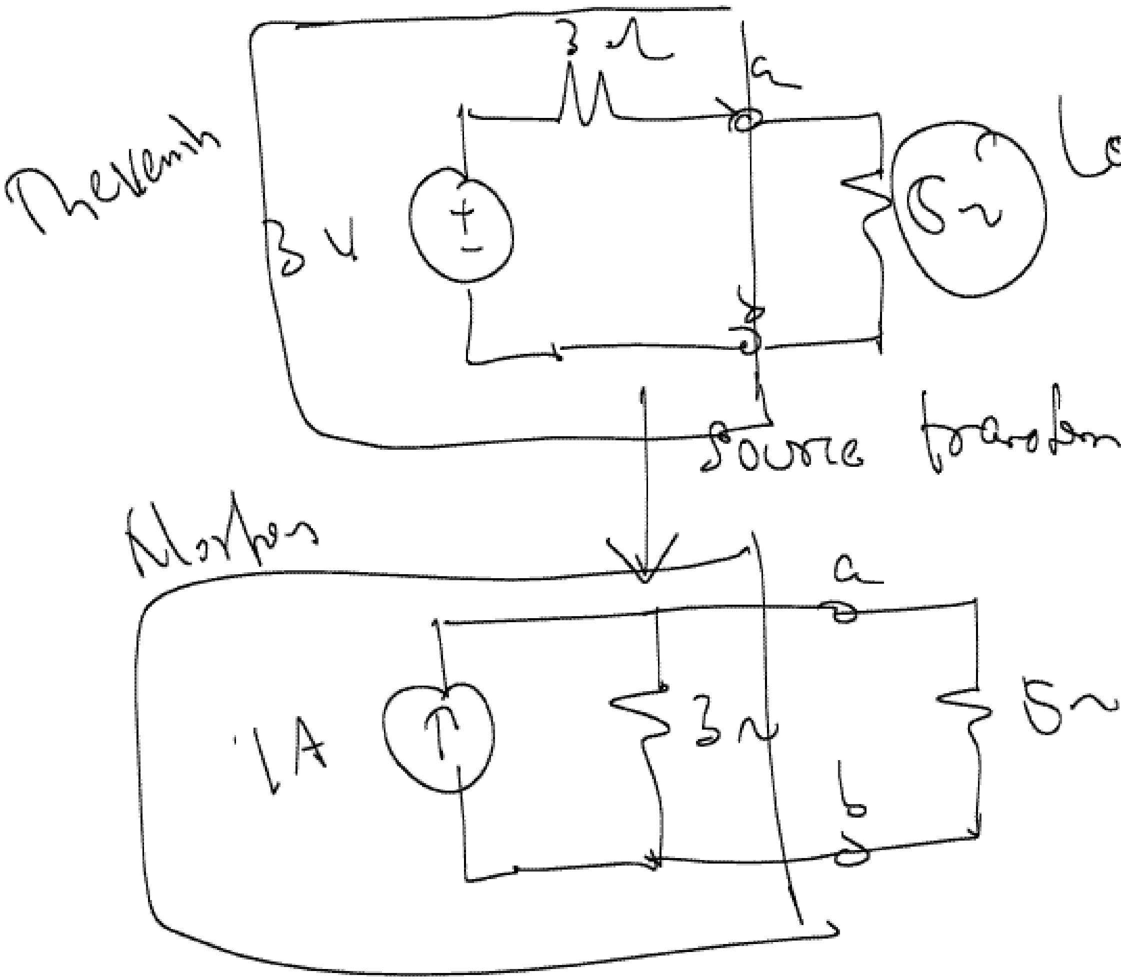
$$V_{oc} = \left(\frac{6\Omega}{6\Omega + 6\Omega}\right) 6V = 3\ V$$

Isc:



$$I_{sc} = 1\ A$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = 3\ \Omega$$



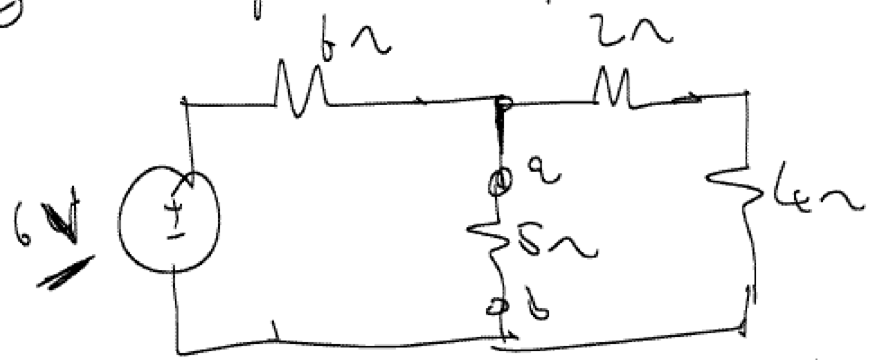
Load!
⇓
usually nonlinear
↓
so you don't include in Thevenin

Note: Two shortcuts to Thevenin!

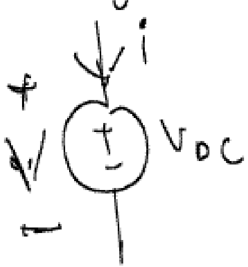
Shortcut 1: If a circuit has **only** independent sources & resistors (i.e. no dependent source)

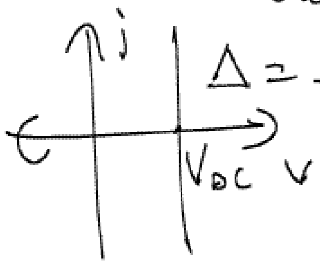
Shortcut: Easily compute R_{Th}

Previous example:



idea: get a "resistance" for this source



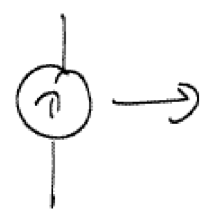


$\Delta = \frac{i}{v}$, $R = \frac{1}{\Delta}$

$\Delta \to \infty$, $R \to 0$

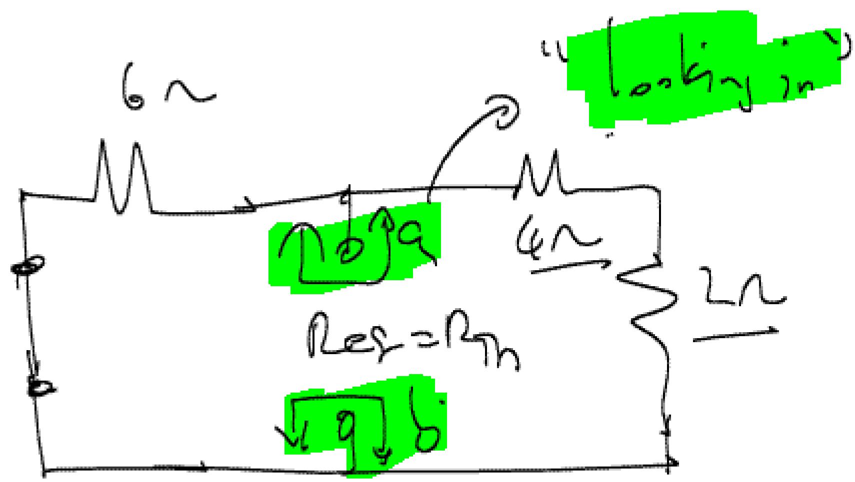
replace voltage sources with short circuit!

Similarly



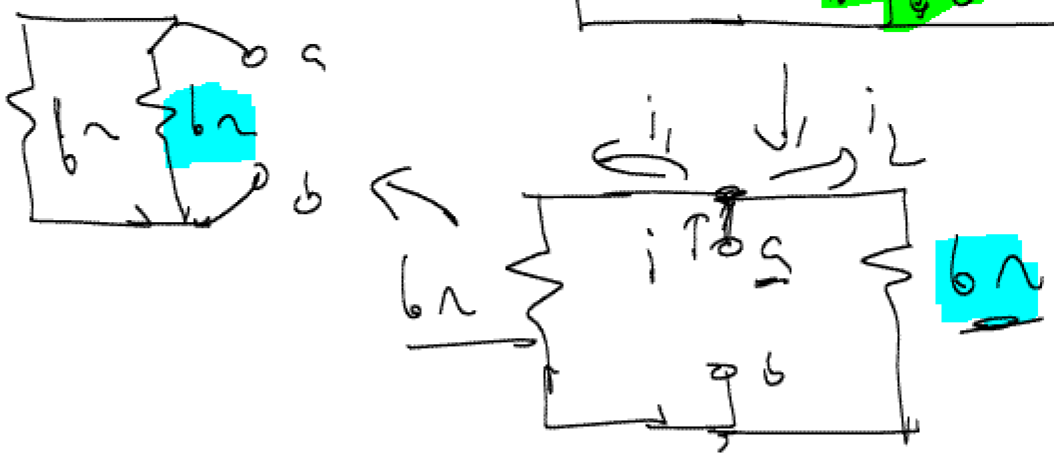
Note! This will not work for dependent sources!

Therefore:



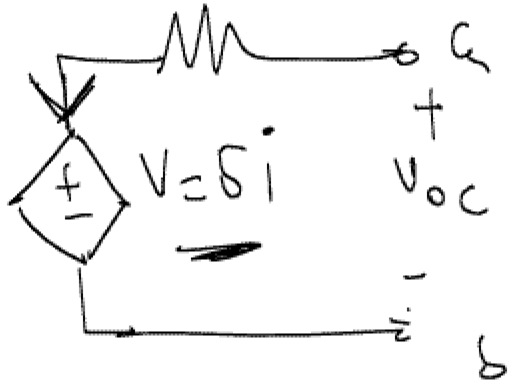
$$R_{Th} = 6\Omega \parallel 6\Omega$$

$$= 3\Omega$$

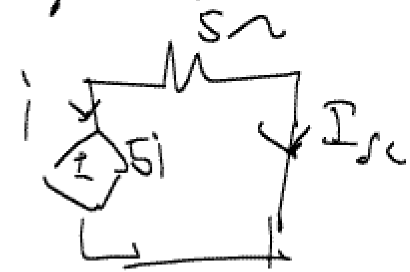


Shortcut 2: A circuit with only dependent sources.

Eg:



Find thevenin equivalent at ab



Voc? i = 0 [open circuit]

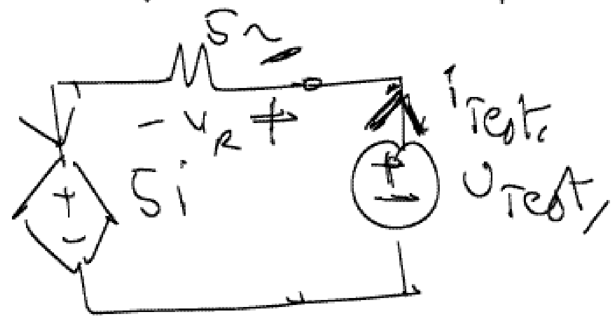
$V_{oc} = 0\ V$

$I_{sc} = i = 0\ A$

$R_{Th} = \frac{V_{oc}}{I_{sc}}$ → undefined

Therefore, we have to apply a test voltage & use it to compute R_{Th}.

$\therefore$ $R_{Th} = 10\,\Omega$



$R_{Th} = \frac{V_{Test}}{i_{Test}}$

$V_R = 5\,i_{Test}$

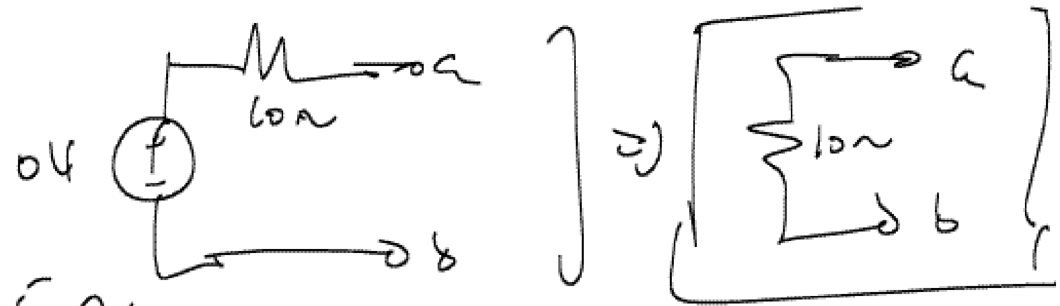
$V_{Test} = V_R + 5i$ [KVL]

$\therefore$ $V_{Test} = 10\,i_{Test}$ $\leftarrow$ $i = i_{Test}$

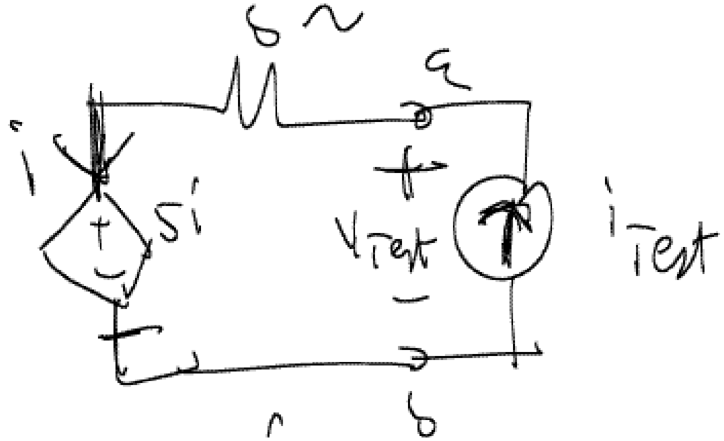
Note: (1) Direction of i_{Test}. Coming out of positive terminal because V_{Test} is powering the circuit!

(2) You can use $V_{Test} = 10\,V$ (or) any other number, except 0 V!

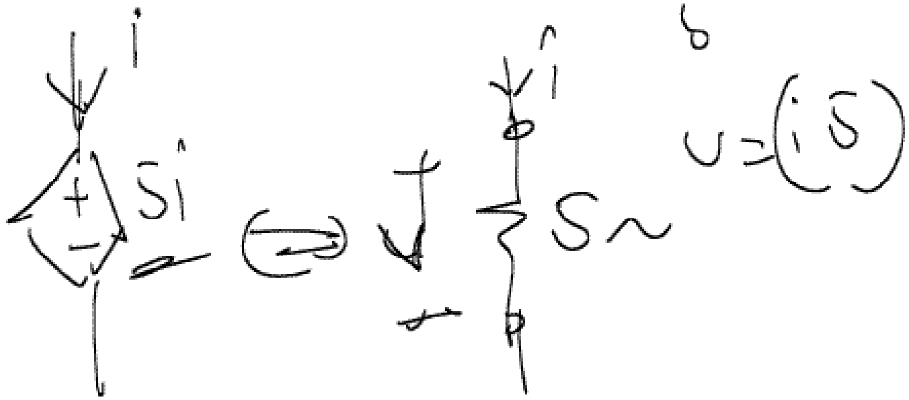
Thevenin equivalent:



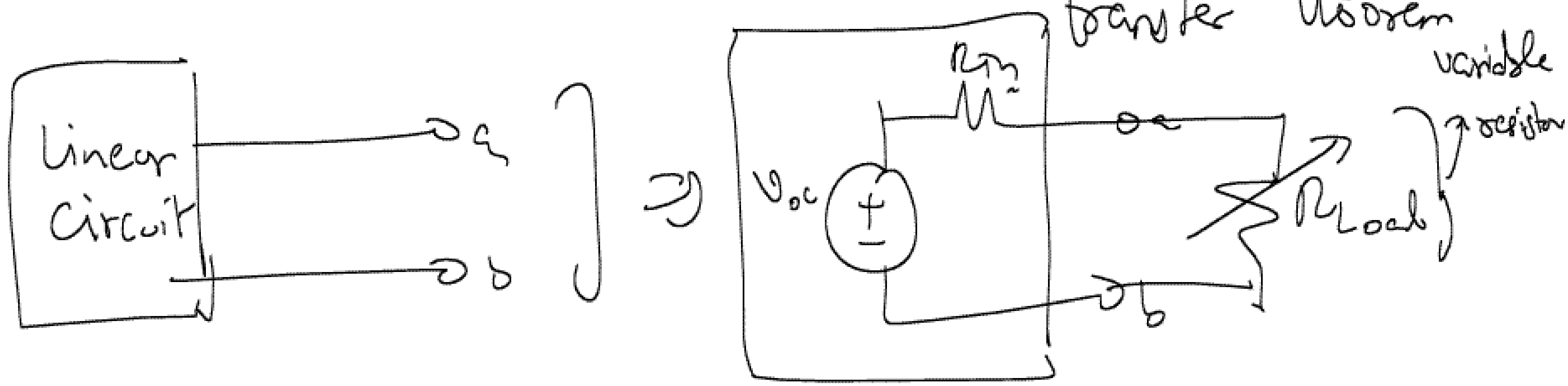
Note (2):



Note:



Last topic in chapter 4: Maximum power transfer theorem

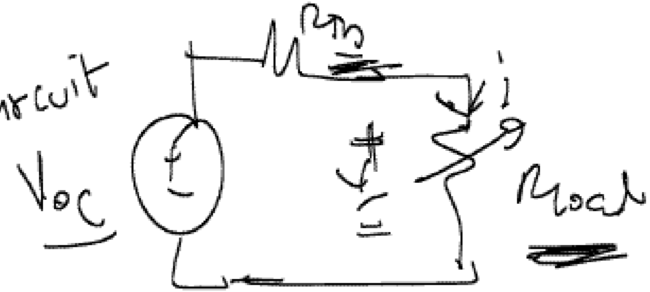


(Q:) What value of R_{Load} (in terms of R_{Th}) so maximum power is dissipated in R_{Load}?

Lets think for a little!

$P_{load} = vi$ (1) $R_{load} = 0 \Rightarrow$ short-circuit

doesn't make sense practically, no device with 0 Ω resistance



(2) $R_{load} \to \infty \Rightarrow$ open circuit

Turns out $\boxed{R_{load} = R_{th}}$ ⟵ MAXIMUM POWER TRANSFER THEOREM

Proof: Calculus: $P = vi = \left\{ \left(\dfrac{R_{load}}{R_{load} + R_{th}} \right) V_{oc} \right\} i$

We also know: $p(R_{load}) = \left(\frac{R_{load}}{R_{load}+R_{th}}\right) V_{oc} \cdot \frac{V}{R_{load}}$

$$p(R_{load}) = \left(\left(\frac{R_{load}}{R_{load}+R_{th}}\right) V_{oc}\right)^2 \frac{1}{R_{load}}$$

Notice: R_{th}, V_{oc} are known, so

$\frac{dp}{dR_{load}} = 0 \xrightarrow{\text{simplify}} \boxed{R_{th} = R_{load}}$

↳ Refer to chapter 4 for details on maths!

Note: Problem 4-86 (HW #5) is good
→ next week

Next time: Application of circuits
↓
RC circuits]