EE 100/42 Spring 2009
Solutions to Homework 6

1.71)
$v_x + 2v_x = 18V$
$v_x = 6V$
$i_x = 6V/2\ \Omega = 3A$
$i_z = 12V/12\ \Omega = 1A$ (i_z defined as flowing down from top node)
$i_y = 3A - 1A = 2A$

2.56)
Using KCL at v_1:

$$5A = \frac{v_1}{10} + \frac{v_1 - v_2}{16} - i_z$$

Using KCL at v_2:

$$3A = \frac{v_2}{20} + \frac{v_2 - v_1}{16} + i_z$$

$v_1 - v_2 = 5i_x = 5(v_1/10) = v_1/2$
Plug in, solve, $v_1 = 64V$
$v_2 = v_1/2 = 32V$
Power at 16 Ω resistor = $(v_1 - v_2)^2/16 = 32^2/16 = 64W$

2.58)
Let the left top node be v_1, right top node be v_2. Use KCL:

$$1A = \frac{v_1}{20} + \frac{v_x}{10}$$

$$0 = \frac{v_2}{5} - \frac{v_x}{10} + \frac{v_x}{5}$$

$v_2 = v_1 - v_x$, plug into #2:
$2v_1 = v_x$
Plug into #1:

$$1A = \frac{v_1}{20} + \frac{2v_1}{10} = \frac{v_1}{20} + \frac{4v_1}{20} = \frac{v_1}{4}$$

$v_1 = 4V$
$R = 4V/1A = 4\ \Omega$

5.54)

$$I_2(-j15 + j10) + 10\angle 180° = 0 \longrightarrow I_2 = j2$$

$$I_1(5 + j15) = 20\angle 0° \longrightarrow I_1 = \frac{2}{5} - \frac{j6}{5}$$

$v_1 = j15(I_1 - I_2) = (6j+18)+30 = 48+6j = 6\sqrt{65}\angle 7.125°$

5.76)

$$I = \frac{V}{Z} = \frac{240\sqrt{2}\angle 50° + 220\sqrt{2}\angle 30°}{1 + j2} = \frac{-51.27 + j104.44}{1 + j2} = \frac{116.35\angle 116.15°}{\sqrt{5}\angle 63.43°} = 52.03\angle 52.72°$$

Power $P = \frac{V_m I_m}{2}\cos(\theta)$

Source A: $P = \frac{240\sqrt{2} * -52.03}{2}\cos(2.72°)$ = -8819.84W

Source B: $P = \frac{220\sqrt{2} * -52.03}{2}\cos(22.72°)$ = 7465.90W

Source A is delivering energy; source B is absorbing energy.

5.89)

a)

Z_{th} = 100+j50

$V_{oc} = 200\angle 0°$

$I_{sc} = V_{oc}/Z_{th} = \frac{200\angle 0°}{50\sqrt{5}\angle 26.57°} = \frac{4}{\sqrt{5}}\angle -13.28°$

Set up Thevenin and Norton equivalent circuits as usual, with Z_{th} in place of R_{th}.

b)

For maximum power with reactive loads, make the load's impedance the complex conjugate of Z_{th}.

Z_{load} = 100-j50 Ω

$$I = \frac{200\angle 0°}{(100 + j50) + (100 - j50)} = \frac{200\angle 0°}{200} = 1\angle 0°$$

$$I_{rms} = \frac{I}{\sqrt{2}}$$

$$P = I_{rms}^2 R = \left(\frac{1}{\sqrt{2}}\right)^2 100 = 50W$$

c)

For maximum power with resistive loads, make the load's resistance the amplitude of Z_{th}.

$R_{load} = \sqrt{100^2 + 50^2} = 50\sqrt{5}$ Ω

$$I = \frac{200\angle 0°}{(100 + j50) + (50\sqrt{5})} = \frac{200\angle 0°}{217\angle 13.28°} = 0.92\angle -13.28°$$

$$I_{rms} = \frac{I}{\sqrt{2}}$$

$$P = I_{rms}^2 R = \left(\frac{0.92}{\sqrt{2}}\right)^2 50\sqrt{5} = 47.32W$$

6.42)

|H(f)|	0.5	2	$1/\sqrt{2}$	$\sqrt{2}$
|H(f)|(db)	-6	6	-3	3

6.53)
Treat the circuit like a voltage divider:

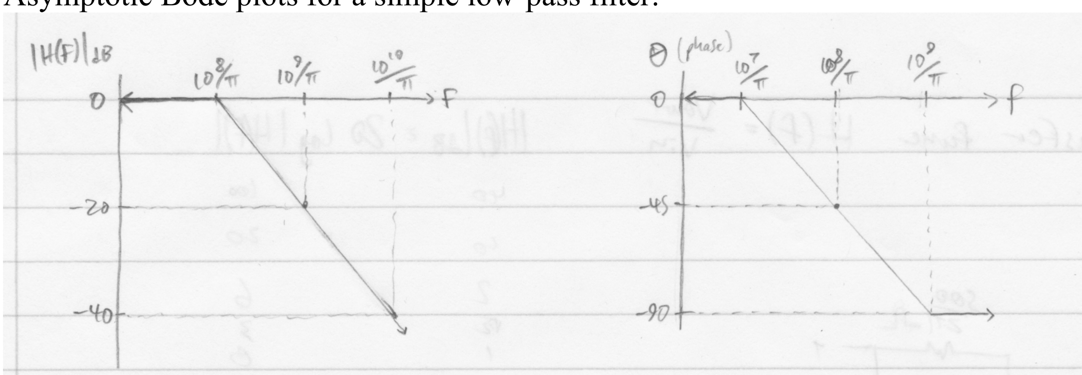
$$V_{out} = V_{in} \frac{1/j\omega 10^{-10}}{50 + 1/j\omega 10^{-10}} = V_{in} \frac{1}{1 + \frac{j50\omega}{10^{10}}} = V_{in} \frac{1}{1 + \frac{j\pi f}{10^{8}}}$$

$(2\pi f = \omega)$

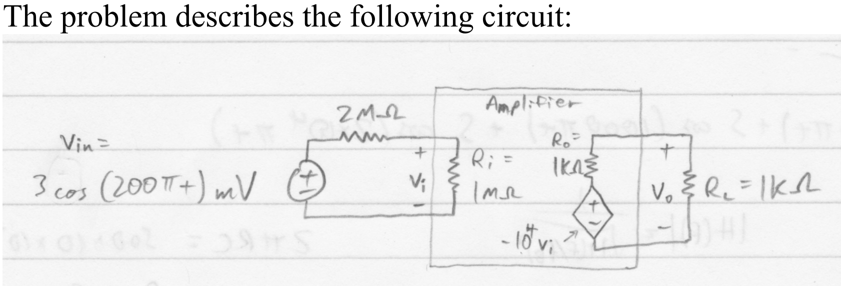
$$\text{H(f)} = \frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{j\pi f}{10^{6}}}$$

corner frequency (break frequency): $\frac{j\pi f_B}{10^8} = j \Rightarrow f_B = \frac{10^8}{\pi}$

Asymptotic Bode plots for a simple low-pass filter:



11.7)
The problem describes the following circuit:



$$V_i = V_{in} \tfrac{1M\Omega}{1M\Omega + 2M\Omega} = 1\cos(200\pi t)\text{ mV}$$

$$V_o = (-10^4)V_i \tfrac{1K\Omega}{1K\Omega + 1K\Omega} = (-10^4)0.5\cos(200\pi t)\text{ mV} = \text{-}5\cos(200\pi t)\text{ V}$$

$$G = \frac{P_o}{P_i} = \frac{V_o^2/R_L}{V_i^2/R_i} = \left(\frac{V_o}{V_i}\right)^2 \frac{R_i}{R_L} = \left(\frac{-5\cos(200\pi t)\,V}{1\cos(200\pi t)\,mV}\right)^2 \frac{1M\Omega}{1K\Omega} = 25 \times 10^9$$

A.4)

Complex number	Polar form	Exponential form
5-j5	$\sqrt{50}\angle -45°$	$\sqrt{50}e^{-j45°} = \sqrt{50}e^{-j0.79}$
-10+j5	$25\sqrt{5}\angle 153.43°$	$25\sqrt{5}e^{j153.43°} = 25\sqrt{5}e^{j2.68}$
-3-j4	$5\angle 233.13°$	$5e^{j233.13°} = 5e^{j4.07}$
-j12	$12\angle -90°$	$12e^{-j90°} = 12e^{-j\frac{\pi}{2}}$