

## Experiment Guide for RC Circuits

### I. Introduction

#### A. Capacitors

A capacitor is a passive electronic component that stores energy in the form of an electrostatic field. The unit of capacitance is the farad (coulomb/volt). Practical capacitor values usually lie in the picofarad ( $1 \text{ pF} = 10^{-12} \text{ F}$ ) to microfarad ( $1 \text{ }\mu\text{F} = 10^{-6} \text{ F}$ ) range.

Recall that a current is a flow of charges. When current flows into one plate of a capacitor, the charges don't pass through (although to maintain local charge balance, an equal number of the same polarity charges leave the other plate of the device) but instead accumulate on that plate, increasing the voltage across the capacitor. The voltage across the capacitor is directly proportional to the charge  $Q$  stored on the plates:

$$Q = CV \quad (\text{Eq. 2})$$

Since  $Q$  is the integration of current over time, we can write:

$$V = \frac{Q}{C} = \frac{\int i(t) dt}{C} \quad (\text{Eq. 3})$$

Differentiating this equation, we obtain the  $I$ - $V$  characteristic equation for a capacitor:

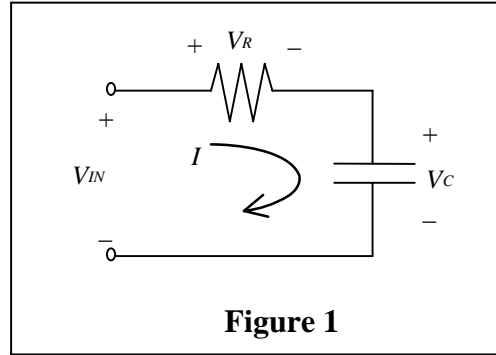
$$i = C \frac{dV}{dt} \quad (\text{Eq. 4})$$

#### B. RC Circuits

An RC (resistor + capacitor) circuit will have an exponential voltage response of the form  $v(t) = A + B e^{-t/RC}$  where constant  $A$  is the final voltage and constant  $B$  is the difference between the initial and the final voltages. ( $e^x$  is  $e$  to the  $x$  power, where  $e = 2.718$ , the base of the natural logarithm.) The product  $RC$  is called the **time constant** (whose units are seconds) and is usually represented by the Greek letter  $\tau$ . When the time has reached a value equal to the time constant,  $\tau$ , then the voltage is  $B e^{-\tau/RC} = B e^{-1} = 0.368 * B$  volts away from the final value  $A$ , or about  $5/8$  of the way from the initial value ( $A + B$ ) to the final value ( $A$ ).

The characteristic “exponential decay” associated with an RC circuit is important to understand, because complicated circuits can oftentimes be modeled simply as resistor and a capacitor. This is especially true in integrated circuits (ICs).

A simple RC circuit is drawn in Figure 1 with currents and voltages defined as shown. Equation 5 is obtained from Kirchhoff's Voltage Law, which states that the algebraic sum of voltage drops around a closed loop is zero. Equation 6 is the defining  $I$ - $V$  characteristic equation for a capacitor (as derived above), and Equation 7 is the defining  $I$ - $V$  characteristic equation for a resistor (Ohm's Law).



$$V_{IN} = V_C + V_R \quad (\text{Eq. 5})$$

$$I = C \frac{dV_C}{dt} \quad (\text{Eq. 6})$$

$$V_R = IR \quad (\text{Eq. 7})$$

By substituting Equations 6 and 7 into Equation 5, the following first-order linear differential equation is obtained:

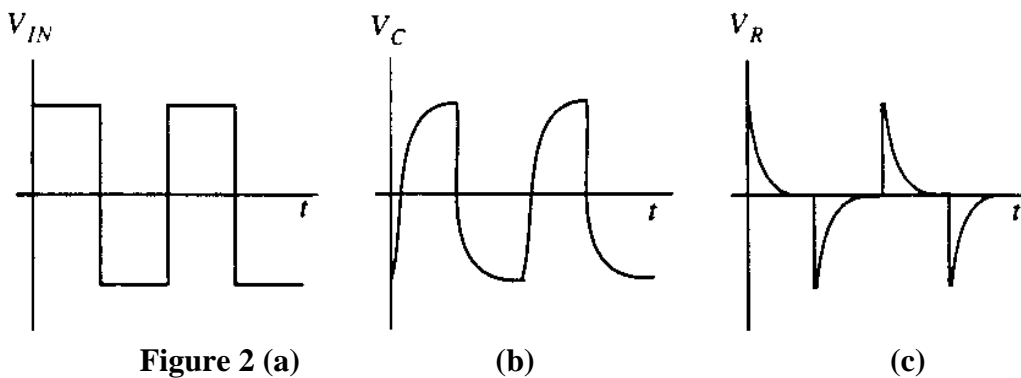
$$V_{IN} = V_C + RC \frac{dV_C}{dt} \quad (\text{Eq. 8})$$

If  $V_{IN}$  is a step function at time  $t=0$ , then  $V_C$  and  $V_R$  are of the form:

$$V_C = A + Be^{-t/RC} \quad (\text{Eq. 9})$$

$$V_R = A' + B'e^{-t/RC} \quad (\text{Eq. 10})$$

If a voltage difference exists across the resistor (*i.e.*  $V_R \neq 0$ ), then current will flow (Eq. 7). This current flows through the capacitor and causes  $V_C$  to change (Eq. 6).  $V_C$  will increase (if  $I > 0$ ) or decrease (if  $I < 0$ ) exponentially with time, until it reaches the value of  $V_{IN}$ , at which time the current goes to zero (since  $V_R = 0$ ). For the square-wave function  $V_{IN}$  as shown in Figure 2a, the responses  $V_C$  and  $V_R$  are shown in Figure 2b and Figure 2c, respectively.

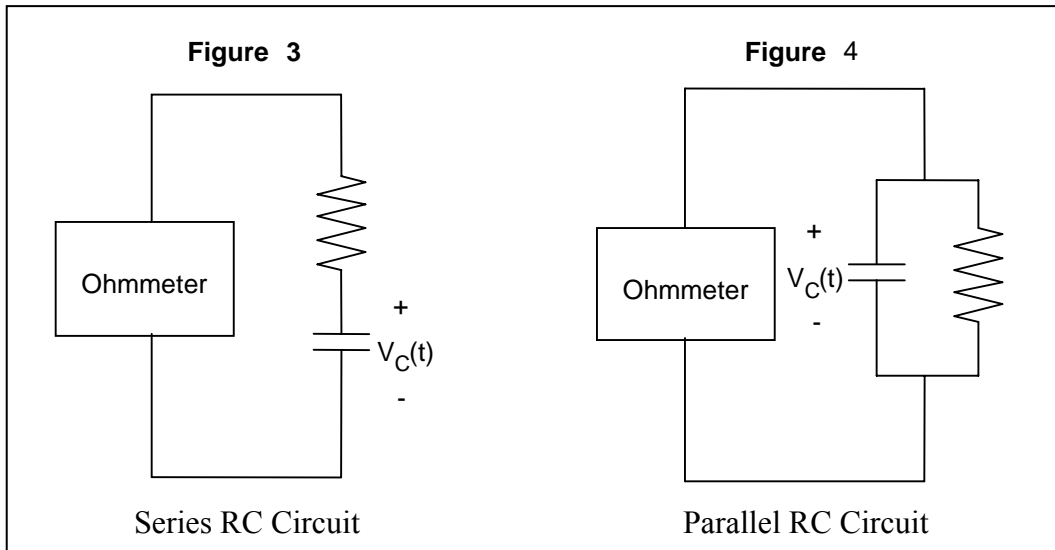


Note that if the frequency of the square wave  $V_{IN}$  is too high (*i.e.* if  $f \gg 1/RC$ ), then  $V_C$  and  $V_R$  will not have enough time to reach their asymptotic values. If the frequency is too low (*i.e.* if  $f \ll 1/RC$ ), the decay time will be very short relative to the period of the waveform and thus the exponential decay will be difficult to observe. As a rough guideline, the period of the square wave should be chosen such that it is approximately equal to  $10RC$ , in order for the responses shown in Figure 2b-c to be readily observed on an oscilloscope.

## II. Hands On

### A. Determining the RC Circuit Configuration

In this part of the experiment, you will make ohmmeter measurements to see if you can discover a method to determine if a resistor and capacitor are connected in series or in parallel.



- (a) Get a resistor and capacitor from your TA.  
 Recall that an ohmmeter has a built-in current source that sends a small current into the circuit under test. The ohmmeter reads the voltage across the circuit under test and determines the resistance of the circuit using Ohm's Law.
- (b) Build the circuit shown in Figure 3. Note that the ohmmeter's current source keeps on charging up the capacitor. (For small values of capacitance, the capacitor will be fully charged almost instantly.)  
**Question 1:** Are you able to measure the value of the resistor? If not, explain the reason why you cannot make the measurement.
- (c) Build the circuit shown in Figure 4. Note that the capacitor stops charging when the current through the resistor is equal to the current from the ohmmeter.  
**Question 2:** Explain how you got your ohmmeter reading for the circuit in Figure 4. Why does it take some time before the ohmmeter's reading stabilizes?

**Question 3:** Given a black box with either a series or parallel RC circuit, can you determine the RC configuration using an ohmmeter? If so, how?

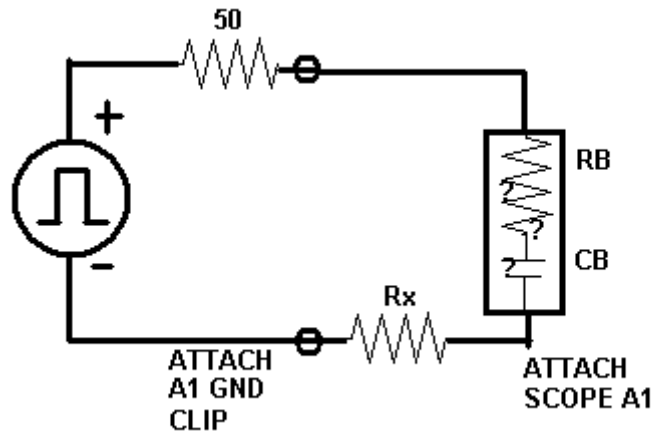
### B. Identifying Physical Values in a Series RC Circuit Black Box and a Parallel RC Circuit Black Box

The TA will give you two “black boxes” (if available). One contains a series RC circuit and the other contains a parallel RC circuit. Determine the basic resistor-capacitor configuration in each black box using an ohmmeter. If you are instructed to build the “black box” yourself, note that you are not allowed access to the node shared by  $R_B$  and  $C_B$ , so you can’t measure  $R_B$  directly. **Please use the breadboard.**

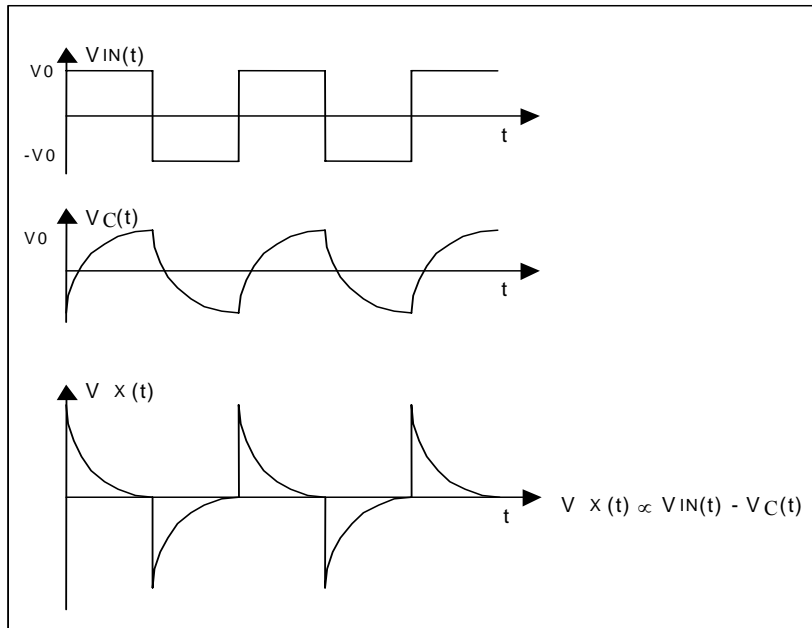
**Question 4:** Determine whether each “block box” is a series or parallel RC circuit.

#### 1) Series RC Circuit Black Box

- (a) Construct the circuit below for the black box that contains the unknown resistor  $R_B$  and capacitor  $C_B$  in series. There is an  $R_S = 50$  ohms source resistor inside the function generator (might be negligible) and  $R_x$  is the external resistor that is suggested by the TA. Measure and record  $R_x$ .

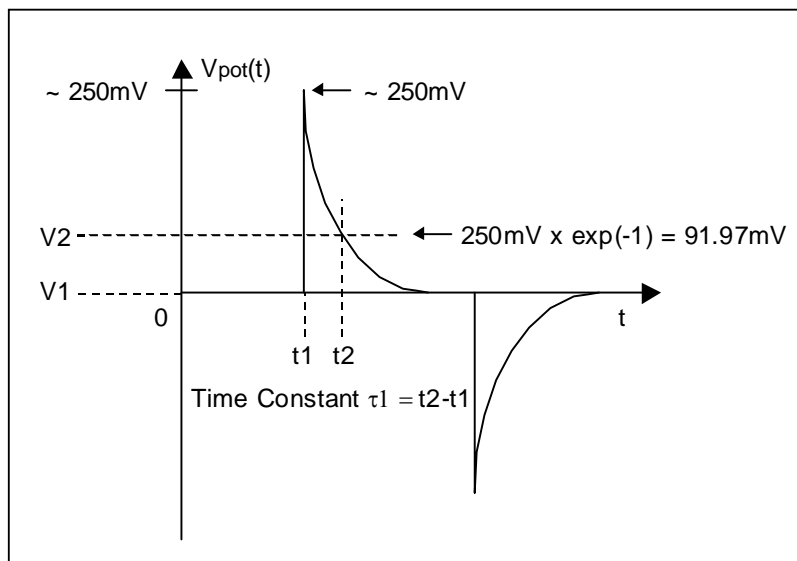


- (b) Set the amplitude of the square wave to about  $5 V_{PP}$  (not critical), with  $0 V_{DC}$  offset. Adjust the frequency of the square wave and the scope viewing scale until you see the exponential characteristic decay. Make sure that you are using the waveform from the OUTPUT terminal of the function generator, not the SYNC terminal.
- (c) Note that the voltage across  $R_x$  follows the shape of  $V_{IN} - V_C$  (try the math and look at the nice pictures).



**Figure 6:** Voltage waveforms in a series RC circuit

- (d) From voltage  $V_X$  across  $R_X$ , you could use Ohm's Law to obtain the current  $I_X$ , but both  $I_X$  and  $V_X$  will have the same time constant. You should trigger on A1 edge and set Main Time Ref to Left, and always use the knobs to zoom in upon your cursor measurements. Your TA should review use of the CURSOR function: Set cursor V1 to ground, set cursor V2 to the peak and record  $\Delta V(A1)$ . Next, move V2 to 0.368 of that value. Move t1 to the start, move t2 to intersect V2.  $\Delta t$  is the time constant.



**Figure 7:** How to measure the time constant  $\tau_1$  on the oscilloscope

**Question 5:** What is the time constant  $\tau_1$ ? What is the value of  $R_X$ ?

- (d) Now increase the value of  $R_X$  by about a factor of ten. Measure the new  $R_{X2}$ .

**Question 6:** What is the time constant  $\tau_2$ ? What is the value of  $R_{X2}$ ?

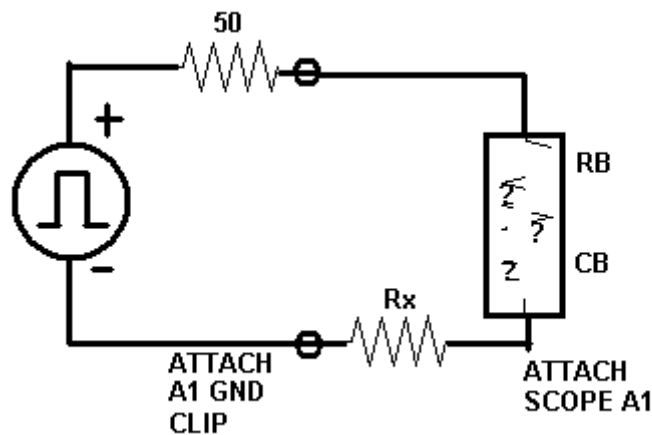
**Question 7:** Solve for the resistance  $R_B$  and capacitance  $C_B$  using the formula for the time constant in both trials.  $\tau = C_B (R_S + R_B + R_X)$ . Ask your TA for the resistance and capacitance values. Are they in good agreement with the values you have obtained experimentally? Explain if there are any significant differences.

## 2) Parallel RC Circuit Black Box

- (a) Build the Parallel “Black Box” using unknown  $R_B$  and  $C_B$  from your TA. Measure the resistance of the circuit inside the black box using the ohmmeter. The rest of the measurements go toward finding the capacitance  $C_B$ .

**Question 8:** What is the measured value of the resistor inside the black box?

- (b) Select a resistor  $R_X$  of a value comparable to the resistor  $R_B$  and construct the circuit below. Measure the time constant of the circuit with  $R_X$ .



**Figure 9:** Setup for finding R and C of an unknown parallel RC circuit

**Question 9:** Measure the time constant of the circuit with  $R_X$ , then solve for the value of the capacitor  $C_B$ . Ask your TA for the values of the resistor and the capacitor inside the black box. Are they in good agreement with the values you have obtained experimentally? Explain if there are any significant differences.