EE 100/42 Spring 2009
Solutions to Midterm One

1. The average power may be found from the phasors $\mathbb{I}$ and $\mathbb{V}$. To find the average power *delivered*, we need to use the current flowing in the direction of increasing voltage; in this case, that current is represented by the phasor $-\mathbb{I}$. Now, it's simply a matter of using the formula for the average power in terms of phasors:

$$\begin{aligned}
P_{av} &= \frac{1}{2}\mathrm{Re}\left\{\mathbb{V}(-\bar{\mathbb{I}})\right\} \\
&= -\frac{1}{2}\mathrm{Re}\left\{5e^{j\frac{\pi}{2}}\overline{(3-4j)}\right\} \\
&= -\frac{1}{2}\mathrm{Re}\left\{5j(3+4j)\right\} \\
&= -\frac{1}{2}\mathrm{Re}\left\{-20+15j\right\} \\
&= 10 \text{ W}
\end{aligned}$$

   where the fact $e^{j\frac{\pi}{2}} = j$ was used in the third equality.

2. It **is not** possible to find the phasor of the signal $2\cos(2t) + 3\cos(3t)$.

   - We can only find phasors of signals with a single frequency.

   For a well-designed circuit with a practical voltage source, the internal resistance R of the source should be much **smaller** than the load resistance.

   - We want the voltage drop across the load resistance to be approximately the same as the voltage source. This can be seen from the voltage divider formula.

   For maximum power transfer from a practical voltage source to a resistive load, the internal resistance R should be much **equal to** the load resistance.

   - This is easily verified by circuit analysis.

   In a circuit driven by AC sources, the average power delivered to a capacitor is **zero**.

   - Capacitors do not dissipate energy, so the average power delivered is zero. Note that the average reactive power is non-zero.

   The internal resistance of a typical antenna source is $\mathbf{10^6}$ Ω.

   - Modeling the antenna source as a very weak, practical voltage source, a large internal resistance will ensure that the load voltage is not close to the antenna voltage unless it is buffered.

3. Since lightbulbs are basically resistors, we can answer the questions by first calculating the value of each resistance. This can be done using the power rating.

$$
\begin{aligned}
P &= iv \\
&= \frac{v^2}{R}
\end{aligned}
$$

The resistances may be found by $R = \frac{v^2}{P}$, where P is the power rating and $v = 100$ V.

$$
\begin{aligned}
R_{50\text{W}} &= 200\ \Omega \\
R_{100\text{W}} &= 100\ \Omega
\end{aligned}
$$

(a) When the lightbulbs are connected in series with a voltage source, the same amount of current flows through each one. To find the lightbulb that dissipates more energy, we simply need to find the one with the greater voltage drop across its terminals (this can be easily seen from $P = iv$, where i is the same for both bulbs). The voltage drops across each bulb may be found using the voltage divider formula,

$$
\begin{aligned}
v_{50\text{W}} &= V_S \frac{200}{200+100} \\
v_{100\text{W}} &= V_S \frac{100}{200+100}
\end{aligned}
$$

where V_S is the 100 V voltage source. Thus, **the 50 W bulb will grow brighter**.

(b) When the lightbulbs are connected in parallel with a voltage source, the voltage drop across both are the same. The lightbulb that dissipates more energy will therefore be the one with the greater amount of current flowing through it. The current flow through each one may be found using the current divider formula,

$$
\begin{aligned}
i_{50\text{W}} &= i_S \frac{100}{200+100} \\
i_{100\text{W}} &= i_S \frac{200}{200+100}
\end{aligned}
$$

where i_S is the current supplied by the voltage source. It can be seen that **the 100 W bulb will grow brighter**.

4. (a) The Thevenin voltage is equal to the open circuit voltage when the berkelistor is removed, in the direction of positive v (see Figure 1. This may be found using the nodal method. Notice that the nodal voltage directly above the voltage source is +10 V.

KCL at node *oc* (upper right-hand corner, where V_{oc} is located):

$$
\begin{aligned}
\frac{10 - V_{oc}}{5} + 3 &= 0 \\
\frac{10}{5} + 3 &= \frac{V_{oc}}{5}
\end{aligned}
$$

$$\Rightarrow V_T = V_{oc} = 25 \text{ V}$$

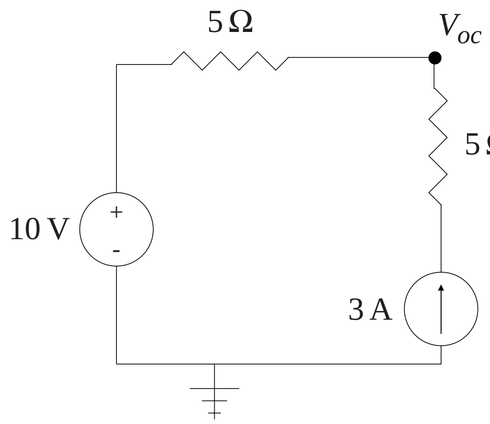

Figure 1: Circuit for Problem 4 with berkelistor removed.

(b) The Thevenin resistance can be easily found by looking into the terminals where the berkelistor was removed, zeroing all of the sources, and then finding the equivalent resistance of the remaining circuit. Zeroing a current source results in an open circuit and zeroing a voltage source results in a short circuit, and the remaining circuit is shown in Figure 2.

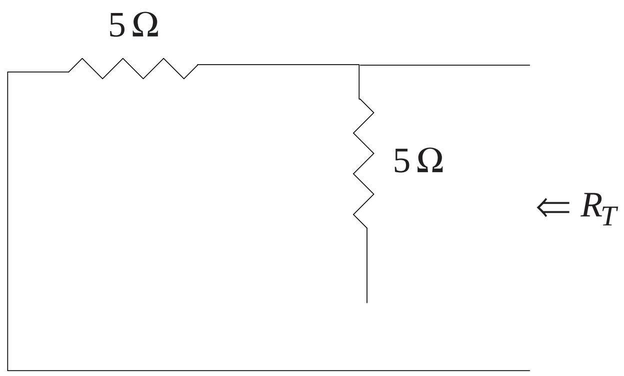

Figure 2: Circuit for Problem 4 with sources zeroed.

The Thevenin resistance is easily seen from the figure to be:

$$R_T = 5\ \Omega$$

(c) The operating point of the berkelistor can be found graphically from the given $i - v$ characteristic (using load line analysis). We first need to find i_{sc}, which can be obtained using V_{oc} and R_T:

$$\begin{aligned} i_{sc} &= \frac{V_{oc}}{R_T} \\ &= \frac{25\ \text{V}}{5\ \Omega} \\ &= 5\ \text{A} \end{aligned}$$

For the load line analysis, we plot i_{sc} and V_{oc}, connect the two points with a line, and find the intersection point (the operating point) between the line and the nonlinear $i-v$ characteristic given. Figure 3 shows the resulting plot of the operating point.

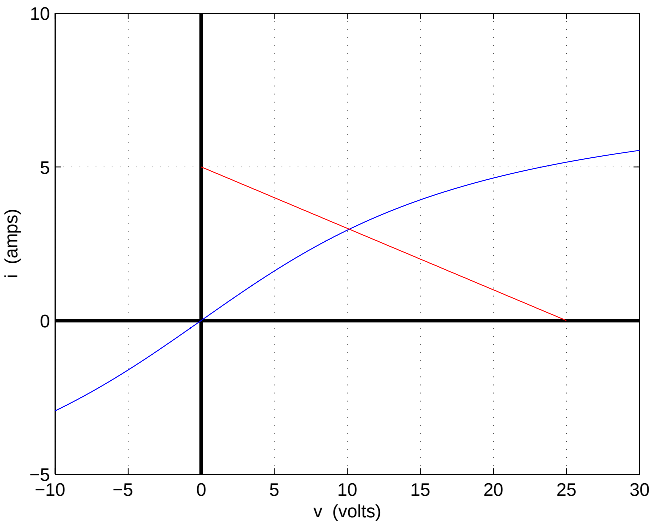

Figure 3: Load line plot for Problem 4. The berkelistor $i-v$ characteristic is in blue and the $i_{sc}-v_{oc}$ plot is in red.

The current drawn by the berkelistor is $i \approx 3$ A.

(d) The voltage drawn by the berkelistor can be read off Figure 3, at the operating point. The voltage is $v \approx 10$ V.

5. (a) We are given the initial charge on the capacitor, from which we can find the initial voltage.

$$\begin{aligned} V_C(0) &= \frac{Q(0)}{C} \\ &= \frac{2 \times 10^{-9} \text{ Coul}}{400 \times 10^{-12} \text{ F}} \\ &= 5 \text{ V} \end{aligned}$$

(b) The asymptotic value of the capacitor voltage may be found from the Thevenin equivalent of the circuit with the capacitor removed. A more conventional depiction of the circuit is given in Figure 4, from which the Thevenin voltage may be found.

KCL at node a, where i_S is the current flowing through the 4 V source in the direction of increasing potential:

$$\begin{aligned} i_S + \frac{v_C - V_a}{R} + \frac{v_C - V_a}{R} + \frac{-V_a}{R} - i_S &= 0 \\ 2\frac{v_C - V_a}{R} - \frac{V_a}{R} &= 0 \end{aligned}$$

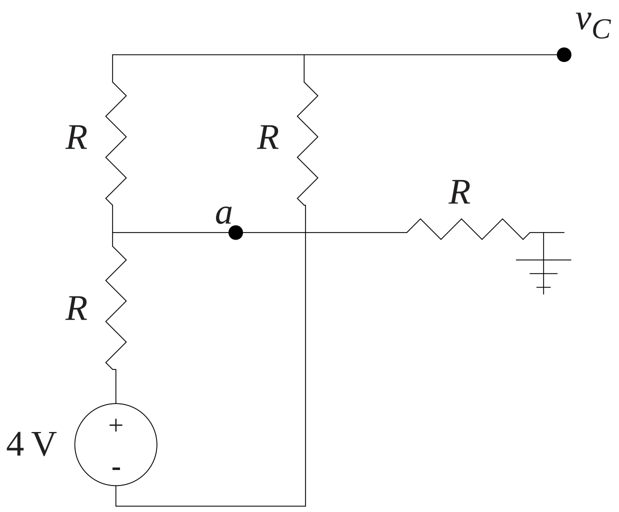

Figure 4: Circuit for Problem 5 with capacitor removed.

KCL at node oc:

$$\frac{V_a - v_C}{R} + \frac{V_a - v_C}{R} = 0$$
$$2\frac{V_a - v_C}{R} = 0$$

The nodal analysis reveals that $v_C = V_a = 0$ V. Thus, $v_C(\infty) = 0$ V.

(c) The time constant is $\tau = R_T C$, where R_T is the Thevenin equivalent resistance of the circuit with the capacitor removed. Zeroing the voltage source, it is easy to see that the Thevenin resistance is $R_T = \frac{3R}{2}$.
Now, the time constant can be found as:

$$\begin{aligned}\tau &= R_T C \\ &= \frac{3}{2}(2 \times 10^3)(400 \times 10^{-12}) \\ &= 1.2 \times 10^{-6} \text{ sec} \\ &= 1.2\ \mu\text{s}\end{aligned}$$

(d) We know the initial voltage, the steady state voltage, and the time constant, which is enough information to plot the time evolution of the capacitor voltage. The plot is given in Figure 5.

6. For both parts in this problem, the steady state output for the input $v_{in}(t) = A\cos(\omega t + \theta)$ is given by the following formula:

$$v_{out}(t) = |H(j\omega)| A \cos\left(\omega t + \theta + \angle H(j\omega)\right)$$

(a) The angular frequency of $v_{in}(t)$ is $\omega_a = 1$ rad/sec. The frequency response plots will give us the magnitude amplification and phase difference between the input

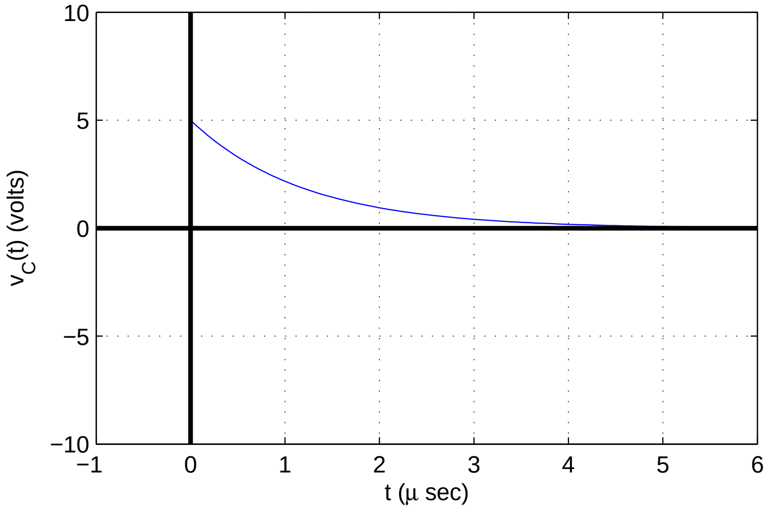

Figure 5: Capacitor voltage versus time for Problem 5.

and the output.

$$\begin{aligned}|H(j\omega_a)| &\approx 11 \text{ dB} \\ &= 10^{\frac{11}{20}} \\ &= 3.55\end{aligned}$$

$$\begin{aligned}\angle H(j\omega_a) &\approx -135^\circ \\ &= -\frac{3\pi}{4}\end{aligned}$$

Note that a magnitude estimate of 10 dB would have produced an amplification ratio of 3.16 (either answer will be acceptable). The corresponding output voltage can now be estimated as (note that $\sin(t) = \cos(t - \frac{\pi}{2})$):

$$\begin{aligned}v_{out}(t) &= |H(j\omega_a)| \cos\left(\omega_a t + 1 - \frac{\pi}{2} + \angle H(j\omega_a)\right) \\ &\approx 3.55 \cos\left(t + 1 - \frac{\pi}{2} - \frac{3\pi}{4}\right) \\ &= 3.55 \sin\left(t + 1 - \frac{3\pi}{4}\right)\end{aligned}$$

(b) The angular frequency of $v_{in}(t)$ is $\omega_b = 0$ rad/sec. The frequency response plots will give us the magnitude amplification and phase difference between the input and the output.

$$
\begin{aligned}
|H(j\omega_b)| &= 0 \text{ dB} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
\angle H(j\omega_b) &= -180^\circ \\
&= -\pi
\end{aligned}
$$

The corresponding output voltage can now be written as:

$$
\begin{aligned}
v_{out}(t) &= |H(j\omega_b)|(10)\cos\left(\omega_b t + \angle H(j\omega_b)\right) \\
&= 1(10)\cos\left(0 - \pi\right) \\
&= -10
\end{aligned}
$$