# EE 100/42 Spring 2009
# Solutions to Practice Midterm

1. (a) By inspection, the period is $T = 3$ ms.

   (b) The DC/average voltage may be found as follows, since the voltage is constant during certain intervals of time:

$$\begin{aligned} V_{DC} &= \frac{1}{T}\int_0^T V(t)dt \\ &= \frac{1}{3\text{ ms}}\left[(1\text{ V})(1\text{ ms}) + (2\text{ V})(1\text{ ms}) + (0\text{ V})(1\text{ ms})\right] \\ &= 1\text{ V} \end{aligned}$$

   (c) The RMS voltage may be found as follows, since the voltage is constant during certain intervals of time:

$$\begin{aligned} V_{rms} &= \sqrt{\frac{1}{T}\int_0^T V(t)^2 dt} \\ &= \sqrt{\frac{1}{3\text{ ms}}\left[(1\text{ V})^2(1\text{ ms}) + (2\text{ V})^2(1\text{ ms}) + (0\text{ V})^2(1\text{ ms})\right]} \\ &= \sqrt{\frac{5\text{ V}^2}{3}} \\ &= \sqrt{\frac{5}{3}}\text{ V} \end{aligned}$$

2. (a)

$$5e^{j\frac{\pi}{2}} \leftrightarrow 5\cos(\omega t + \frac{\pi}{2})$$

   (b)

$$\begin{aligned} 3 + 4j &= (3^2 + 4^2)^{\frac{1}{2}} e^{j\tan^{-1}\left(\frac{4}{3}\right)} \\ &\leftrightarrow 5\cos(\omega t + \tan^{-1}\left(\frac{4}{3}\right)) \end{aligned}$$

   (c)

$$\begin{aligned} 3\cos(\omega t) + 4\sin(\omega t) &\leftrightarrow 3e^{j0} + 4e^{-j\frac{\pi}{2}} \\ &= 3 - 4j \end{aligned}$$

   (d)

$$\begin{aligned} \sqrt{2}\sin(\omega t - 45^\circ) &= \sqrt{2}\cos(\omega t - \frac{3\pi}{4}) \\ &\leftrightarrow \sqrt{2}e^{-j\frac{3\pi}{4}} \end{aligned}$$

3. The internal resistance $R$ of a practical current source is in **parallel** with the source.

   For a well-designed circuit with a practical current source, this internal resistance $R$ should be much **larger** than the load resistance.

   A circuit element that requires an external power supple is called **passive**.

   We **cannot** find the Thevenin equivalent of a circuit containing diodes.

   The input resistance of an ammeter is **very small**.

   An oscilloscope can easily **not be used** to measure magnetic field strength.

4. (a) The internal resistance of a practical battery model is in series with the voltage source. A realistic model of a practical voltmeter has an internal resistance in parallel with an ideal voltmeter. The circuit diagram is given in Figure 1.

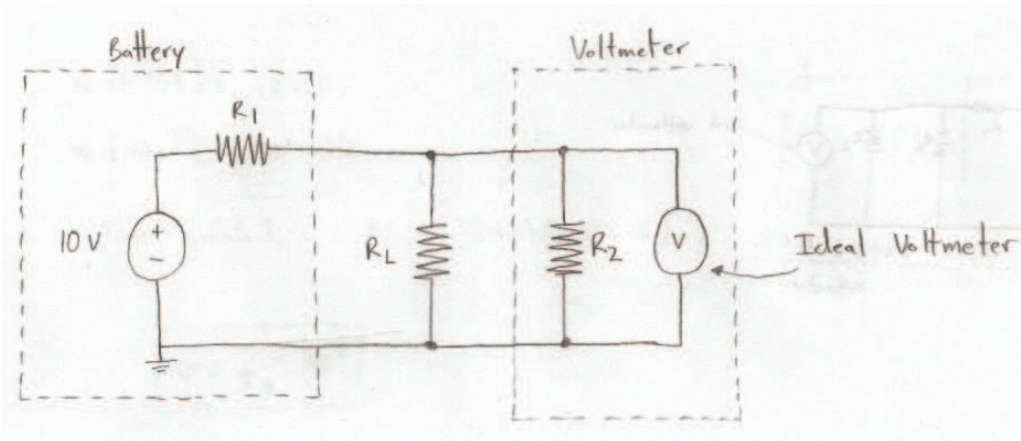


Figure 1: Circuit diagram for Problem 4.

   (b) The measured voltage can be found by reducing the parallel combination of $R_L$ and $R_2$ into an equivalent resistance, and then applying the voltage divider formula.

$$
\begin{aligned}
V &= (10\text{ V})\frac{(R_L \parallel R_2)}{R_1 + (R_L \parallel R_2)} \\
&= 10\frac{\frac{R_L R_2}{R_L+R_2}}{R_1 + \frac{R_L R_2}{R_L+R_2}}\text{ V} \\
&= 10\frac{R_L R_2}{R_1 R_L + R_1 R_2 + R_L R_2}\text{ V}
\end{aligned}
$$

   (c) Ideally, you would want $R_1 << R_L$ so the load experiences a voltage drop that is close to the open-circuit voltage.

   (d) Ideally, you would want $R_2 >> R_L$ so the voltmeter does not perturb the original circuit too much.

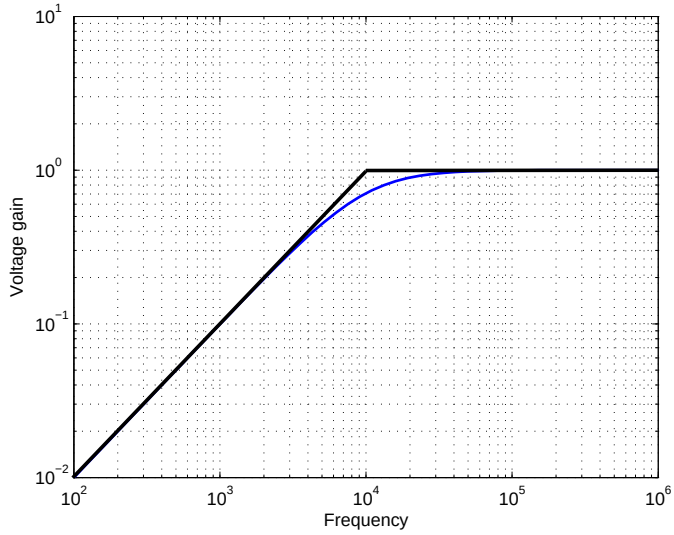


Figure 2: Frequency response $H(\omega)$. The asymptotic approximation is in black and the actual plot is in blue.

5. (a) A current of $\mathbb{I}_{in}$ flows through the capacitor and the resistance $R_2$. To find the current $\mathbb{I}_{out}$, we can use the current divider formula, where $Z_L$ is the impedance of the inductor.

$$\begin{aligned}\mathbb{I}_{out} &= \mathbb{I}_{in}\frac{Z_L}{Z_L + R_1} \\ &= \mathbb{I}_{in}\frac{j\omega L}{j\omega L + R_1}\end{aligned}$$

(b) The frequency response function is:

$$\begin{aligned}H(\omega) &= \frac{j\omega L}{R_1 + j\omega L} \\ &= \frac{j\omega \frac{L}{R_1}}{j\omega \frac{L}{R_1} + 1} \\ &= \frac{j\omega 10^{-4}}{j\omega 10^{-4} + 1}\end{aligned}$$

The voltage gain is given in Figure 2.

(c) This is a **high-pass** filter.

6. Assign ground as the node at the bottom of the circuit.

(a) KCL at node a, using the sum of the currents **into** the node:

$$1 + \frac{0 - V_a}{1} + \frac{V_b - V_a}{1} = 0$$

This can be rewritten as:

$$2V_a - V_b = 1$$

(b) KCL at node b, using the sum of the currents **into** the node:

$$-I + \frac{0 - V_b}{1} + \frac{V_a - V_b}{1} = 0$$

This can be rewritten as:

$$-V_a + 2V_b = -I$$

(c) The extra equation that is required is the relationship between the node voltages and the voltage source (recall that we had to introduce an extra unknown $I$ to account for the current through the voltage source).

$$\begin{aligned} V_b - 0 &= 10 \text{ V} \\ V_b &= 10 \text{ V} \end{aligned}$$

Now we can use the equation from Part (a) to solve for $V_a$:

$$\begin{aligned} V_a &= \frac{V_b + 1}{2} \text{ V} \\ V_a &= \frac{11}{2} \text{ V} \end{aligned}$$

7. (a) Pulling out the capacitor allows the Thevenin equivalent circuit to be found. The first step is finding $V_{oc}$, or the open-circuit voltage across the terminals after the capacitor is removed (using the sign convention for $v_c(t)$). Noticing that no current flows through $R_3$ when the capacitor is removed, the expression for $V_{oc}$ may be obtained by applying the voltage divider formula:

$$\begin{aligned} V_{oc} &= v_s \frac{R_2}{R_1 + R_2} \\ &= v_s \frac{2 \ k\Omega}{4 \ k\Omega} \\ &= \frac{1}{2} v_s \end{aligned}$$

Since $V_{oc} = v_T(t)$, then the Thevenin voltage is:

$$v_T(t) = \begin{cases} -2 \text{ V} & t \leq 0 \\ 3 \text{ V} & t > 0 \end{cases}$$

The Thevenin resistance may be found by finding the short circuit current and then finding $\frac{V_{oc}}{i_{sc}}$. For this problem, it will be easier to find the equivalent resistance looking into the terminals with all of the sources zeroed out.

$$\begin{aligned} R_T &= R_3 + \frac{R_1 R_2}{R_1 + R_2} \\ &= 3000 \ \Omega + \frac{2000^2 \ \Omega^2}{4000 \ \Omega} \\ &= 4 \ k\Omega \end{aligned}$$

(b) The time constant for the Thevenin equivalent circuit is

$$\begin{aligned}\tau &= R_T C \\ &= (6000)(2.5 \times 10^{-6}) \text{ sec} \\ &= 15 \text{ ms.}\end{aligned}$$

The steady state voltage for $v_c$ is equal to $v_T$ for $t > 0$. The initial voltage across the capacitor is $v_T$ for $t \leq 0$, assuming that the system had sufficient time to equilibrate. This information, along with the time constant, gives the expression for the voltage across the capacitor for $t > 0$:

$$\begin{aligned}v_C(t) &= 5 + (-8)e^{-\frac{t}{\tau}} \\ &= 5 - 8e^{-\frac{t}{0.015}}\end{aligned}$$

We expect that the solution will reach the steady state in approximately 3 time constants. The graph is given in Figure 3.

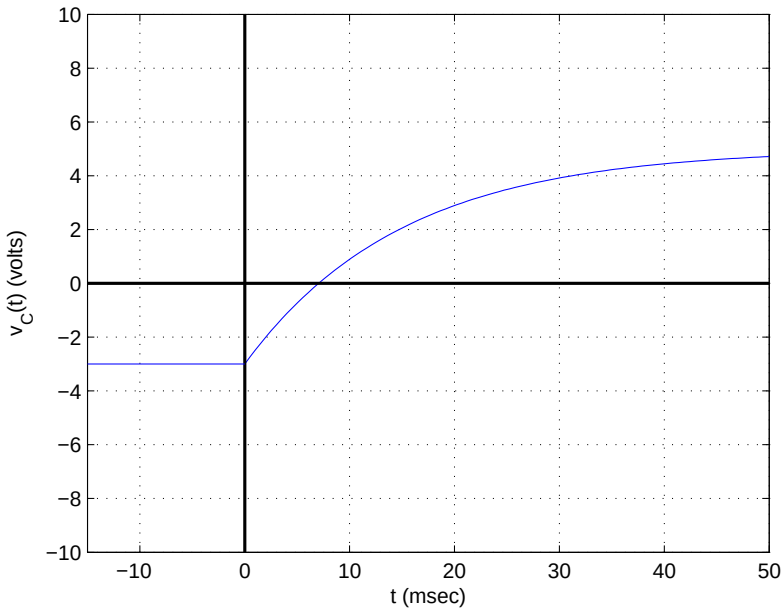


Figure 3: $v_C(t)$ for Problem 7.

8. There are only four possible resistances that may be constructed from two resistors:

$$R_1$$
$$R_2$$
$$R_1 + R_2$$
$$\frac{R_1 R_2}{R_1 + R_2}$$

The series combination means that $R_1 + R_2 = 18\ \Omega$ since it is the greatest resistance possible. Plugging in the other two choices for combinations of $R_1$ and $R_2$ leaves the following:

$$R_1 = 6\ \Omega$$
$$R_2 = 12\ \Omega$$