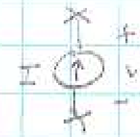
(1) # Final Review 8/10/05

Overall Review & Tips

A) Nodal Analysis

1.)  is i always 0?

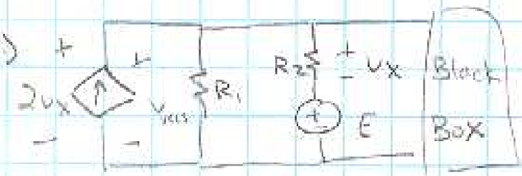
NO

 is v always 0?

NO

2.) $P = IV \rightarrow P = I^2R$ or $P = V^2/R$

<u>ONLY</u> for resistors

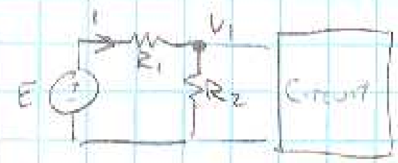
3.) 

$P_{VCCS} = i \cdot v$

$i = 2v_x \quad v_{VCCS} \neq v_x$

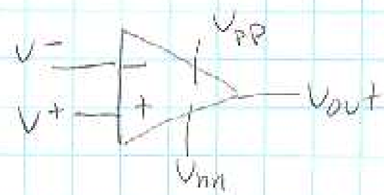
$P \neq (2v_x) \cdot v_x$

4.) $i = \frac{\Delta V}{R}$



$i = \frac{E - v_1}{R_1}$ not $i = \frac{E}{R_1}$

B.) Op-Amps

1.) 

$i_P = 0$

$i_N = 0$

$V_{out} = A(v^+ - v^-)$

} ALWAYS

2.) Op-Amps on the final may not be idle.

3.) If ideal (linear region): $v^+ = v^-$ & $A \rightarrow \infty$

4.) If voltage/current source is either directly or indirectly connected to $v^- \rightarrow$ op-amp is inverting & $A_v < 0$

If connected to $v^+ \rightarrow$ non-inverting & $A_v > 0$

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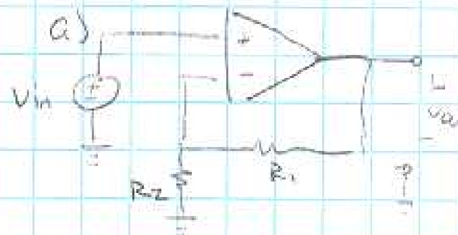
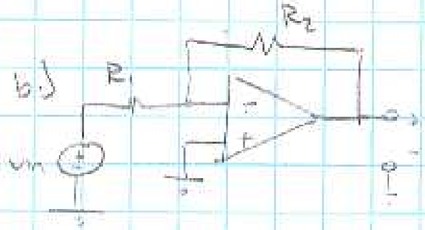
5.) Op-Amps **ALWAYS** amplify. → $|A_v| \geq 1$

$0 < |A_v| < 1$ ← Never

6.) Typical op-amps:



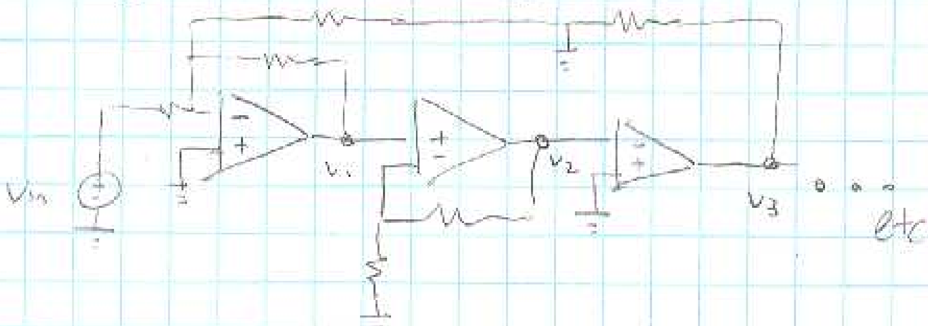


Non-Inverting
$A_v = 1 + R_1/R_2$

Inverting
$A_v = -R_2/R_1$

Note: Equations do not hold in cascades

7.) Cascade (like 14.18)



Let $A_1 = v_1/v_{in}$
$A_2 = v_2/v_1$
$A_3 = v_3/v_2$
... etc.
$A_i = v_i/v_{i-1}$

a.) $|A_i| \geq 1$ for all i

b.) $A_{overall} = \frac{v_{out}}{v_{in}} = A_1 A_2 A_3 \ldots$

8.) For all diode problems:

a.) Assume ideal ⇒ $v^+ = v^-$ → Calculate v_{out}

b.) If $v_{out} > v_{pp}$, then set $v_{out} = v_{pp}$ & recalculate v^+ & v^-

c.) " $v_{out} < v_{nn}$ " " $v_{out} = v_{nn}$ " " "

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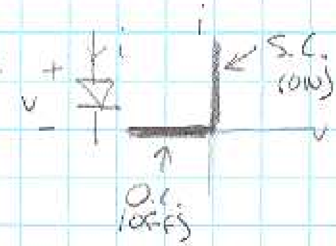
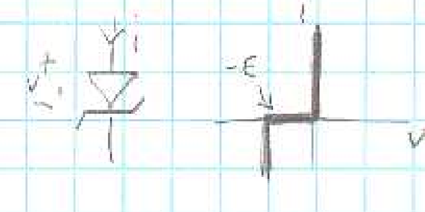
C.) Diodes

1.) Ideal Diode:



Zener Diode:



2) Method:

a) Guess each diode as either on or off.

b.) ON = S.C. ⇒ $v_D = 0$; OFF = O.C. v_D is unknown ← calculate

c.) Calculate v_D for all off diodes

d.) If $v_D < 0$ → Open circuit region ⇒ Assumption correct

e.) If $v_D > 0$ ⇒ Short circuit region → Assumption wrong

3) Good guesses:

if i → &  Guess ON

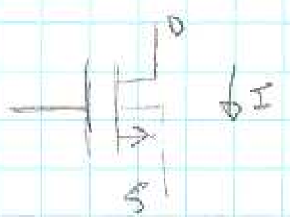
if i → &  Guess OFF

4) Zener Diodes in the breakdown region ⇒

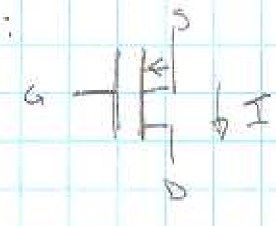
Treat diode as voltage source.

D.) Transistors

1.) NMOS:



PMOS:



2) Operation Regions & Equations

a) Cutoff : $i_D = 0$ for $v_{GS} \le V_{t0}$ (12.1)

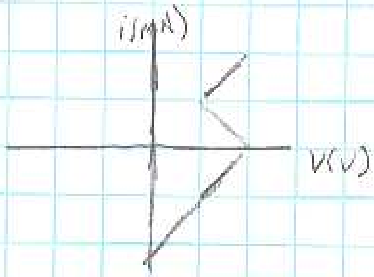
b.) Triode : $i_D = K[2(v_{GS} - V_{t0})v_{DS} - v_{DS}^2]$ for $v_{DS} < v_{GS} - V_{t0}$ & $v_{GS} \ge V_{t0}$ (12.2)

c.) Saturation : $i_D = K(v_{GS} - V_{t0})^2$ for $v_{GS} \ge V_{t0}$ & $v_{DS} \ge v_{GS} - V_{t0}$ (12.4)

d.) Boundary (Triode & Saturation) ⇒ $v_{DS} = v_{GS} - V_{t0}$

e.) $K = \left(\frac{W}{L}\right)\frac{KP}{2}$

E.) Nonlinear Circuits



1.) Equations & Equilibrium

$i = -C\,dv/dt$

$d/dt = 0 \rightarrow i = 0 \rightarrow$ Eq.

$i > 0 \rightarrow v \downarrow$

$i < 0 \rightarrow v \uparrow$

$v = -L\,di/dt$

$d/dt = 0 \rightarrow v = 0 \rightarrow$ Eq

$v > 0 \rightarrow i \downarrow$

$v < 0 \rightarrow i \uparrow$

2.) Important! Look @ labels!

$v_C = v$, $v_L = v$, $i_C = -i$, $i_L = -i$

3.) When finding v_f or i_f, extrapolate the lines forward to the proper equilibrium condition.

$C \rightarrow i = 0$ (x-axis) $\qquad L \rightarrow v = 0$ (y-axis)

Note: if in negative slope region, extrapolate backwards to eq.

4.) For capacitors, i_f always $= 0$

For inductors v_f always $= 0$

5.) $\text{Slope} = \frac{\Delta i}{\Delta v} = \frac{1}{R}$, so

$$\tau_C = RC = \frac{\Delta V}{\Delta I} C = \frac{C}{\text{slope}} \; ; \quad \tau_L = \frac{L}{R} = \frac{\Delta I}{\Delta V} L = (\text{slope})(L)$$

6.) Always find: $i_0, v_0, i_f, v_f, R, C(\text{or } L), \tau$, & t_0

& insert

$$x(t) = x_f + (x_0 - x_f)\exp\left[\frac{-(t - t_0)}{\tau}\right]$$

7) (i) If not @ eq pt & not heading to one, calculate a switching time

(ii) If heading towards an eq pt, do not calculate a switching time (it will be ∞).

8.) Shapes of curves?

Slope of Region	What is x doing?	Draw
Negative	Increasing	
Negative	Decreasing	
Positive	Increasing	
Positive	Decreasing	

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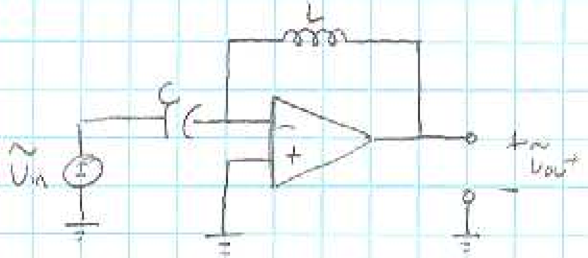
Problems

1.) Op-Amps & AC Analysis



$\tilde{V}_{in} = \sin(\omega t)$

$\omega = 1000$ rad/sec

$L = 1$ mH

$C = 1$ mF

Use AC analysis to find Vout/Vin.

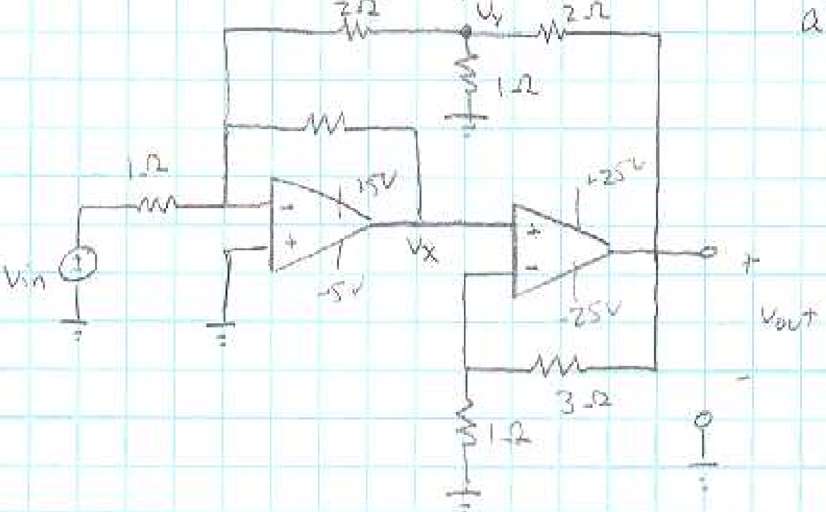
Solution: $Z_C = \dfrac{1}{j\omega C} = \dfrac{-j}{\omega C}$ $\quad Z_L = j\omega L$

Nodal Analysis: @ V^- $\quad \dfrac{(V_{in} - 0)}{1/j\omega C} = (V_{in})(j\omega C) = \dfrac{0 - V_{out}}{j\omega L}$

$$\frac{V_{out}}{V_{in}} = -j^2\omega^2 LC = -(-1)(1000\text{rad/s})^2 \left(.001 \frac{V\cdot s}{A}\right)\left(.001 \frac{A\cdot s}{V}\right)$$

$$\boxed{= 1}$$

2.) Cascading Op-Amp



a.) Assume both op-amps are ideal & find Vout / Vin

b.) Find Vx / Vin

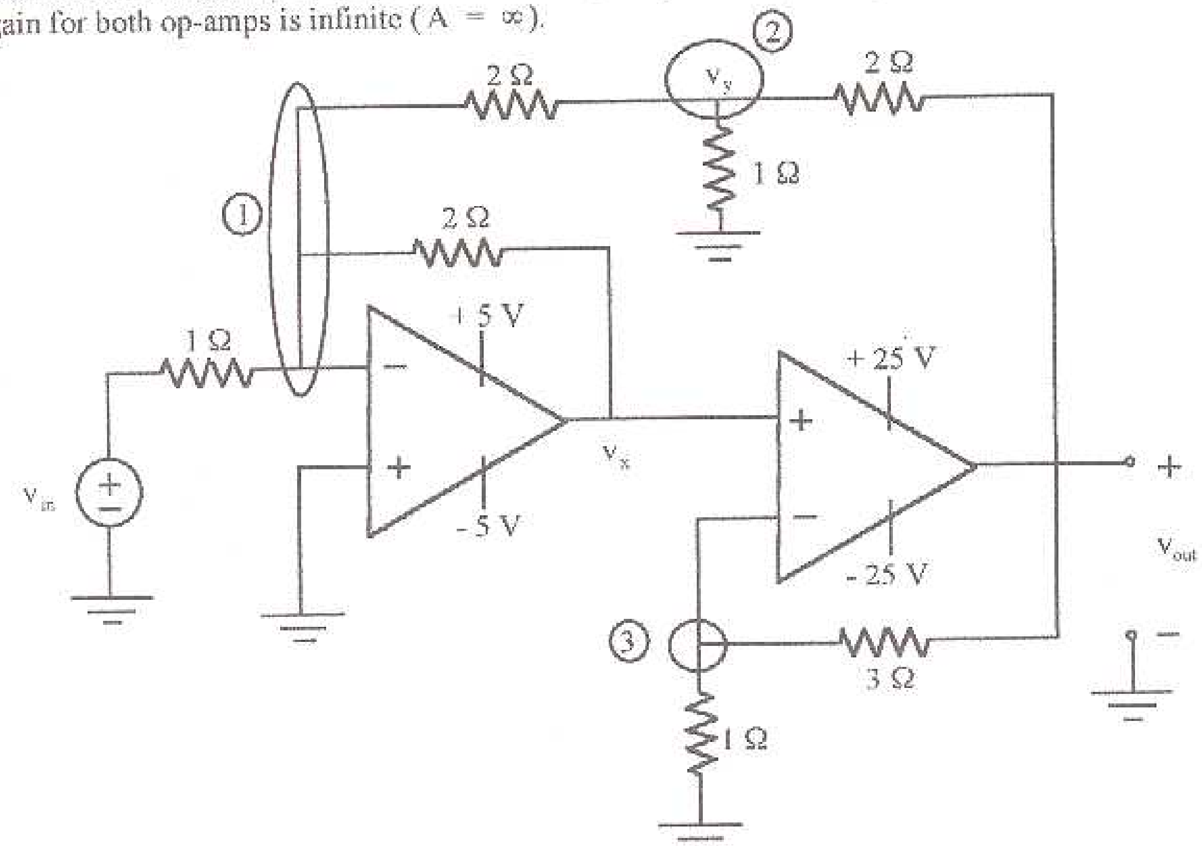
c.) $V_{in} = 3V$, Find Vout

d.) $V_{in} = -6V$, Find Vout

e.) $V_{in} = 30V$, Find Vout

(7)

Consider the following dual op-amp circuit. Each op-amp has a separate power supply. Assume the open loop gain for both op-amps is infinite (A = ∞).



a.) Is the above circuit inverting or non-inverting? Answer this by inspection only. Explain your logic.

Soln: As shown in lecture the label "inverting" or "non-inverting" is dependent on where the input voltage is connected to the op-amp. v_{in} is connected to V_N of the first op-amp, making the first op-amp inverting. The output voltage of the first op-amp is the input of the second op-amp. v_x is connected to V_P of the second op-amp, making this op-amp non-inverting. Thus, overall the input voltage is inverted by the first op-amp and so the overall circuit is **INVERTING**.

b.) Assuming both op-amps are working in the linear region, find the overall closed loop gain: $\frac{v_{out}}{v_{in}}$.

(Hint: it may be useful to find the relationship between v_{out} and v_y or v_x)

Soln: Since the op-amps are ideal we can assume that $V_P = V_N$.

Nodal analysis at node 3: $\frac{v_{out} - v_x}{3} = \frac{v_x}{1}$, so $v_{out} = 4v_x$.

Nodal analysis at node 2: $\frac{0 - v_y}{2} = \frac{v_y - v_{out}}{2} + \frac{v_y}{1}$, so $v_{out} = 4v_y$

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Node 1 is at 0 V because V_p is grounded. Nodal analysis at node 1: $\frac{v_{in}-0}{1}=\frac{0-v_x}{2}+\frac{0-v_y}{2}$.

Substituting from what we got from nodes 2 and 3: $\frac{v_{in}-0}{1}=\frac{-1}{2}\left(\frac{v_{out}}{4}\right)-\frac{1}{2}\left(\frac{v_{out}}{4}\right)$.

so: $\boxed{\frac{v_{out}}{v_{in}}=-4}$

c.) Using the same assumption, find the closed loop gain of the first op-amp: $\frac{vx}{vin}$.

Soln: Use the node 1 analysis, replace v_y with v_{out} and then replace v_{out} with v_{in}.

$$\frac{v_{in}}{1}=\frac{-v_x}{2}-\frac{v_{out}}{8},\ v_{out}=-4v_{in},\ \frac{v_{in}}{1}=\frac{-v_x}{2}+\frac{4v_{in}}{8}.$$

Rearrange: $\boxed{\frac{v_x}{v_{in}}=-1}$

d.) If $v_{in}=3$ V, find v_{out}.

Soln: Simple insert into our answers above,
BUT WE MUST MAKE SURE TO CHECK FOR SATURATION.
If $v_{in}=3$ V, then $v_x=-3$V (op-amp 1 not saturated), and $v_{out}=-12$V (op-amp 2 not saturated).

So: $\boxed{v_{out}=-12V}$

e.) If $v_{in}=-6$ V, find v_{out}.
Soln: If $v_{in}=-6$ V, then $v_x=+6$V, but the op-amp is saturated, so $v_x=+5$V. Op-amp 2 will not saturate. $\boxed{v_{out}=+20V}$

f.) If $v_{in}=30$ V, find v_{out}.

Soln: If $v_{in}=30$ V, then $v_x=-30$V, but the op-amp is saturated, so $v_x=-5$V. Op-amp 2 will not saturate. $\boxed{v_{out}=-20V}$

g.) What range of values of v_{in} will saturate the second op-amp?
Explain how you came up with your answer.

Soln: A voltage greater than +6.25V or less than -6.25V would be required as a value of v_x to saturate the second op-amp. But because of the different power supplies attached to each op-amp, this can never occur. When the first op-amp saturates, the largest voltage it outputs has a magnitude of 5V. This means that the overall circuit cannot output more than +20V or less than -20V. In other words,

<u>NO VALUES OF AN INPUT VOLTAGE WILL SATURATE THE SECOND OP-AMP.</u>

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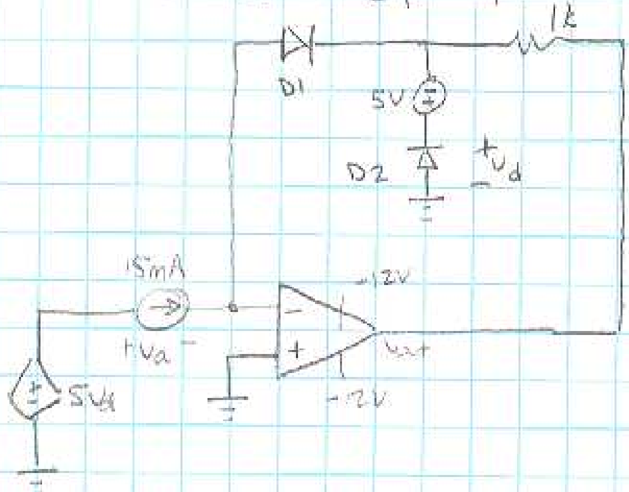
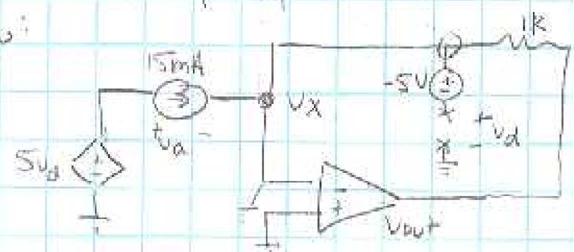
3) Diodes (+ Op-Amp)



All diodes are ideal
find v_a.

Solution: Guess state of diodes
D1 seems on & D2 seems off
Assume op-amp is ideal at first.

Redraw:



$V_x = V^- = V^+ = 0$

$i_{1k} = 15mA \quad V_{1k} = 15V$

$V_{out} = 0 - 15V = \underline{-15V}$

<u>Saturation</u>

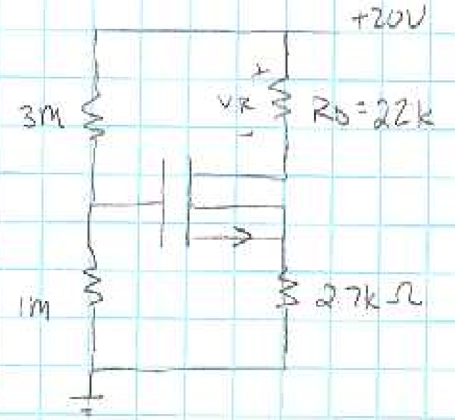
$V_{out} = -12V$

$\frac{V^- - V_{out}}{1k} = 15mA \Rightarrow V^- + 12 = 15 \Rightarrow V^- = 3V$

$V^- = V_d - 5 \Rightarrow V_d = V^- + 5 = 8V$

$V_a = 5V_d - V_x = 5(8) - 3 = \boxed{37V}$

4) Transistors



$KP = 50\mu A/V^2$
$V_{to} = 2V$
$L = 10\mu m$
$W = 40\mu m$

Find i_{DS}, V_D, V_S, V_G, V_R, V_{GS}, V_{DS}, V_{DG}

Solution: Label NMOS:

$V_G = 20V \cdot \frac{1M}{1M+3M} \rightarrow \boxed{G = 5V}$ $\qquad V_{GS} = 5 - V_S$

$i_D = i_{DS} = V_S / 27k\Omega$

$K = \frac{W}{L}\left(\frac{KP}{2}\right) = \left(\frac{400}{10}\right)\left(\frac{50\times10^{-6} A/V^2}{2}\right) = 0.001\ A/V^2$

$i_D = K V_{DS}^2$

$V_{DS} = V_{GS} - V_{tO} = 5 - V_S - 2 = 3 - V_S$

$\frac{V_S}{27k} = .001(3 - V_S)^2$

Solve $\rightarrow$ $V_S = 2.1149\ V$ or $4.2554\ V$

if $V_S = 4.2554 \rightarrow V_{GS} = 5 - V_S = 0.7446 \rightarrow < V_{TO}$? <u>WRONG</u>

$V_S = 2.1149V \rightarrow V_{GS} = 2.8851V \geq V_{to} \rightarrow$ <u>ON</u>

Saturation? is $V_{DS} \geq V_{GS} - V_{TO}$?

$V_{DS} \geq 2.8851 - 2 = 0.8851V$

$I_{DS} = {}^{V_S}/_{27k} = 783.2963\mu A \rightarrow V_R = 17.2325\ V$

$V_D = 20 - 19.5824 = 2.7675\ V$

$V_{DS} = 0.6526$

$0.6526 < 0.8851$

NOT IN SATURATION REGION

$\rightarrow$ TRIODE

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$$i_D = K\left[2(v_{GS}-V_{T0})v_{DS} - v_{DS}^2\right]$$

$$v_{GS}-V_{T0} = 3 - v_S, \quad v_{DS} = v_D - v_S = 20 - (22k)i_{DS} - v_S$$

$$v_{DS} = 20 - \frac{22 v_S}{2.7} - v_S = 20 - \frac{24.7 v_S}{2.7}$$

$$\frac{v_S}{2700} = 0.001\left[2(3-v_S)\left(20 - \frac{24.7v_S}{2.7}\right) - \left(20 - \frac{24.7v_S}{2.7}\right)^2\right]$$

Solve: $v_S = 2.1046V$ or $v_S = 2.0345$

v_S	$v_{GS}-V_{T0} = 3 - v_S$		$v_{DS} = 20 - 24.7v_S/2.7$	VALID?
2.1046	0.8954	>	0.7468	NO
2.0345	0.9655	<	1.3881	YES

So, in conclusion

$v_G = 5V$, $v_S = 2.0345V$, $v_D = 3.4226V$
$v_{GS} = 2.9655V$, $v_{DS} = 1.3881V$, $v_{DG} = 1.5774V$

$i_{DS} = 753.5185\ \mu A$

$v_R = 16.5774\ V$

Try again w/ $R_D = 2.7k\Omega$

→ Saturation Region, so

$v_G = 5V$, $v_S = 2.1149V$, $v_D = 16.3185V$
$v_{GS} = 2.8851V$, $v_{DS} = 14.2036V$, $v_{DG} = 11.3185$

$i_{DS} = 783.2963\ \mu A$ $\quad v_R = 3.6815V$

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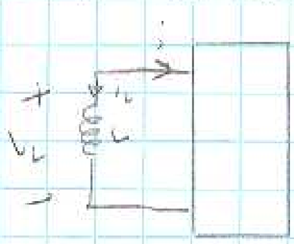
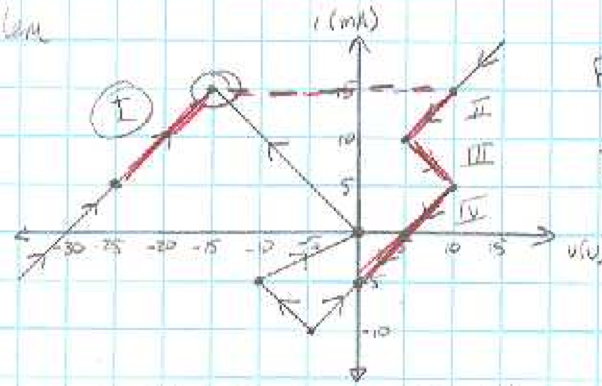
5) Non-linear Problem



L = 1 H



Find & graph $v(t)$ & $i(t)$
Start @ $v_L(0) = -25$

$\frac{-v}{L} = \frac{di}{dt}$ $\quad \frac{di}{dt} = 0 \rightarrow$ Eq @ $v = 0$ $\quad v > 0,\ i \downarrow\ ;\ v < 0\ i \uparrow$

Impasse Pt @ (-15V, 15mA) → jumps to (10V, 15mA)

Region I: $v_0 = -25$, $v_\infty = 0$ $\quad R = 1000\,\Omega$ $\quad \tau = 0.001s$
$i_0 = 5$, $i_\infty = 30$ $\quad L = 1H$ $\quad t_0 = 0$
$v(t) = -25\exp[-1000t]$
$i(t) = 30 - 25\exp[-1000t]$ $\quad$ for $0 \le t \le t_1$
t_1 @ $v = -15V$ $\quad t_1 = \ln(5/3)/1000$

Region II: $v_0 = 10$ $\quad v_\infty = 0$ $\quad R = 1000$ $\quad \tau = .001s$
$i_0 = 15$ $\quad i_\infty = 5$ $\quad L = 1H$ $\quad t_1 = \ln(5/3)/1000$
$v(t) = 10\exp[-1000(t-t_1)]$
$i(t) = 5 + 10\exp[-1000(t-t_1)]$ $\quad$ for $t_1 \le t \le t_2$
t_2 @ $V = 5$ $\quad t_2 = t_1 + \ln 2/1000$ $= \ln(10/3)/1000$

Region III: $v_0 = 5$ $\quad v_\infty = 0$ $\quad R = 1000$ $\quad \tau = .001s$
$i_0 = 10$ $\quad i_\infty = 15$ $\quad L = 1H$
$v(t) = 5\exp[+1000(t-t_2)]$
$i(t) = 15 - 5\exp[+1000(t-t_2)]$ $\quad$ for $t_2 \le t \le t_3$
t_3 @ $V = 10$ $\quad t_3 = t_2 + \ln 2/1000$ $= \ln(20/3)/1000$

Region IV: $v_0 = 10$ $\quad v_\infty = 0$ $\quad R = 1000$ $\quad \tau = .001s$
$i_0 = 5$ $\quad i_\infty = -5$ $\quad L = 1H$
$v(t) = 10\exp[-1000(t-t_3)]$
$i(t) = -5 + 10\exp[-1000(t-t_3)]$ $\quad$ for $t \ge t_3$

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Plots of v(t) & i(t)