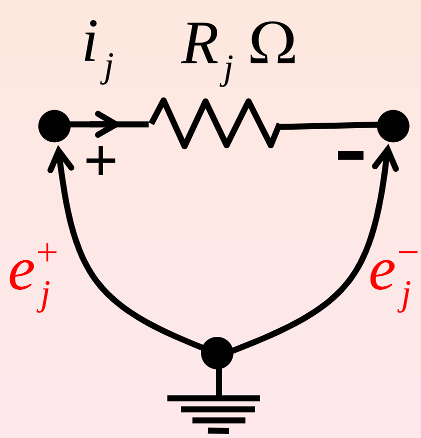
Node Voltage Method

This simplest among many circuit analysis methods is applicable only for connected circuits N made of linear 2-terminal resistors and current sources. The only variables in the **linear** equations are the n-1 node voltages e_1, e_2, …, e_{n-1} for an n-node circuit.

Step 1. Choose an arbitrary datum node and label the remaining nodes consecutively ①, ②, …, (**n-1**), and let e_1, e_2, …, e_{n-1} be node-to-datum voltages.

Step 2. Express the current of each resistor R_j via Ohm's law in terms of 2 node-to-datum voltages:



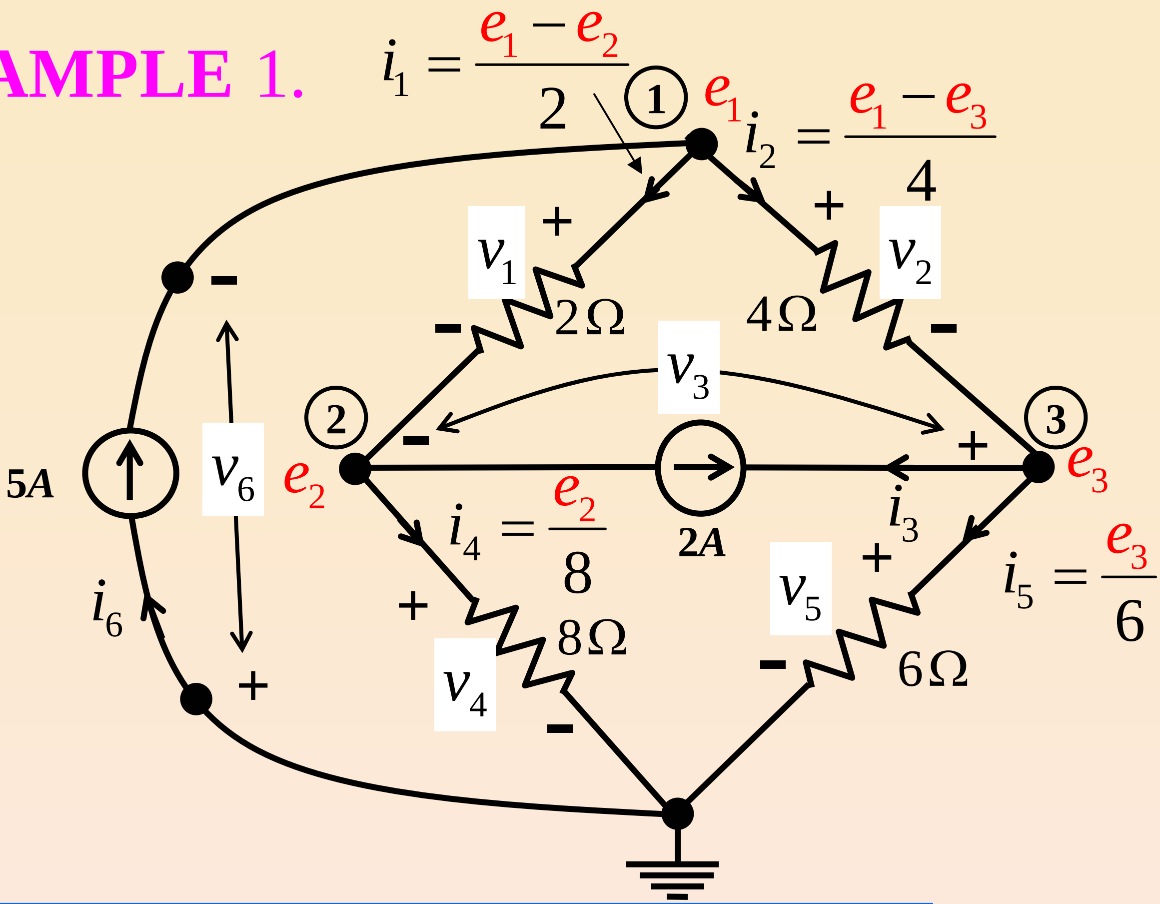
$$i_j = \frac{e_j^+ - e_j^-}{R_j} \quad (1)$$

Step 3. Apply KCL to each node ①, ②, …, (**n-1**) with each resistor current i_j expressed in terms of e_j^+ and e_j^-.

Step 4. Solve the (n-1) independent linear equations for e_1, e_2, …, e_{n-1}.

Step 5. Solve for the resistor currents via Eq.(1).

EXAMPLE 1.



$$\text{KCL at } ① : \quad \frac{(e_1 - e_2)}{2} + \frac{(e_1 - e_3)}{4} = 5 \qquad (2)$$

$$\text{KCL at } ② : \quad \frac{-(e_1 - e_2)}{2} + \frac{e_2}{8} = -2 \qquad (3)$$

$$\text{KCL at } ③ : \quad \frac{-(e_1 - e_3)}{4} + \frac{e_3}{6} = 2 \qquad (4)$$

Recast Eqs. (2), (3), and (4) in matrix form :

Node voltage Equations

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{5}{8} & 0 \\ -\frac{1}{4} & 0 & \frac{5}{12} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix} \qquad (5)$$

Solving eq. (5) by Cramer's Rule (or any other method) :

$$\Delta \triangleq \det\begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{5}{8} & 0 \\ -\frac{1}{4} & 0 & \frac{5}{12} \end{bmatrix} = \frac{20}{384} \qquad (6)$$

(a) To solve for e_1, replace column 1 of matrix from (5) and calculate

$$\Delta_1 \triangleq \det\begin{bmatrix} 5 & -\frac{1}{2} & -\frac{1}{4} \\ -2 & \frac{5}{8} & 0 \\ 2 & 0 & \frac{5}{12} \end{bmatrix} = \frac{115}{96} \qquad (7)$$

Cramer's Rule $\Rightarrow$

$$e_1 = \frac{\Delta_1}{\Delta} = \left(\frac{115}{96}\right) \Big/ \left(\frac{20}{384}\right) = 23V \qquad (8)$$

(b) To solve for e_2, replace column 2 of matrix from (5) and calculate

$$\Delta_2 \triangleq \det\begin{bmatrix} \frac{3}{4} & 5 & -\frac{1}{4} \\ -\frac{1}{2} & -2 & 0 \\ -\frac{1}{4} & 2 & \frac{5}{12} \end{bmatrix} = \frac{76}{96} \qquad (9)$$

Cramer's Rule $\Rightarrow$

$$e_2 = \frac{\Delta_2}{\Delta} = \left(\frac{76}{96}\right) \Big/ \left(\frac{20}{384}\right) = 15.2V$$

(c) To solve for e_3, replace column 3 of matrix from (5) and calculate

$$\Delta_3 \triangleq \det\begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & 5 \\ -\frac{1}{2} & \frac{5}{8} & -2 \\ -\frac{1}{4} & 0 & \frac{2}{12} \end{bmatrix} = \frac{93}{96} \qquad (10)$$

Cramer's Rule $\Rightarrow$

$$e_3 = \frac{\Delta_3}{\Delta} = \left(\frac{93}{96}\right) \Big/ \left(\frac{20}{384}\right) = 18.6V \qquad (11)$$

Once the node-to-datum voltages $\{e_1, e_2, e_3\}$ are found, all resistor currents and voltages are found trivially via KVL, KCL, and Ohm's law :

$$v_1 = e_1 - e_2 = 7.8\,V \qquad , i_1 = \frac{7.8}{2} = 3.9\,A$$

$$v_2 = e_1 - e_3 = 4.4\,V \qquad , i_2 = \frac{4.4}{4} = 1.1\,A$$

$$v_3 = e_3 - e_2 = 3.4\,V \qquad , i_3 = -2\,A$$

$$v_4 = e_2 = 15.2\,V \qquad , i_4 = \frac{15.2}{8} = 1.9\,A$$

$$v_5 = e_3 = 18.6\,V \qquad , i_5 = \frac{18.6}{6} = 3.1\,A$$

$$v_6 = -e_1 = -23\,V \qquad , i_6 = 5\,A$$

Verification of Solution

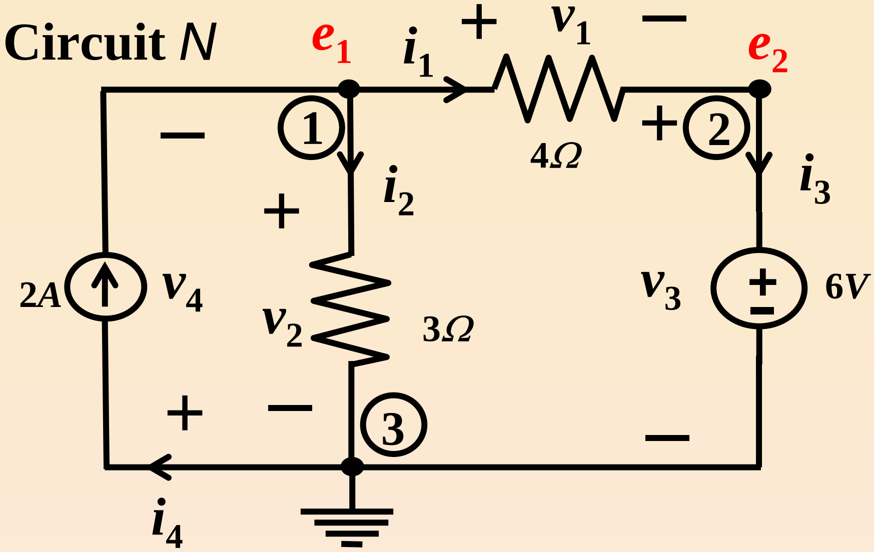
Apply Tellegen's Theorem :

$$\sum_{j=1}^{6} v_j\, i_j = (v_1\, i_1) + (v_2\, i_2) + (v_3\, i_3) + (v_4\, i_4) + (v_5\, i_5) + (v_6\, i_6)$$

$$= (7.8)(3.9) + (4.4)(1.1) + (3.4)(-2) + (15.2)(1.9) + (18.6)(3.1) + (-23)(5)$$

$$\stackrel{?}{=} 0$$

Modified Node Voltage Method



$$i_1 = \frac{e_1 - e_2}{4}$$

$$i_2 = \frac{e_1}{3}$$

Step 1.

When the circuit contains "α" voltages $v_{s_1}, v_{s_2}, \cdots, v_{s_\alpha}$, use their associated currents $i_{s_1}, i_{s_2}, \cdots, i_{s_\alpha}$ when applying KCL.

KCL at ① : $$\frac{e_1}{3} + \frac{(e_1 - e_2)}{4} = 2 \qquad (1)$$

KCL at ② : $$-\frac{(e_1 - e_2)}{4} + i_3 = 0 \qquad (2)$$

Step 2.

For each voltage source v_{s_j}, add an equation $e_j^+ - e_j^- = v_{s_j}$.

$$e_2 = 6 \qquad (3)$$

Step 3.

Solve the $(n\text{-}1) + \alpha$ equations for $e_1, e_2, \cdots e_{n-1}, i_{s_1}, i_{s_2}, \cdots i_{s_\alpha}$.

Substituting (3) into (1), we obtain :

$$\frac{e_1}{3} + \frac{(e_1 - 6)}{4} = 2 \quad \Rightarrow \quad \boxed{e_1 = 6V} \qquad (4)$$

Substituting (4) into (2), we obtain :

$$\boxed{i_3 = 0} \qquad (5)$$

Explicit matrix form of Node Voltage Equations

Assumption: Circuit N contains ***only linear*** resistors and independent current sources which do not form cut sets.

Step 1. Delete all "β" current sources from N and draw the **reduced** digraph G of the remaining pure resistor circuit. Assume G has "n" nodes and "b" branches.

Step 2. Pick a datum node and label the node-to-datum voltages $\{ e_1, e_2, \ldots, e_{n-1} \}$, and derive the **reduced-incidence matrix A**. Define the "branch admittance matrix" $\mathbf{Y}_b$ and independent current source vector $\mathbf{i}_s$ as follow:

$$\underbrace{\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_b \end{bmatrix}}_{\mathbf{i}} = \underbrace{\begin{bmatrix} Y_1 & 0 & 0 & \cdots & 0 \\ 0 & Y_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & Y_b \end{bmatrix}}_{\mathbf{Y}_b} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_b \end{bmatrix}}_{\mathbf{v}} \quad (1), \qquad \mathbf{i}_s \triangleq \begin{bmatrix} i_{s_1} \\ i_{s_2} \\ \vdots \\ i_{s_\beta} \end{bmatrix} \quad (2)$$

where $Y_j \triangleq \dfrac{1}{R_j}$, $R_j =$ resistance of branch j.

i_{s_m} = algebraic sum of all current sources **entering** node ⓜ, $m = 1, 2, \ldots, n-1$.

Step 3. Form the Node voltage equation

$$\mathbf{Y}_n \mathbf{e} = \mathbf{i}_s \qquad (3)$$

where $\mathbf{Y}_n \triangleq \mathbf{A}\mathbf{Y}_b\mathbf{A}^T$ is called the **node-admittance matrix**.

Deriving the Node Voltage Equation $\left(\mathbf{Y}_n \mathbf{e} = \mathbf{i}_s\right)$

The "β " current sources can be deleted since they can be trivially accounted for by representing their net contribution at each node Ⓜ by the algebraic sum of all current sources **entering** node Ⓜ, $m = 1, 2, \ldots, n\text{-}1$. The KCL equations therefore takes the "augmented" form

$$\mathbf{A}\,\mathbf{i} = \mathbf{i}_s \tag{4}$$

Substituting (1) for **i** in (4), we obtain

$$\mathbf{A}\,\mathbf{Y}_b \mathbf{v} = \mathbf{i}_s \tag{5}$$

Substituting KVL

$$\mathbf{v} = \mathbf{A}^T\,\mathbf{e} \tag{6}$$

for **v** in (5), we obtain

$$\underbrace{\left(\mathbf{A}\,\mathbf{Y}_b \mathbf{A}^T\right)}_{\mathbf{Y}_n} \mathbf{e} = \mathbf{i}_s \tag{7}$$

■

Writing Node-Admittance Matrix $\mathbf{Y}_n$ By Inspection

Node Votage Equation :

$$\underbrace{\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1,n-1} \\ Y_{21} & Y_{22} & \cdots & Y_{2,n-1} \\ \vdots & \vdots & \cdots & \vdots \\ Y_{n-1,1} & Y_{n-1,2} & \cdots & Y_{n-1,n-1} \end{bmatrix}}_{\mathbf{Y}_n} \underbrace{\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{n-1} \end{bmatrix}}_{\mathbf{e}} = \underbrace{\begin{bmatrix} i_{s_1} \\ i_{s_2} \\ \vdots \\ i_{s_{n-1}} \end{bmatrix}}_{\mathbf{i}_s} \qquad (8)$$

Diagonal Elements of $\mathbf{Y}_n$

Y_{mm} = sum of admittances $Y_j \triangleq \dfrac{1}{R_j}$ of all resistors connected to node (m) , $m = 1,\ 2,\ \ldots,\ n-1$

Off-Diagonal Elements of $\mathbf{Y}_n$

Y_{jk} = - (sum of admittances $Y_j \triangleq \dfrac{1}{R_j}$ of all resistors connected across node (j) and node (k))

Symmetry Property:

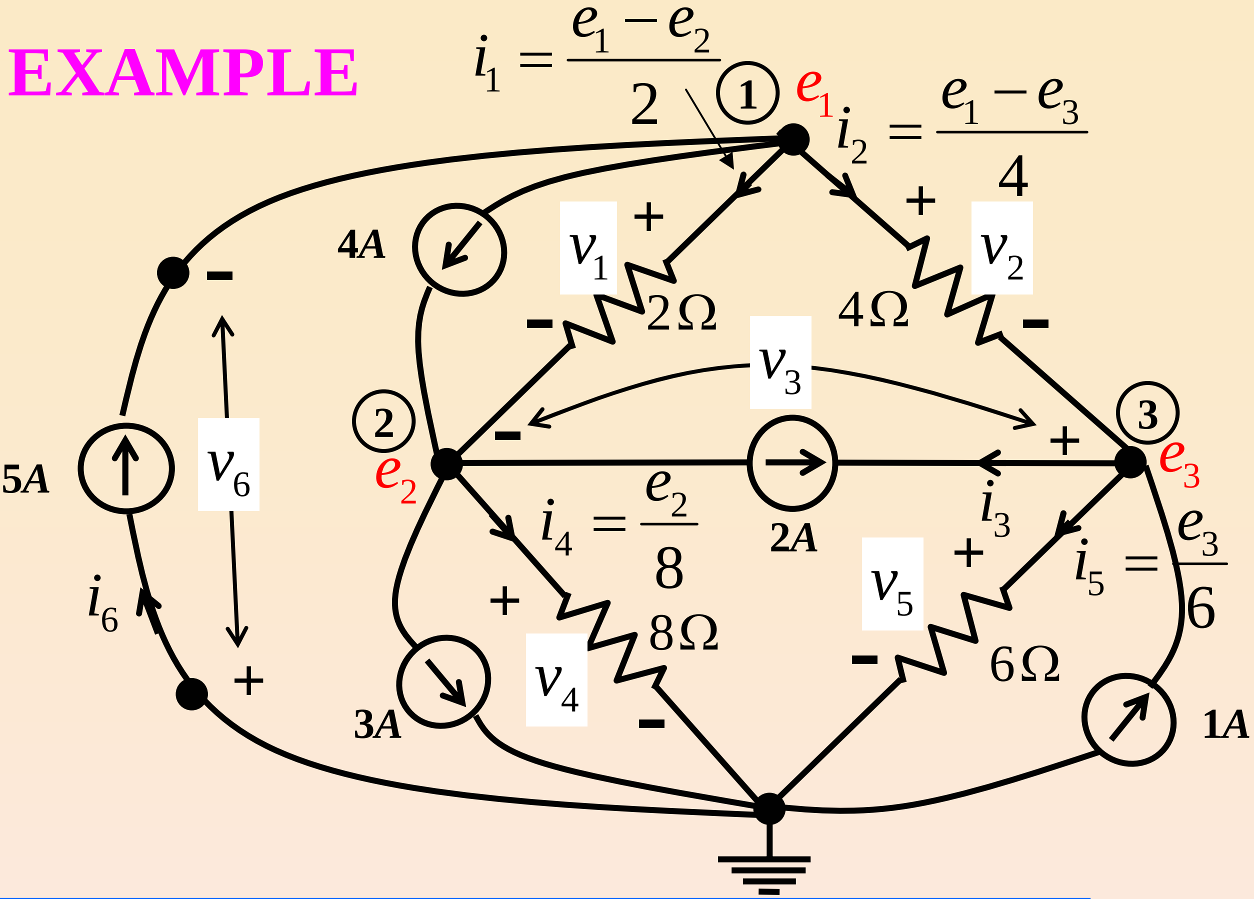
$\mathbf{Y}_n$ is a symmetric matrix, i.e., $Y_{jk} = Y_{kj}$

Proof :

Since $\mathbf{Y}_b$ in (1) is a diagonal matrix, $\mathbf{Y}_b = \mathbf{Y}_b^T$

$$\mathbf{Y}_n^T = \left(\mathbf{A}\mathbf{Y}_b\mathbf{A}^T\right)^T = \mathbf{A}\mathbf{Y}_b^T\mathbf{A}^T = \mathbf{A}\mathbf{Y}_b\mathbf{A}^T = \mathbf{Y}_n \qquad \blacksquare$$

EXAMPLE



KCL at ① : $\frac{(e_1 - e_2)}{2} + \frac{(e_1 - e_3)}{4} = 5 - 4$ (2)

KCL at ② : $\frac{-(e_1 - e_2)}{2} + \frac{e_2}{8} = 4 - 2 - 3$ (3)

KCL at ③ : $\frac{-(e_1 - e_3)}{4} + \frac{e_3}{6} = 2 + 1$ (4)

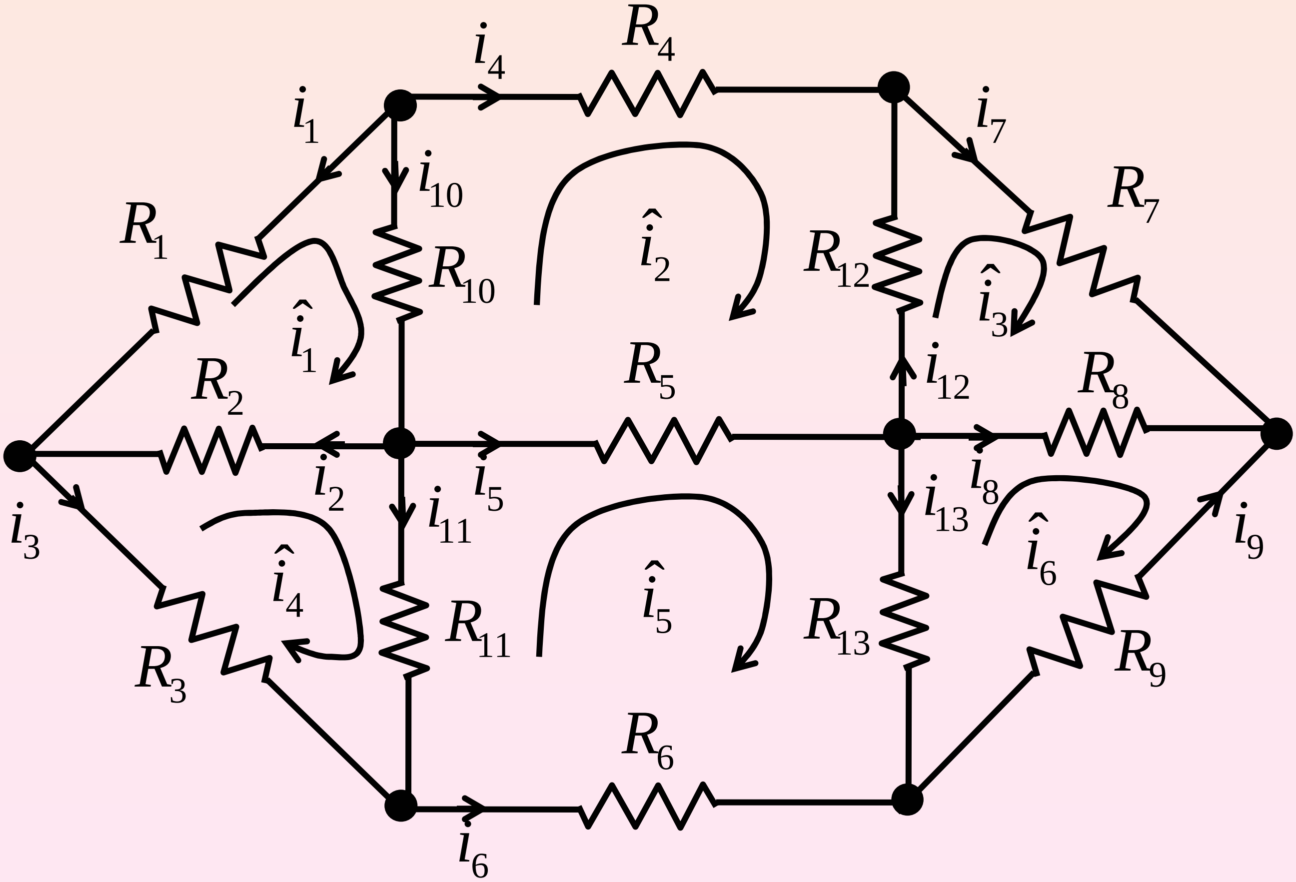
Matrix Form :

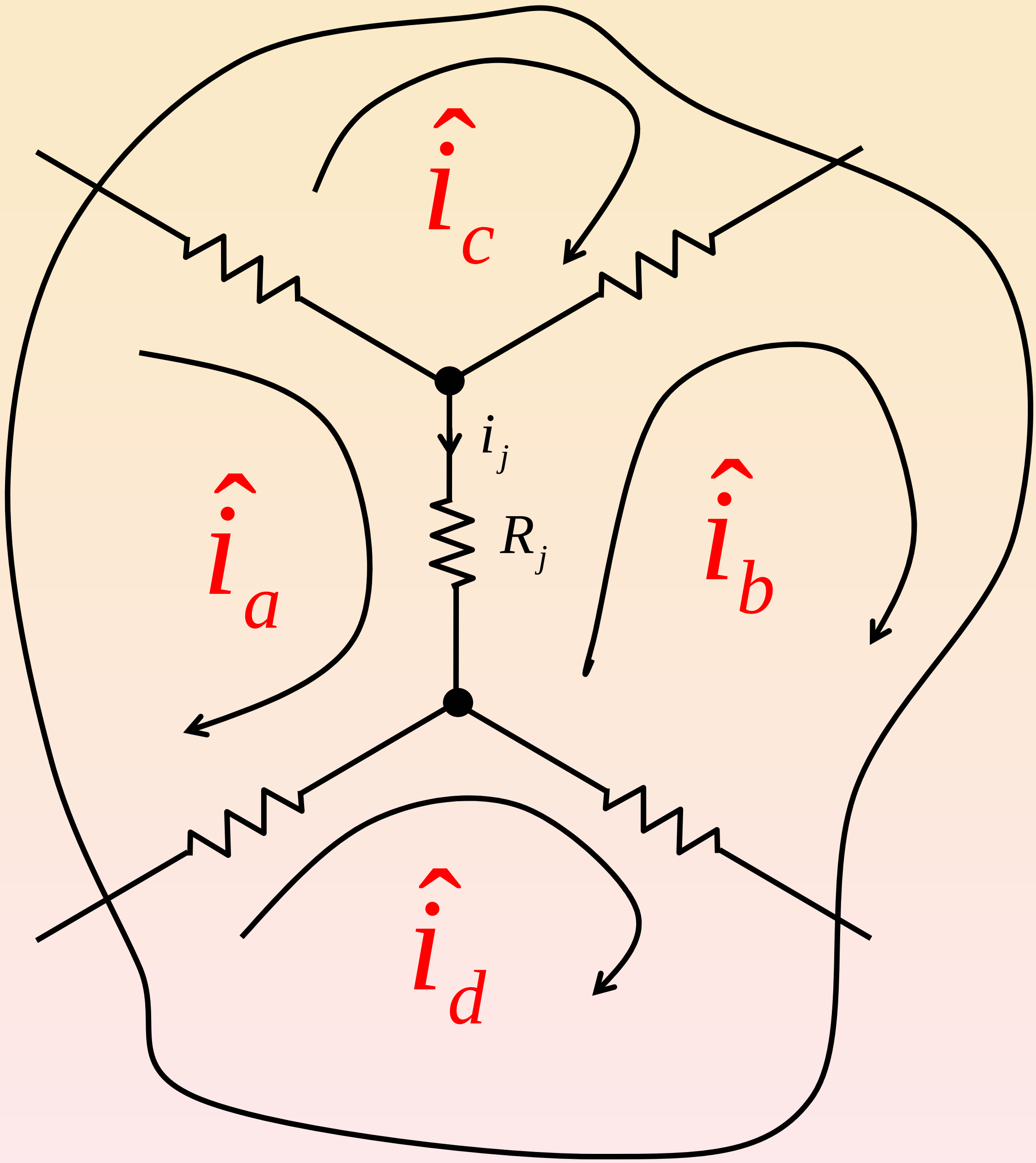
Node Equations

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{5}{8} & 0 \\ -\frac{1}{4} & 0 & \frac{5}{12} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} \quad (5)$$

Mesh Current Method

The next simplest among many circuit analysis methods is applicable only for connected **planar** circuit N (with a **planar** digraph) made of 2-terminal **linear** resistors and voltage sources. The only variables in the equations are " l " **conceptual mesh** currents $\hat{i}_{m_1}, \hat{i}_{m_2}, \cdots, \hat{i}_{m_l}$ circulating in the " l " meshes in a **clockwise** direction (by convention) :





$$i_j = \hat{i}_a - \hat{i}_b$$

Mesh Current Method

The next simplest among many circuit analysis methods is applicable only for connected **planar** circuits made of 2-terminal **linear** resistors and voltage sources. The only variables in the associated "**mesh-current equations**" are " l " **mesh current** $\hat{i}_{m_1}$, $\hat{i}_{m_2}, \cdots, \hat{i}_{m_l}$ which we **define** to be circulating in the " l " meshes in a **clockwise** direction (by convention). Unlike node-to-datum voltages in the node voltage method which are physical in the sense they can be measured by a volt meter, the "mesh" currents are **abstract** variables introduced mathematically for writing a set of equations whose solution can be used to find each resistor current i_j trivially via

$$i_j = \hat{i}_a - \hat{i}_b$$

Where $\hat{i}_a$ (resp., $\hat{i}_b$) is the circulating current flowing through R_j in the same (resp., opposite) direction as the reference current i_j.

Mesh Current Equations

$$\begin{bmatrix} 6 & 0 & -2 \\ 0 & 14 & -8 \\ -2 & -8 & 10 \end{bmatrix} \begin{bmatrix} \hat{i}_1 \\ \hat{i}_2 \\ \hat{i}_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix} \quad (1)$$

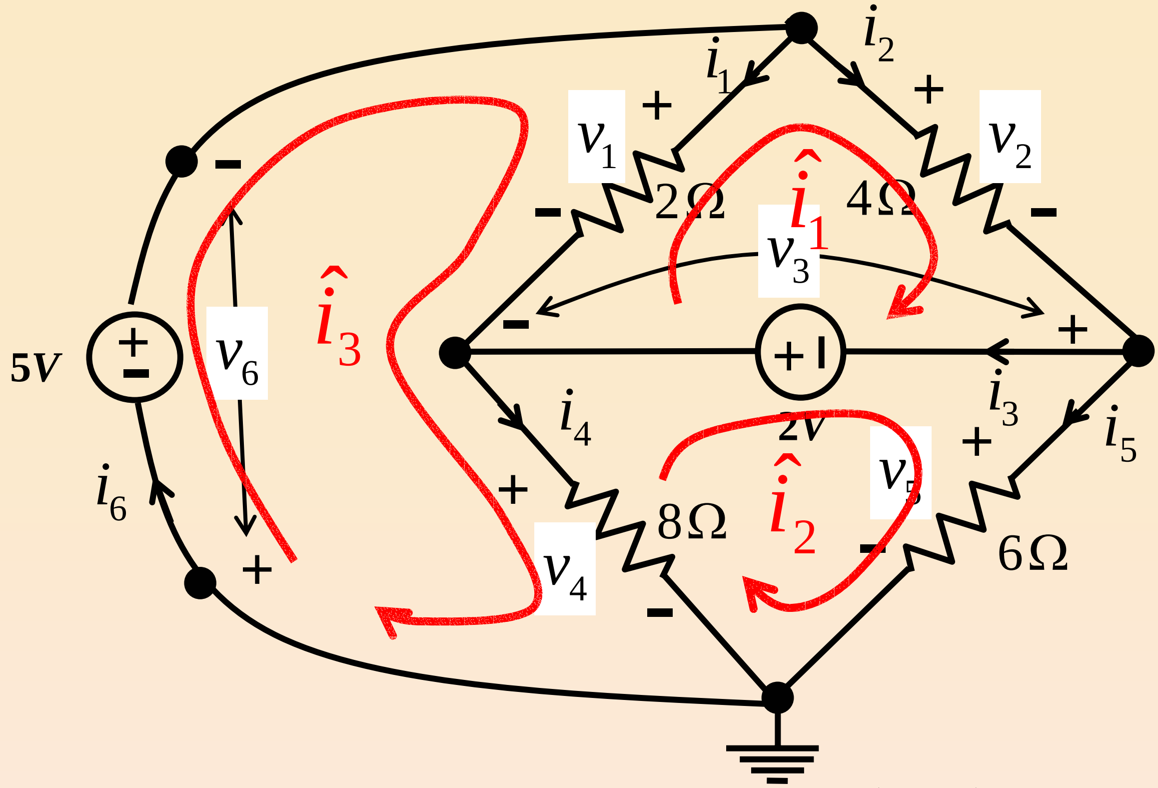
Then calculate :

$$\hat{i}_1 = \frac{13}{20}, \quad \hat{i}_2 = \frac{8}{20}, \quad \hat{i}_3 = \frac{19}{20} \quad (2)$$

$$\left.\begin{aligned}
i_1 &= \hat{i}_3 - \hat{i}_1 = \frac{3}{10}, & v_1 &= 2i_1 = \frac{6}{10} \\
i_2 &= \hat{i}_1 = \frac{13}{20}, & v_2 &= 4i_2 = \frac{13}{5} \\
i_3 &= \hat{i}_1 - \hat{i}_2 = \frac{5}{20}, & v_3 &= -2 \\
i_4 &= \hat{i}_3 - \hat{i}_2 = \frac{11}{20}, & v_4 &= 8i_4 = \frac{22}{5} \\
i_5 &= \hat{i}_2 = \frac{8}{20}, & v_5 &= 6i_5 = \frac{12}{5} \\
i_6 &= \hat{i}_3 = \frac{19}{20}, & v_6 &= -5
\end{aligned}\right\} \quad (3)$$

Verification by Tellegen's Theorem :

$$\sum_{j=1}^{6} v_j i_j = \left(\frac{6}{10}\right)\left(\frac{3}{10}\right) + \left(\frac{13}{5}\right)\left(\frac{13}{20}\right) + (-2)\left(\frac{5}{20}\right) + \left(\frac{22}{5}\right)\left(\frac{11}{20}\right) + \left(\frac{12}{5}\right)\left(\frac{8}{20}\right) + (-5)\left(\frac{19}{20}\right) \overset{?}{=} 0$$



$$i_1 = \hat{i}_3 - \hat{i}_1 \quad , \quad v_1 = 2\left(\hat{i}_3 - \hat{i}_1\right)$$

$$i_2 = \hat{i}_1 \quad , \quad v_2 = 4\hat{i}_1$$

$$i_3 = \hat{i}_1 - \hat{i}_2 \quad , \quad v_3 = -2$$

$$i_4 = \hat{i}_3 - \hat{i}_2 \quad , \quad v_4 = 8\left(\hat{i}_3 - \hat{i}_2\right)$$

$$i_5 = \hat{i}_2 \quad , \quad v_5 = 6\hat{i}_2$$

$$i_6 = \hat{i}_3 \quad , \quad v_6 = -5$$

Loop equation around mesh 1:

$$-v_1 + v_2 + v_3 = 0 \quad \Rightarrow \quad -2(\hat{i}_3 - \hat{i}_1) + 4\hat{i}_1 - 2 = 0$$

$$\Rightarrow \quad 6\hat{i}_1 - 2\hat{i}_3 = 2 \qquad (1)$$

Loop equation around mesh 2:

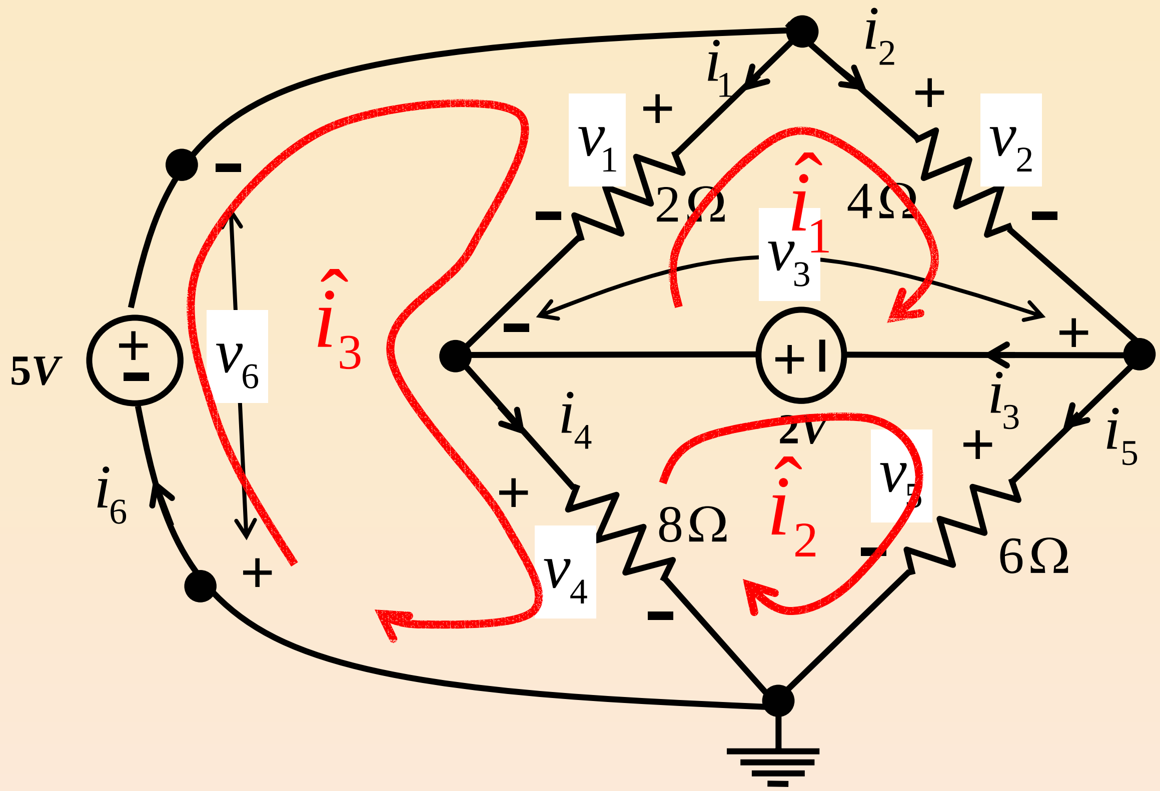
$$-v_3 + v_5 - v_4 = 0 \quad \Rightarrow \quad -(-2) + 6\hat{i}_2 - 8(\hat{i}_3 - \hat{i}_2) = 0$$

$$\Rightarrow \quad 14\hat{i}_2 - 8\hat{i}_3 = -2 \qquad (2)$$

Loop equation around mesh 3:

$$v_6 + v_1 + v_4 = 0 \quad \Rightarrow \quad -5 + 2(\hat{i}_3 - \hat{i}_1) + 8(\hat{i}_3 - \hat{i}_2) = 0$$

$$\Rightarrow \quad -2\hat{i}_1 - 8\hat{i}_2 + 10\hat{i}_2 = 5 \qquad (3)$$



Mesh 1: $6\hat{i}_1 - 2\hat{i}_3 = 2$ (1)

Mesh 2: $14\hat{i}_2 - 8\hat{i}_3 = -2$ (2)

Mesh 3: $-2\hat{i}_1 - 8\hat{i}_2 + 10\hat{i}_3 = 5$ (3)

Solving $\hat{i}_1$ from (1) $\Rightarrow$ $\hat{i}_1 = \frac{1}{3}\hat{i}_3 - \frac{2}{3}$ (4)

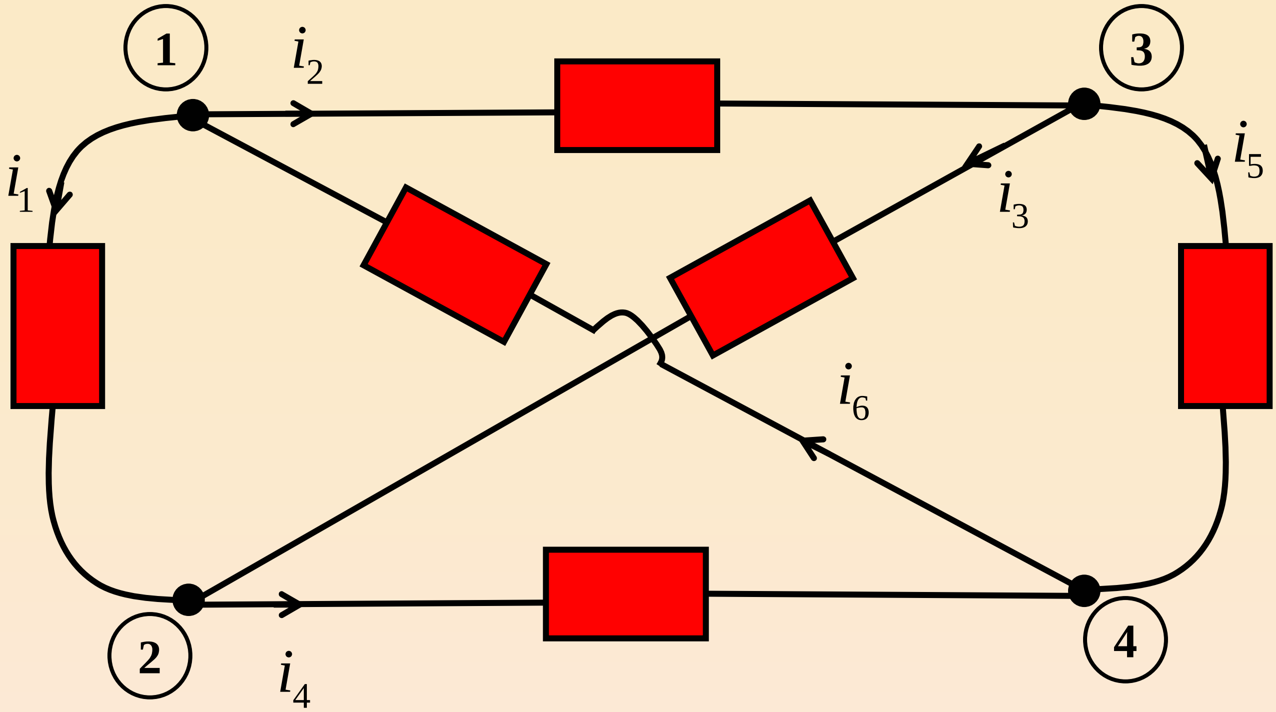
Solving $\hat{i}_2$ from (2) $\Rightarrow$ $\hat{i}_2 = \frac{4}{7}\hat{i}_3 - \frac{1}{7}$ (5)

Substituting (4) and (5) into (3) $\Rightarrow$

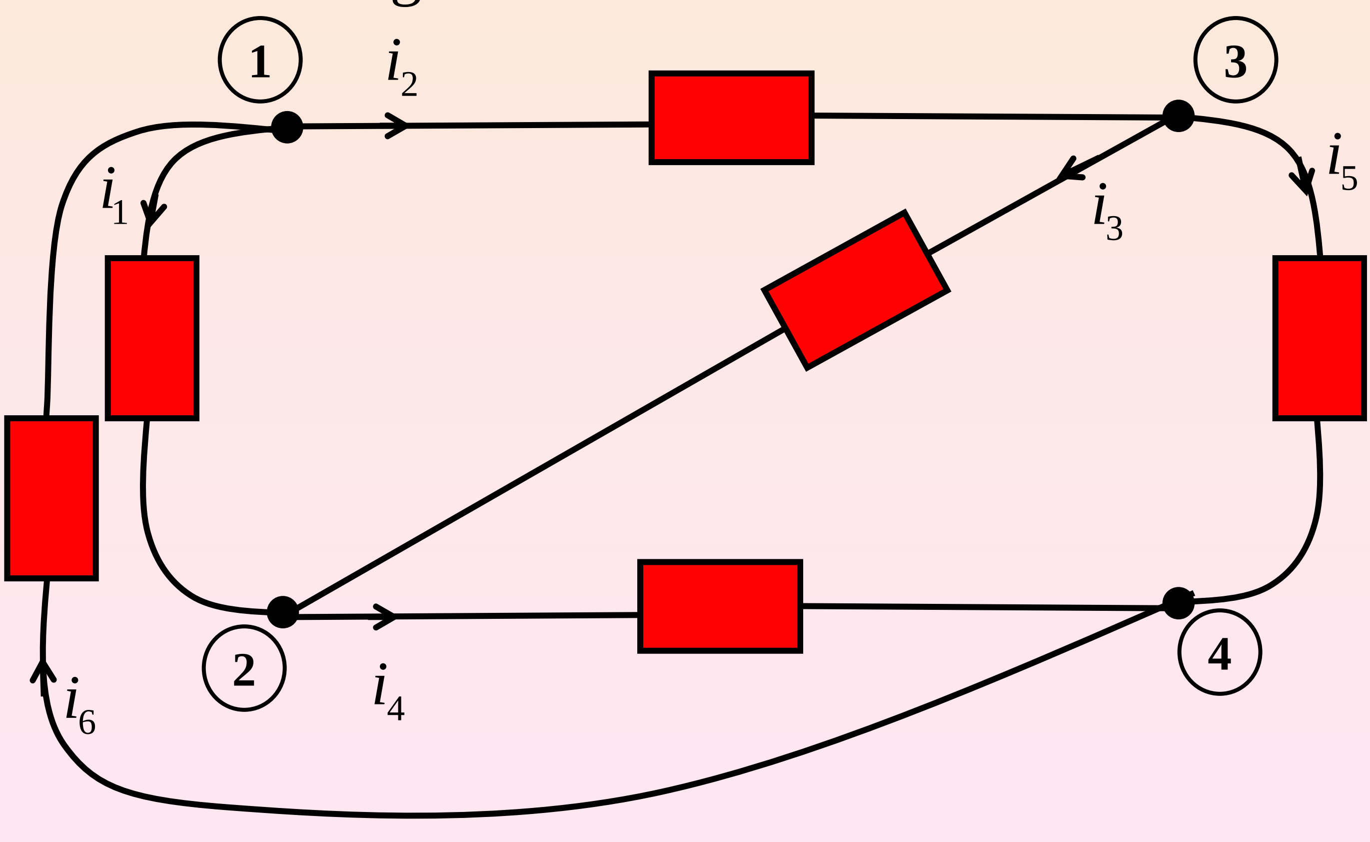
$$\hat{i}_3 = \frac{19}{12} A \qquad (6)$$

(5) and (6) $\Rightarrow$ $\hat{i}_2 = \frac{8}{20} A$ (7)

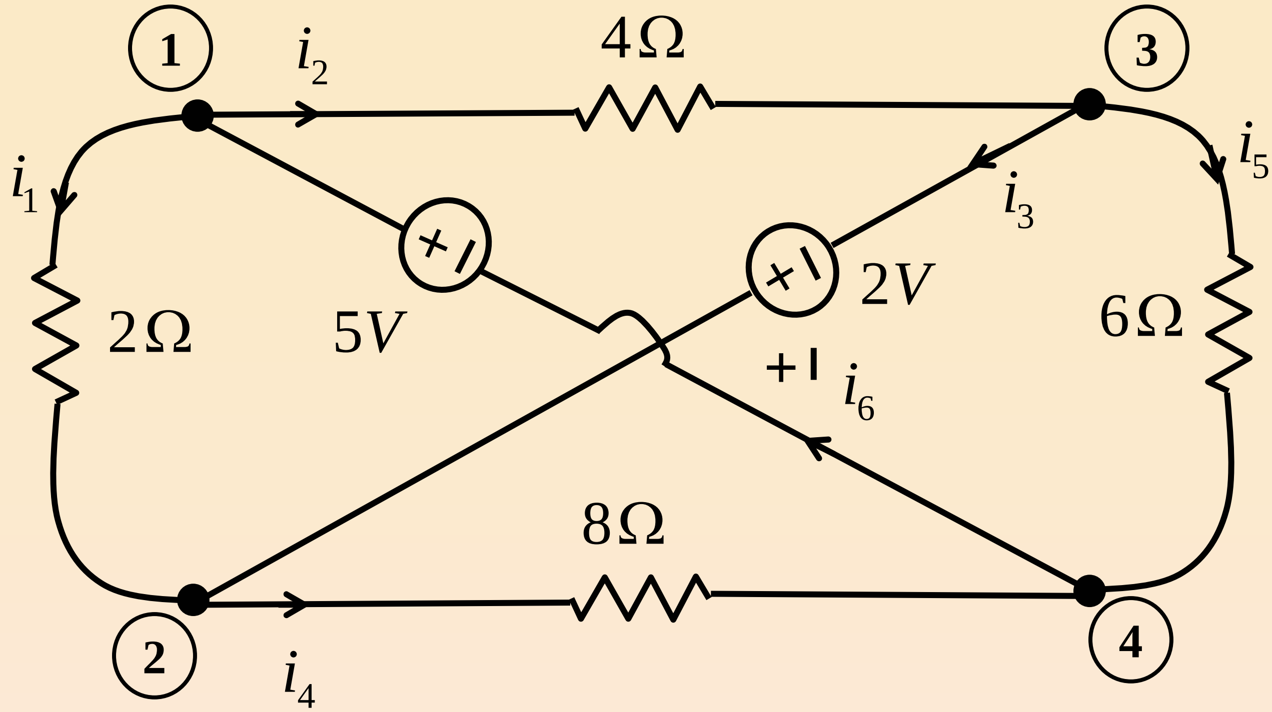
(4) and (6) $\Rightarrow$ $\hat{i}_1 = \frac{13}{20} A$ (8)



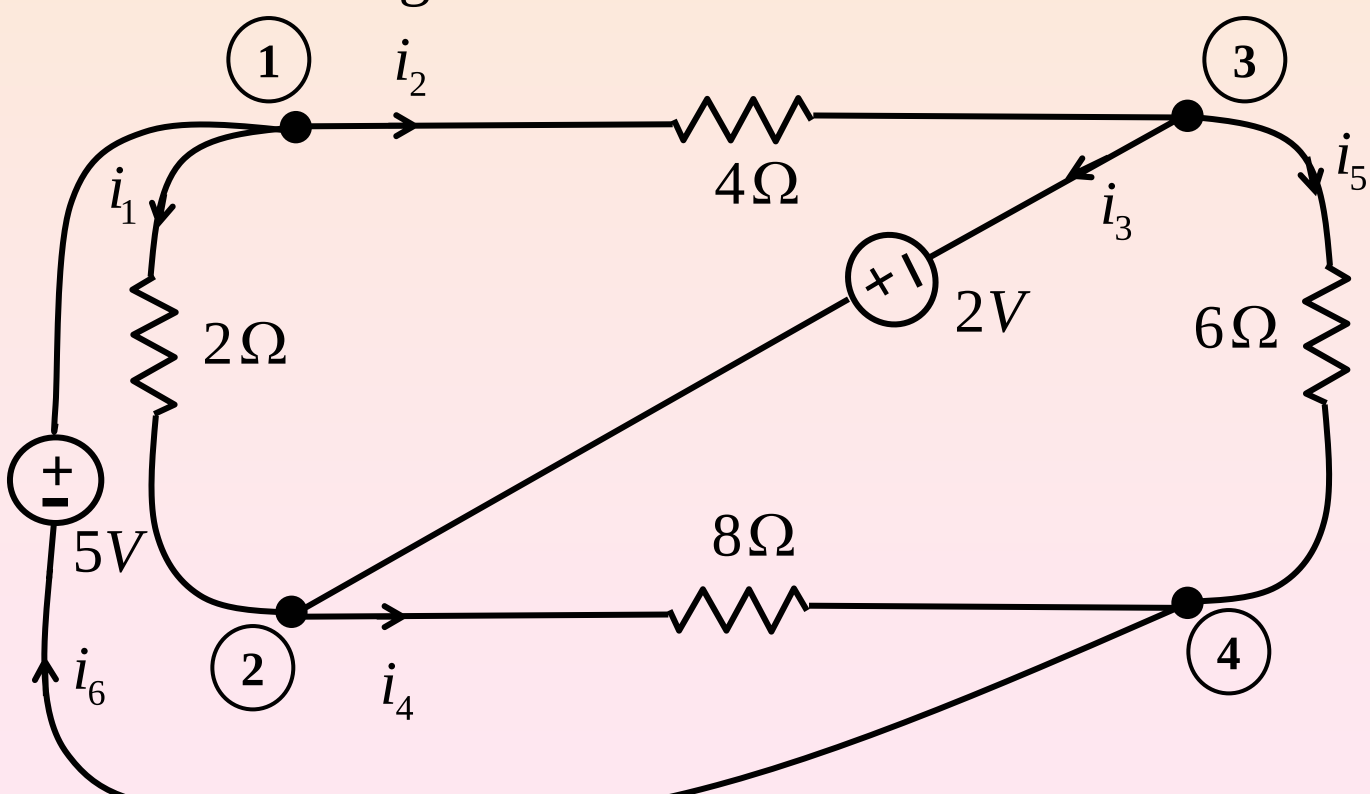
We can redraw this circuit so that there are no intersecting branches.



Hence the above circuit is **planar** and it is possible to formulate mesh current equations.



We can redraw this circuit so that there are no intersecting branches.



Hence the above circuit is **planar** and it is possible to formulate mesh current equations.

All branch voltages and currents can be trivially calculated from e_1 and i_3.

$$v_1 = e_1 - e_2 = 0\,V \qquad , \; i_1 = \frac{v_1}{4} = 0\;A$$

$$v_2 = e_1 = 6\,V \qquad , \; i_2 = \frac{v_2}{3} = 2\;A$$

$$v_3 = e_2 = 6\,V \qquad , \; i_3 = 0\;A$$

$$v_4 = -e_1 = -6\,V \qquad , \; i_4 = 2\;A$$

Verification of Solution by Tellegen's Theorem :

$$\sum_{j=1}^{4} v_j\, i_j = (v_1\, i_1) + (v_2\, i_2) + (v_3\, i_3) + (v_4\, i_4)$$

$$= (0)(0) + (6)(2) + (6)(0) + (-6)(2)$$

$$\stackrel{?}{=} 0$$

Note:

The unknown variables in the modified node voltage method consist of the usual n-1 node-to-datum voltages, plus the unknown currents associated with the voltage sources.

Hence, if there are "α" voltage sources, the modified node voltage method would consist of (n-1)+α independent linear equations involving (n-1)+α unknown variables

$$\{\underbrace{e_1, e_2, \cdots, e_{n-1}}_{(n\text{-}1)\text{ node-to-datum variables}}, \underbrace{i_{s_1}, i_{s_2}, \cdots i_{s_\alpha}}_{\alpha\text{ current variables}}\}.$$

Conservation of Electrical Energy

The algebraic sum of electrical **energy** flowing into all devices in a connected circuit is zero for all times $t > -\infty$.

Proof.

Tellegen's Theorem $\Rightarrow$

$$\sum_{j=1}^{b} \int_{-\infty}^{t} v_j(t)\, i_j(t)\, dt = 0$$

for all t. ■

$$\det \mathbf{A}_1 = \underbrace{\left| \begin{array}{cc|cccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ \hline 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -3 & 0 & 0 \\ 6 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right|}_{\mathbf{A}_1} \triangleq \Delta_1$$

$$
\det \mathbf{A}_4 = \left|\begin{array}{cc:cccc:cccc}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
\hdashline
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hdashline
0 & 0 & 1 & 0 & 0 & 0 & -4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 \\
0 & 0 & 0 & 6 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right| \triangleq \Delta_4
$$

$$
\underbrace{\qquad\qquad\qquad}_{\mathbf{A}_4}
$$

$$\det \mathbf{A}_9 = \underbrace{\left|\begin{array}{cc|cccc|cccc}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
\hline
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 1 & 0 & 0 & 0 & -4 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -3 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1
\end{array}\right|}_{\mathbf{A}_9} \triangleq \Delta_9$$

$$e_1 = \frac{\Delta_1}{\Delta}$$

$$= \frac{1}{\Delta}\left\{\begin{array}{c} a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31} + a_{41}A_{41} + a_{51}A_{51} \\ + a_{61}A_{61} + a_{71}A_{71} + a_{81}A_{81} + a_{91}A_{91} + a_{10,1}A_{10,1} \end{array}\right\}$$

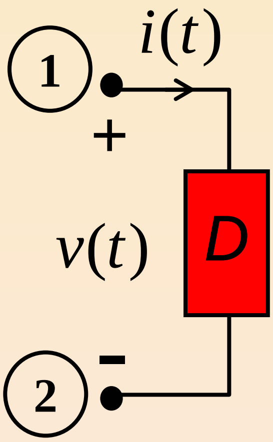
$$= \frac{1}{\Delta}\left(a_{91}A_{91} + a_{10,1}A_{10,1}\right), \quad \text{because } a_{11} = a_{21} = \cdots a_{81} = 0$$

$$= \underbrace{\frac{A_{91}}{\Delta}}_{k_{11}}\left(a_{91}\right) + \underbrace{\frac{A_{10,1}}{\Delta}}_{k_{12}}\left(a_{10,1}\right)$$

$$= k_{11} \bullet 6 + k_{12} \bullet 2$$

$$= k_{11} \bullet v_{s1} + k_{12} \bullet i_{s1}$$

Instantaneous Power of a 2-terminal device



$$p(t) \triangleq v(t)\,i(t)$$

Under Associated Reference Convention,

$$p(t) > 0 \quad \text{at} \quad t = T_1$$

means $p(T_1)$ Watts of power enters (flows into) D at $t = T_1$.

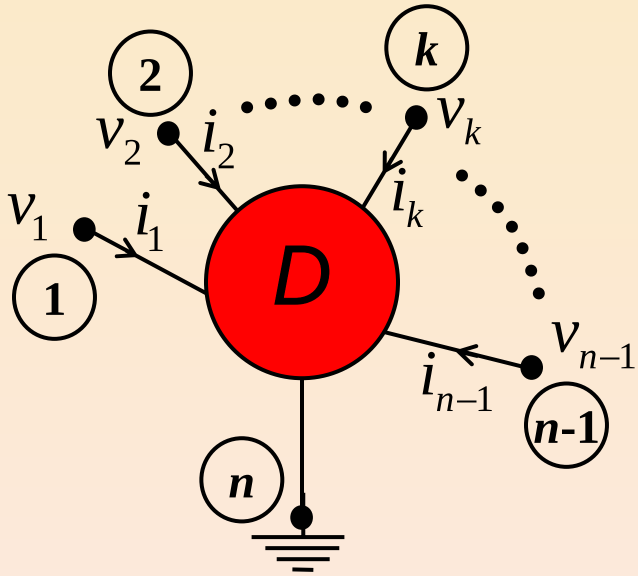
$$p(t) < 0 \quad \text{at} \quad t = T_2$$

means $p(T_2)$ Watts of power leaves (flows out of) D at $t = T_2$.

Energy entering D from time T_1 to T_2 :

$$W_{T_1 - T_2} = \int_{T_1}^{T_2} v(t)\,i(t)\,dt$$

Instantaneous Power of an *n*-terminal device



$$p(t) = \sum_{j=1}^{n-1} v_j(t)\, i_j(t)$$

Energy entering D from time T_1 to T_2 :

$$W_{T_1-T_2} = \sum_{j=1}^{n-1} \int_{T_1}^{T_2} v_j(t)\, i_j(t)\, dt$$

Tellegen’s Theorem has many deep applications. For this course, it can be used to check whether your answers in homework problems, midterm and final exams are correct.