

Lecture 10

07-19-04

Midterm: Distributed during breaks

Average: 87%

(1) Suppose HW are: 90%

Midterms + HWs + Oth/Class Part. + LAB

→ Class Average @ the end: 92.8%

FINAL will have to be ~ 61%

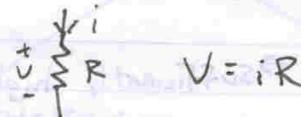
to get the average down to a B+

Today: 6.1-6.3 & 7.1-7.2

Wed: 6.4, 6.5 ← mutual inductance, 7.3-7.7

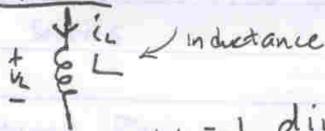
Chap 6 - Inductors, Capacitors, Mutual Inductance

Recall:



$V = iR$

Inductor:



$$V_L = L \frac{di_L}{dt}$$

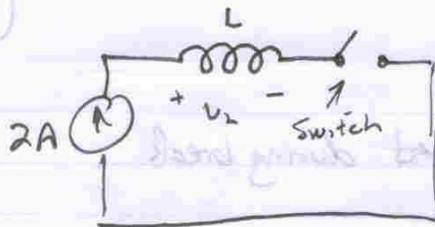
L: unit = Henries (H)

Stuff about inductors

① Stores energy in a magnetic field

② Inductors maintain current through discontinuities

(2)



@ $t = t_1$, Switch opened

$$v_L = L \frac{di_L}{dt}$$

Physically, what happens @ $t = t_1$?

@ $t = t_1^-$ ← just before t_1

$$i_L = 2A$$

@ $t = t_1^+$ → i_L is 0?

↑ instant after

$t = t_1^+$

Question: what happens to $\frac{di}{dt}$?



$$\frac{di}{dt} \approx \frac{\Delta i}{\Delta t} = \frac{\text{finite}}{\approx 0} \rightarrow -\infty$$

This would mean $v_L = -\infty$

But physically, this cannot happen

Inductor maintains current through discontinuities:

$$i(t=t_1^-) = i(t=t_1^+) = 2A$$

Like a spark plug → sufficiently high voltage produced
→ spark is seen as a current path is created

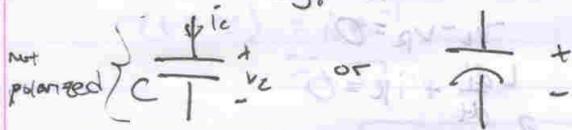
through the air

(A)

(3)

Capacitors

- Stores energy in an electric field



$$i_c = C \frac{dv_c}{dt}$$

Capacitance: Farads (F)

There is a current (according to Maxwell's equations) called the displacement current that travels across the capacitor

Skipping 6-3 for the moment → jump to Chap. 7

What happens in a 1st order RC circuit or RL circuit?

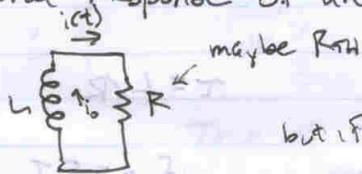
- Find voltages & currents
RC & RL Circuits

(2.1)

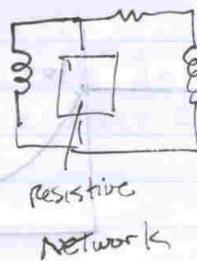
Natural Response
Circuit behavior w/ no sources

Forced Response
Circuit behavior w/ independent sources

Natural Response of an RL circuit



but if!



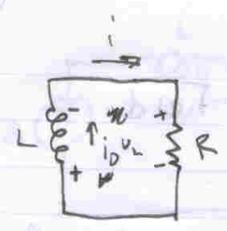
Much harder

→ Ch. 8

→ we are skipping

(3)

(4)



Use KVL:

$$v_L - v_R = 0$$

$$L \frac{di}{dt} + iR = 0$$

Linear, 1st Order, Ordinary differential Eq.

$$L \frac{di}{dt} = -iR \rightarrow \frac{di}{i} = -\frac{R}{L} dt$$

$$\int_{i_0}^i \frac{di}{i} = \int_{t_0}^t -\frac{R}{L} dt \Rightarrow \ln(i) = -\frac{R}{L} (t - t_0) + C$$

$$\ln\left(\frac{i}{i_0}\right) = -\frac{R}{L} (t - t_0)$$

$$i(t) = i_0 e^{-\frac{R}{L} (t - t_0)}$$

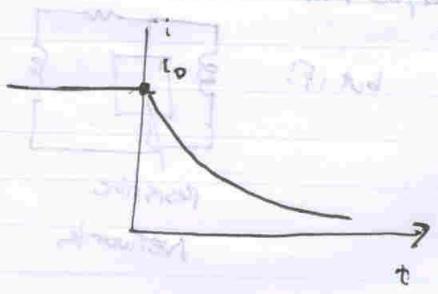
Sanity Check: (1) $i(t=0) = i_0 e^{-R/L(0)} = i_0$

$$\frac{di}{dt} = -\frac{iR}{L}$$

$\frac{dy}{dx} = y \rightarrow y(x) = e^x$ or $y(x) = 0$

Unique Sol'n

@ $t \rightarrow \infty \quad i \rightarrow 0$



$$\tau = L/R$$

$$\text{if } t = 5\tau \quad i = i_0 e^{-5} \approx 0$$

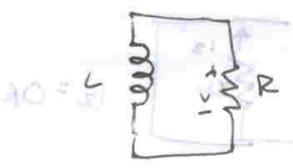
4

5

$\tau = \frac{L}{R} \rightarrow$ Time Constant

$i(t = 5\tau) \rightarrow 0$

$i(t = \tau) = i_0 e^{-1}$
 $\approx .37 i_0$

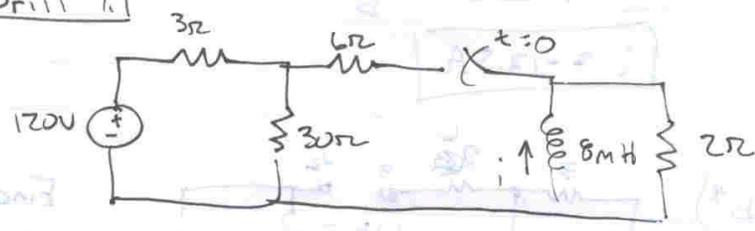


$v(t) = i(t)R$
 $i_0 R e^{-t/\tau}$

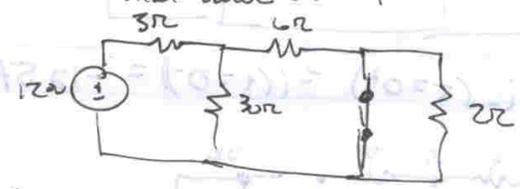
What if $R \rightarrow 0$ $i(t) = i_0$



Drill 7.1



Calculate initial value of i



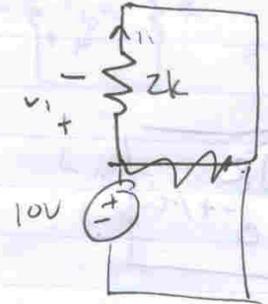
$v = L \frac{di}{dt}$ if i is constant $\frac{di}{dt} = 0$, so $v = 0$

Thus at steady state an inductor models a short circuit

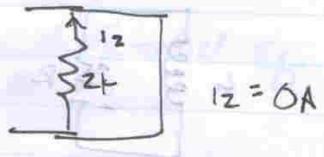
V1.451

6

Note!



$v_1 = 10V$
 $i_1 = 70$
 $i_1 = \frac{10V}{2k} \cdot 10$

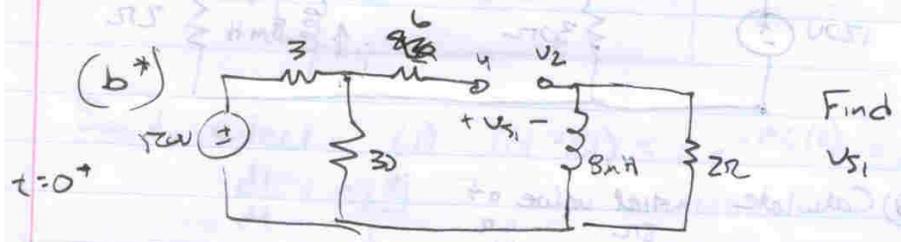


$i_2 = 0A$

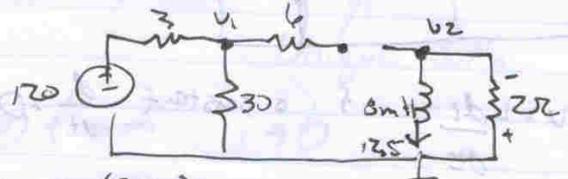
Back to problem



$i = -12.5A$



$i_L(t=0^+) = i(t=0^-) = -12.5A$



$v_1 = 120 \left(\frac{30}{30+3} \right) = 109.1V$

$v_2 = 0 - 2(12.5) = -25V$

$v_{S1} = v_1 - v_2 = 109.1 - (-25) = 134.1V$