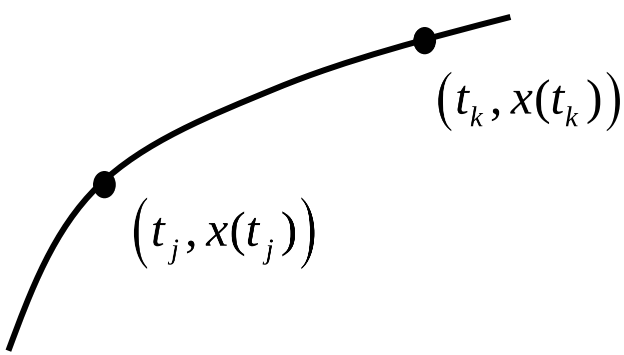
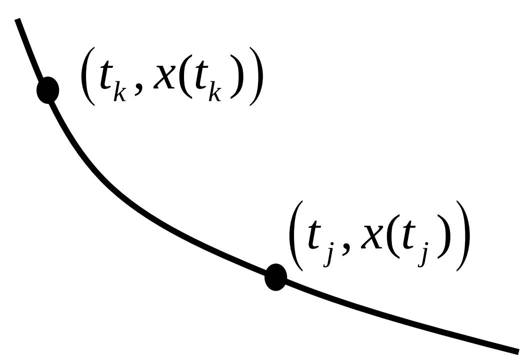
Elapsed Time Formula

$$t_k - t_j = \tau \ln \frac{x(t_j) - x(t_\infty)}{x(t_k) - x(t_\infty)}$$

$\left(t_j, x(t_j)\right)$, $\left(t_k, x(t_k)\right)$ — any point lying on an exponential waveform with positive or negative τ





1st-order Linear Time-Invariant Circuits Driven by DC Sources

State Equation

$$\dot{x} = -\frac{x}{\tau} + \frac{x(t_\infty)}{\tau}$$

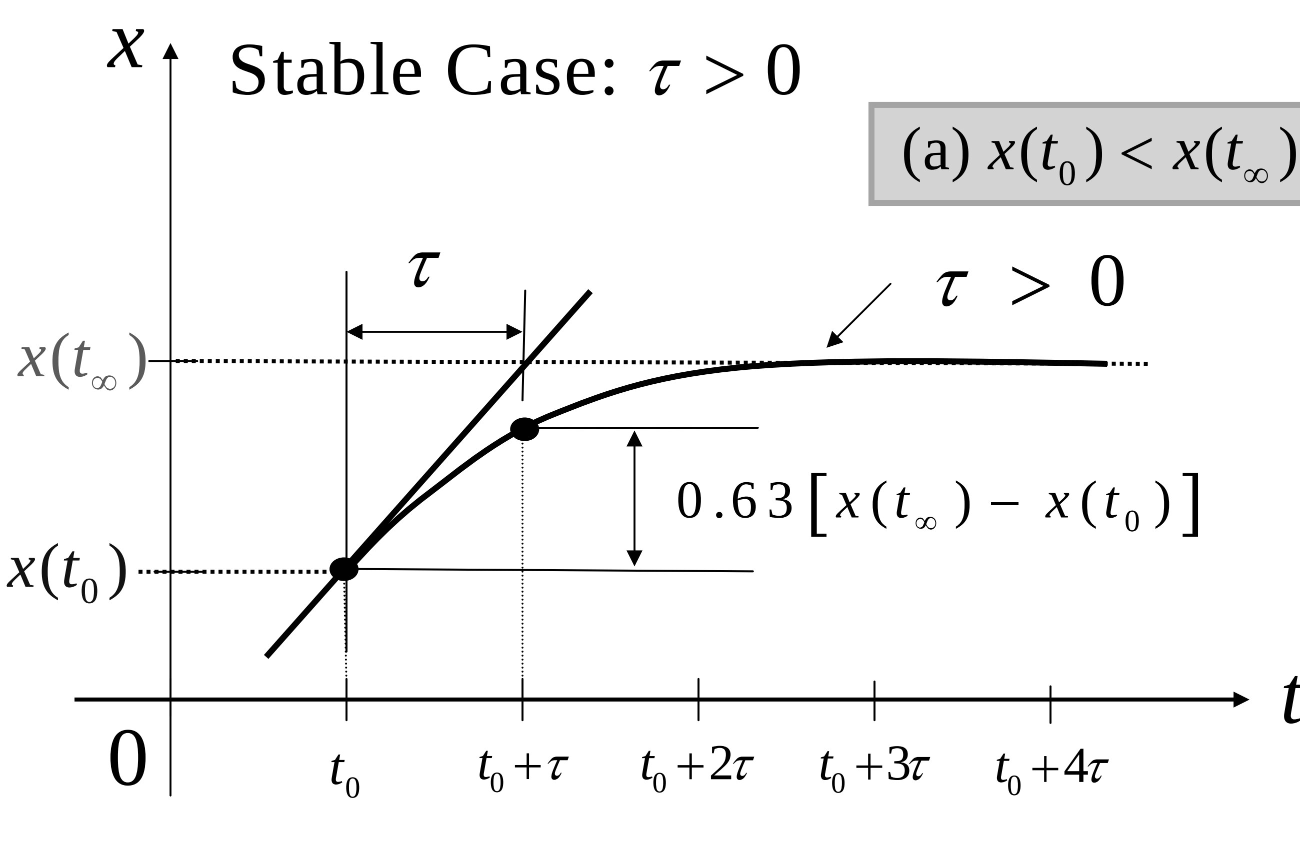
τ = time constant

$x(t_\infty)$ = Equilibrium Point

$x(t_0)$ = Initial State at $t = t_0$

Solution:

$$x(t) = x(t_\infty) + \left[x(t_0) - x(t_\infty)\right] e^{-\frac{(t-t_0)}{\tau}}$$



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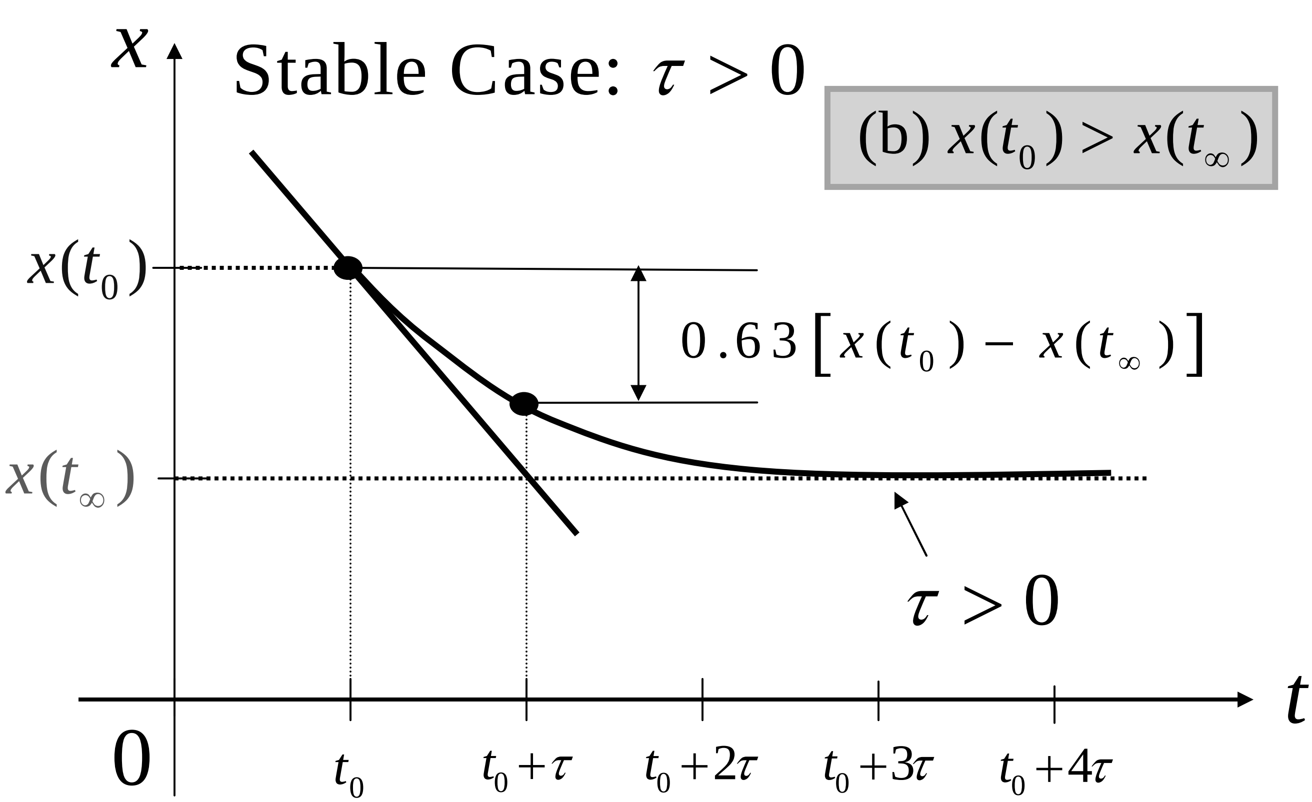
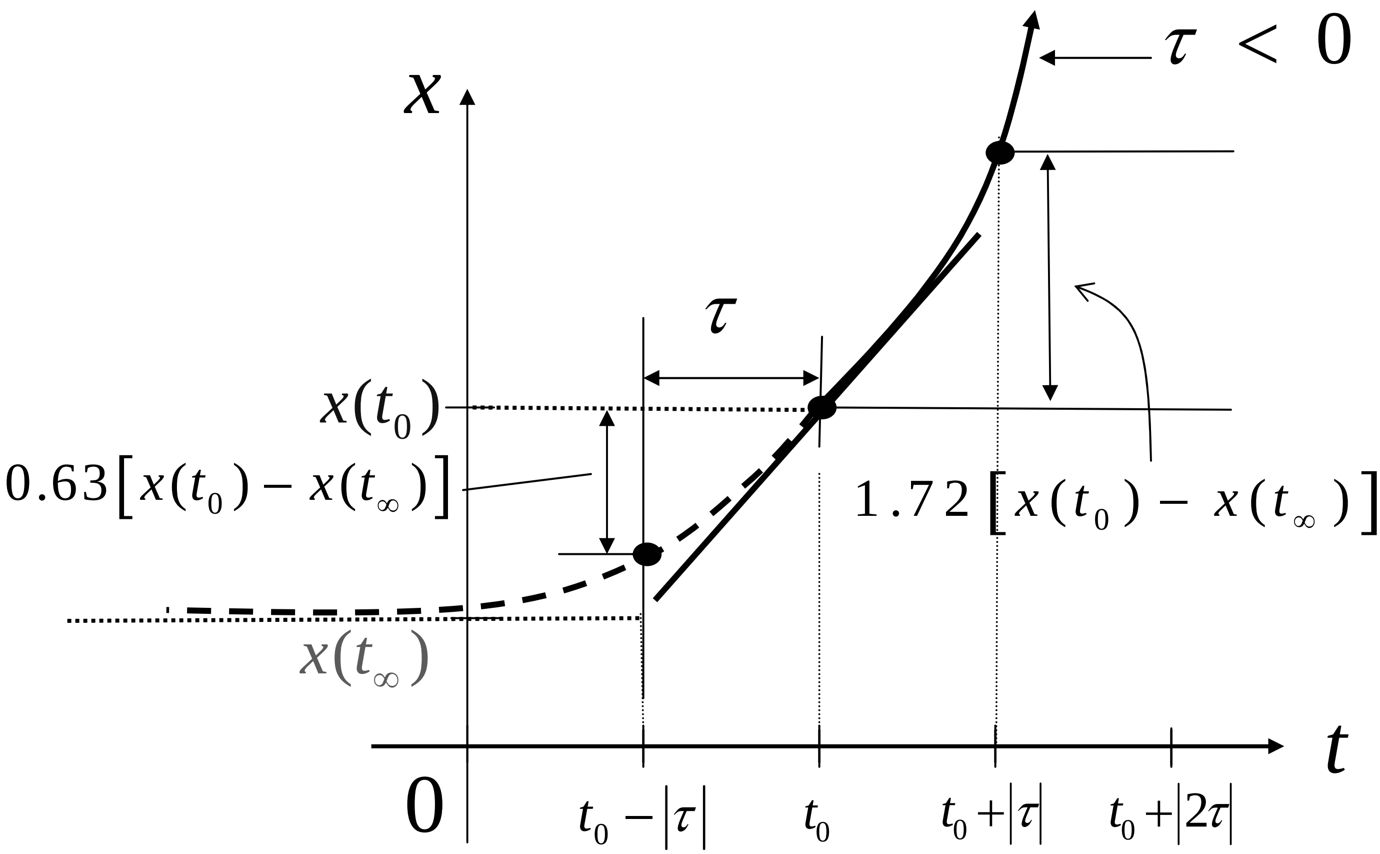
Stable Case: $\tau > 0$



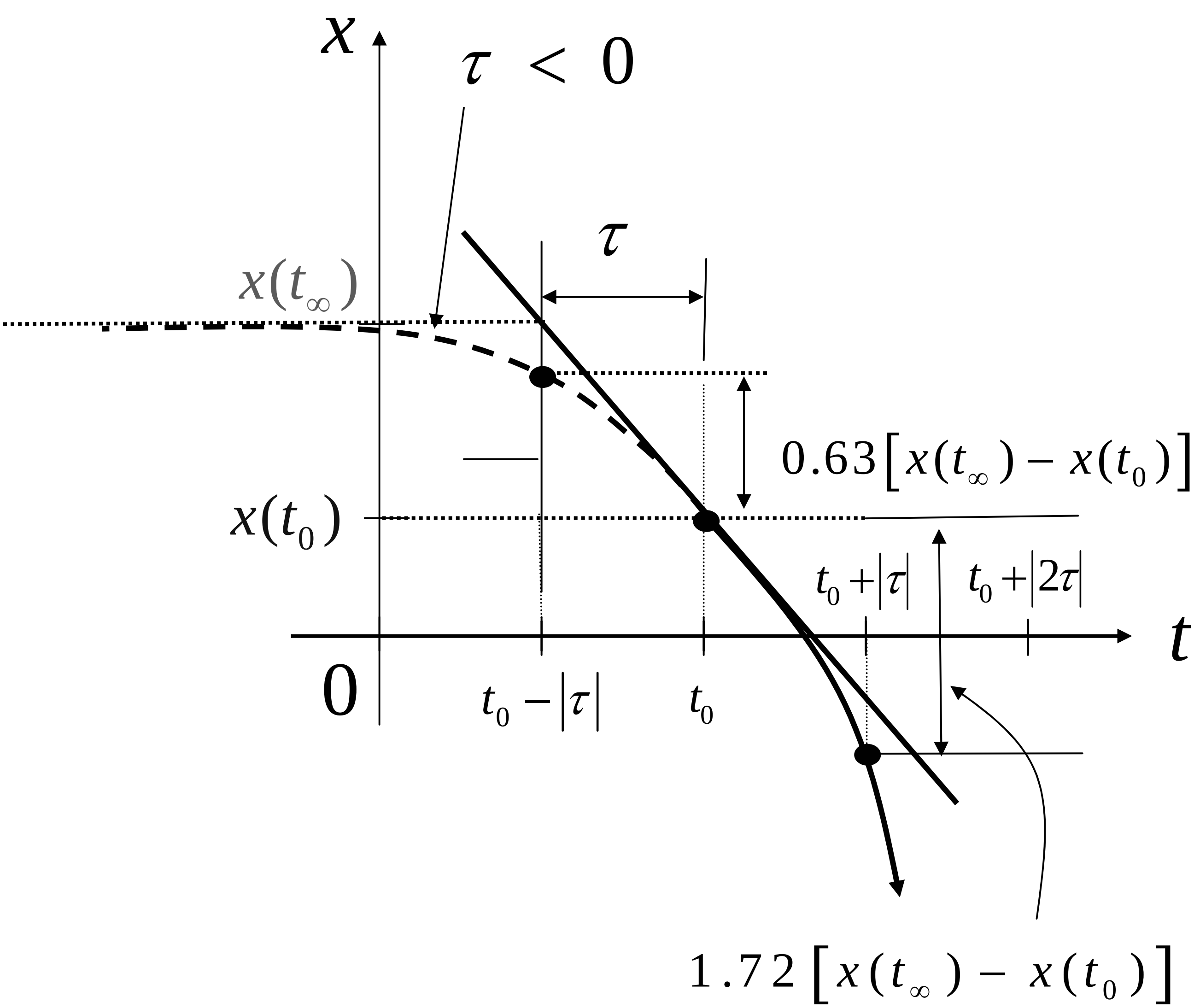
Untable Case: $\tau < 0$

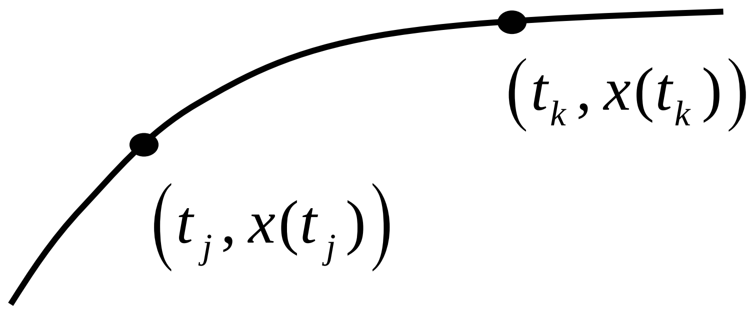
(a) $x(t_0) > x(t_\infty)$



Untable Case: $\tau < 0$

(b) $x(t_0) < x(t_\infty)$





$$x(t_j) - x(t_\infty) = \left[x(t_0) - x(t_\infty)\right] e^{-\frac{(t_j - t_0)}{\tau}}$$

$$x(t_k) - x(t_\infty) = \left[x(t_0) - x(t_\infty)\right] e^{-\frac{(t_k - t_0)}{\tau}}$$

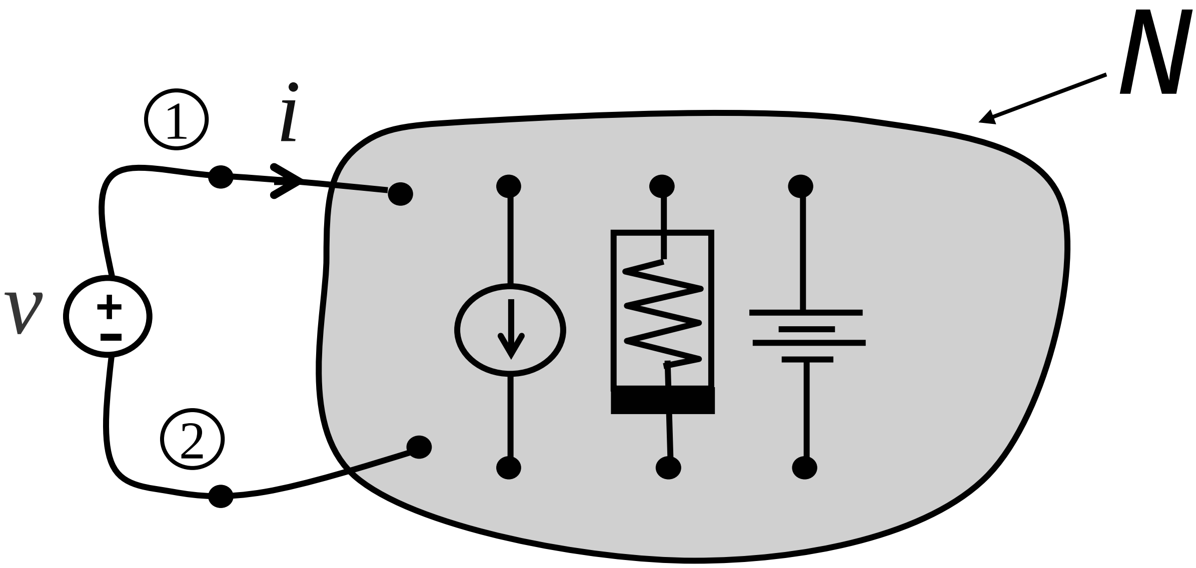
$$\frac{x(t_j) - x(t_\infty)}{x(t_k) - x(t_\infty)} = \frac{e^{-\frac{(t_j - t_0)}{\tau}}}{e^{-\frac{(t_k - t_0)}{\tau}}} = e^{\frac{(t_k - t_j)}{\tau}}$$

$$\ln\left[\frac{x(t_j) - x(t_\infty)}{x(t_k) - x(t_\infty)}\right] = \frac{(t_k - t_j)}{\tau}$$

$$\Rightarrow$$

$$\boxed{t_k - t_j = \tau \ln\left[\frac{x(t_j) - x(t_\infty)}{x(t_k) - x(t_\infty)}\right]}$$

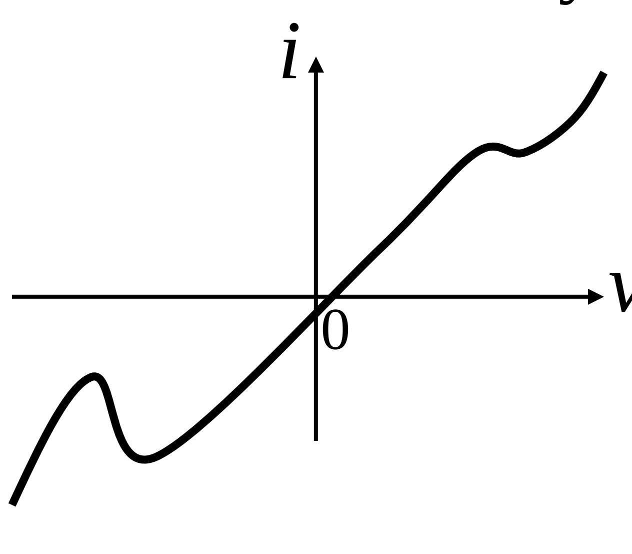
Driving-Point Characteristic



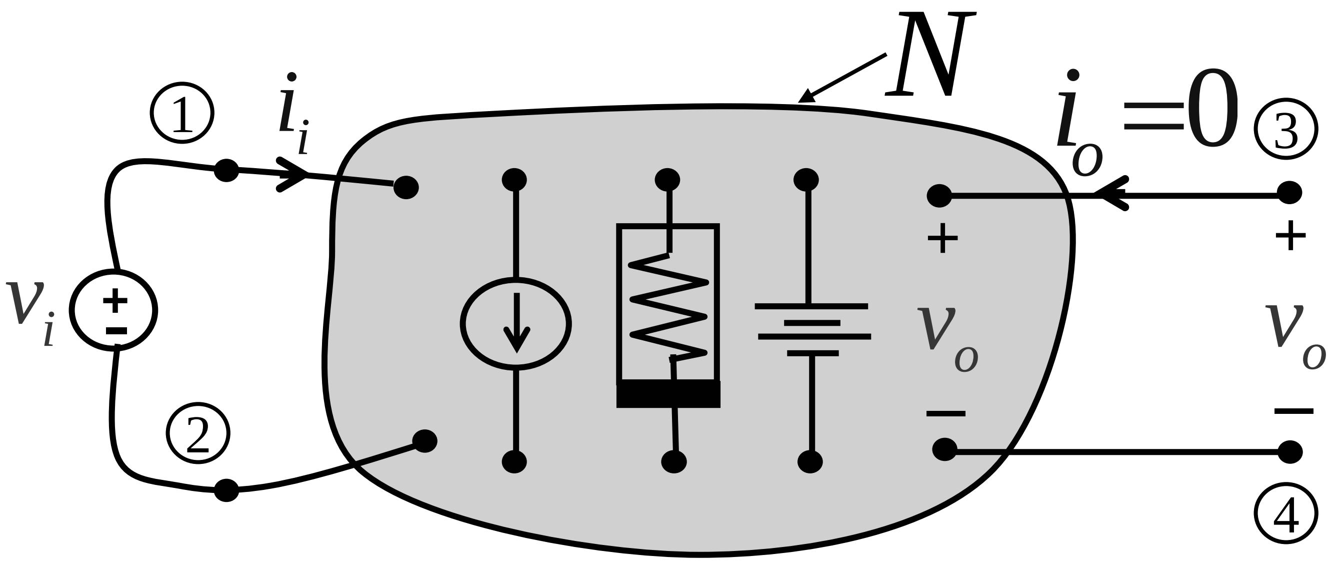
The 2 nodes { ① , ② } where the voltage source is connected are called **driving-point terminals.**

The *i*-vs.-*v* driving-point **characteristic** is the set of all (*i*,*v*) which simultaneously satisfy:

1. KCL
2. KVL
3. Constitutive Relation of all elements inside ***N***



Transfer Characteristic

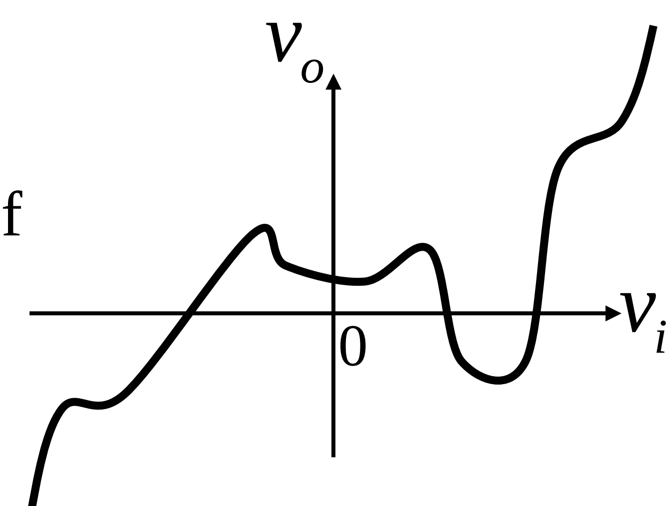


Nodes ① and ② are called driving-point terminals.

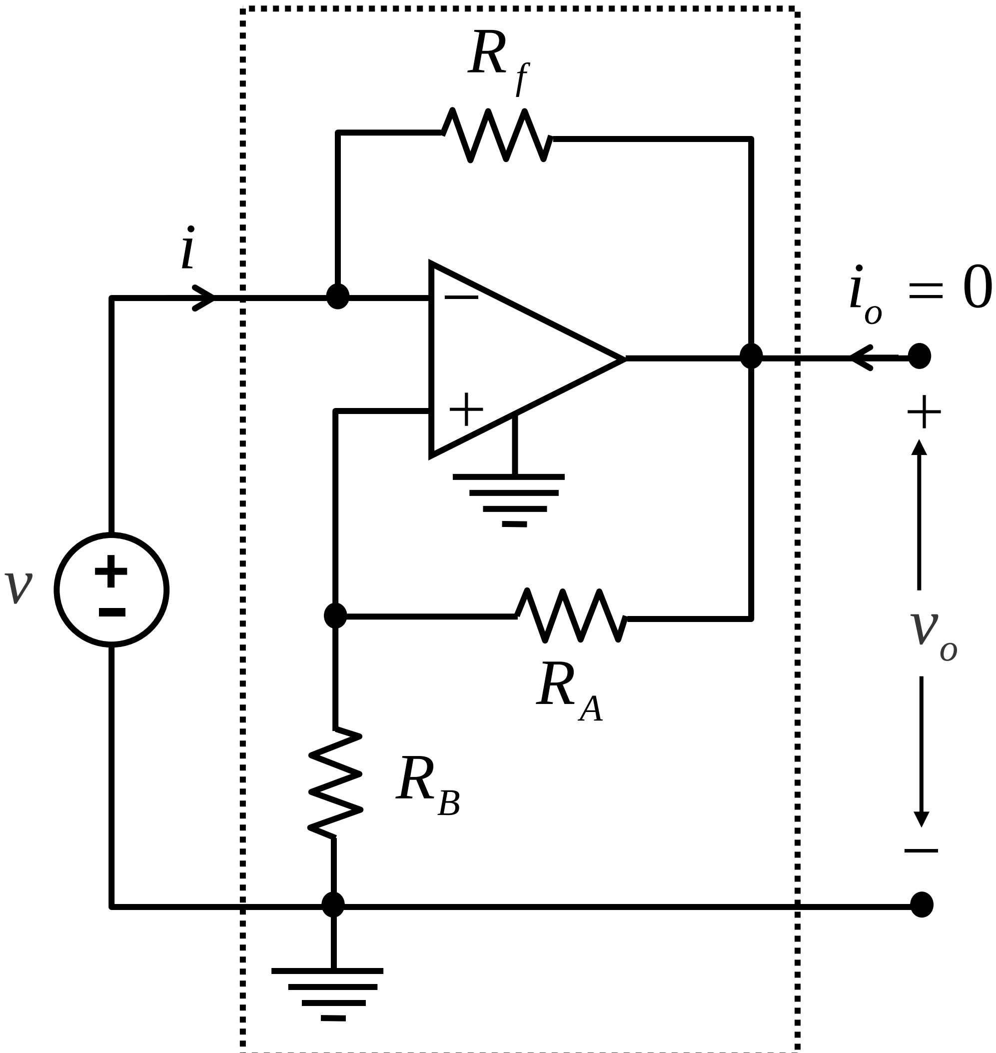
Remark:

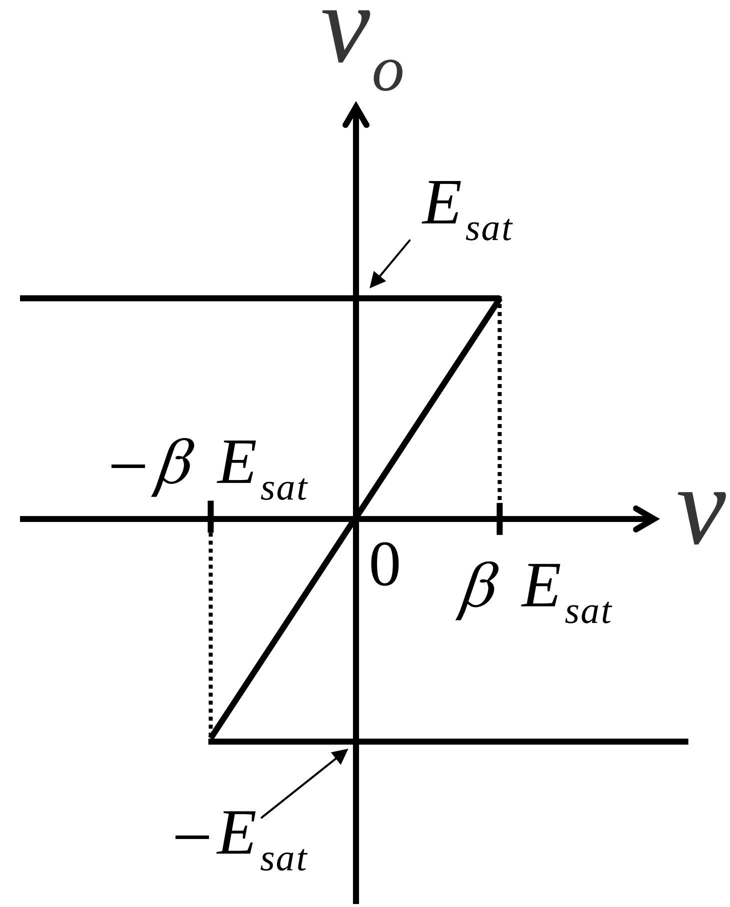
The v_o-vs.-v_i **transfer characteristic** is the set of all (v_i, v_o) which simultaneously satisfy:

1. KCL
2. KVL
3. v - i characteristics of all elements inside ***N***
4. $i_o = 0$ (no-loading condition)

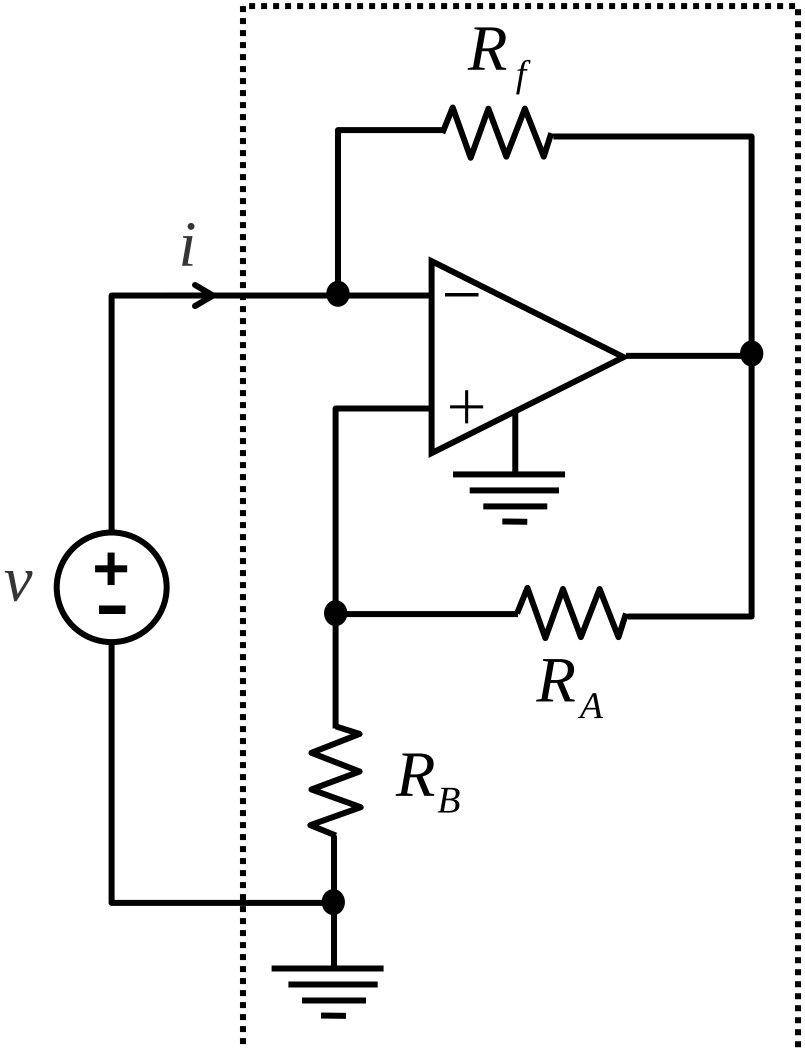


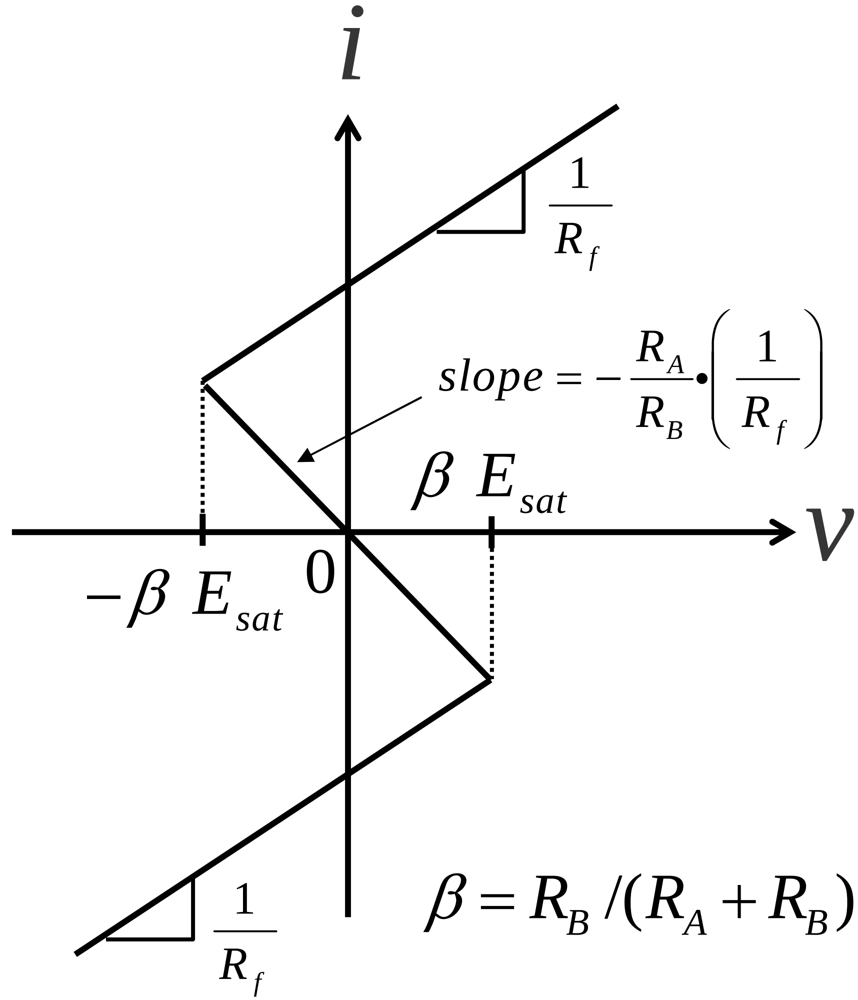
TC (Transfer) Plot





DP (Driving-Point) Plot





$$\beta = R_B / (R_A + R_B)$$