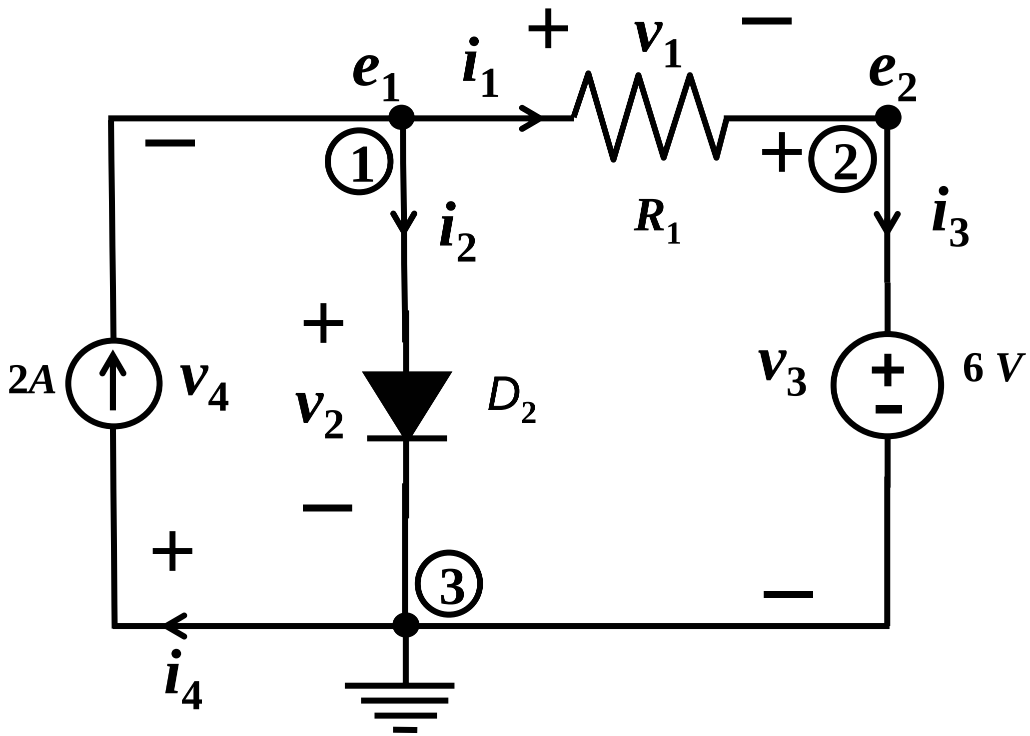
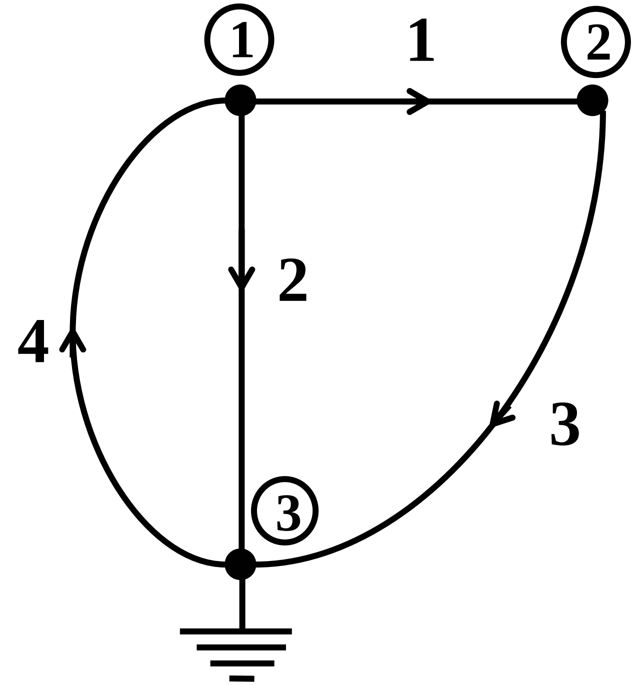
Circuit N



Digraph G



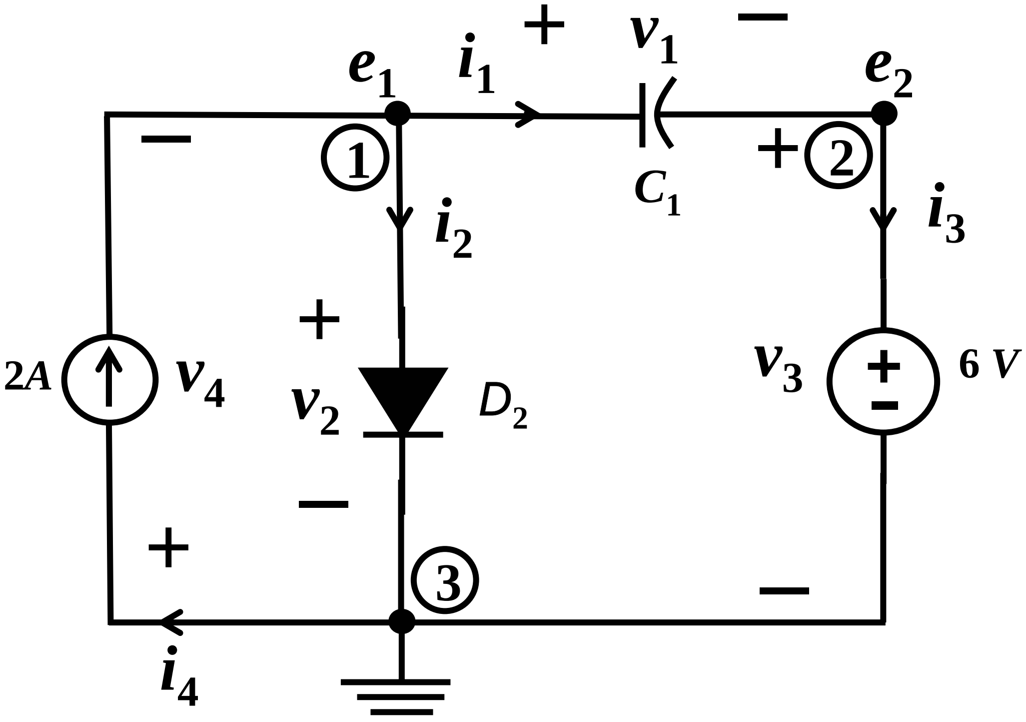
Reduced Incidence Matrix A

branch number

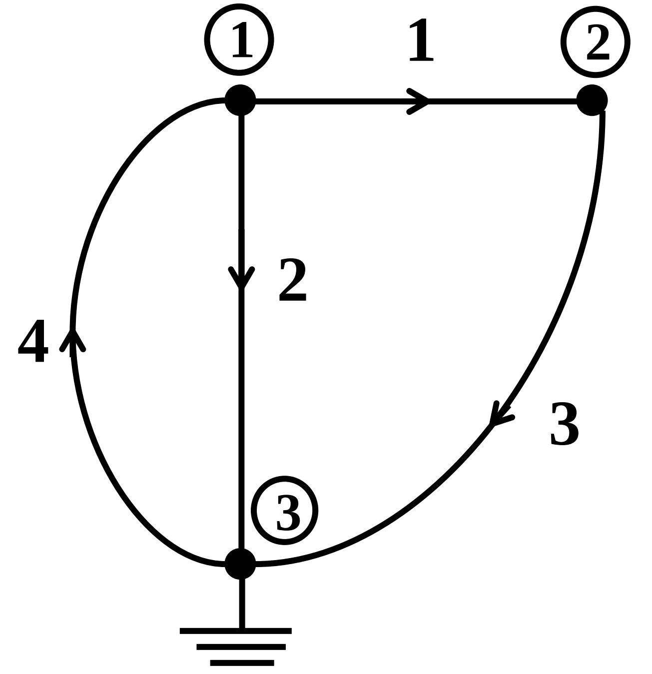
node number

	1	2	3	4
①	1	1	0	–1
②	–1	0	1	0

Circuit *N*



Digraph *G*



Reduced Incidence Matrix A

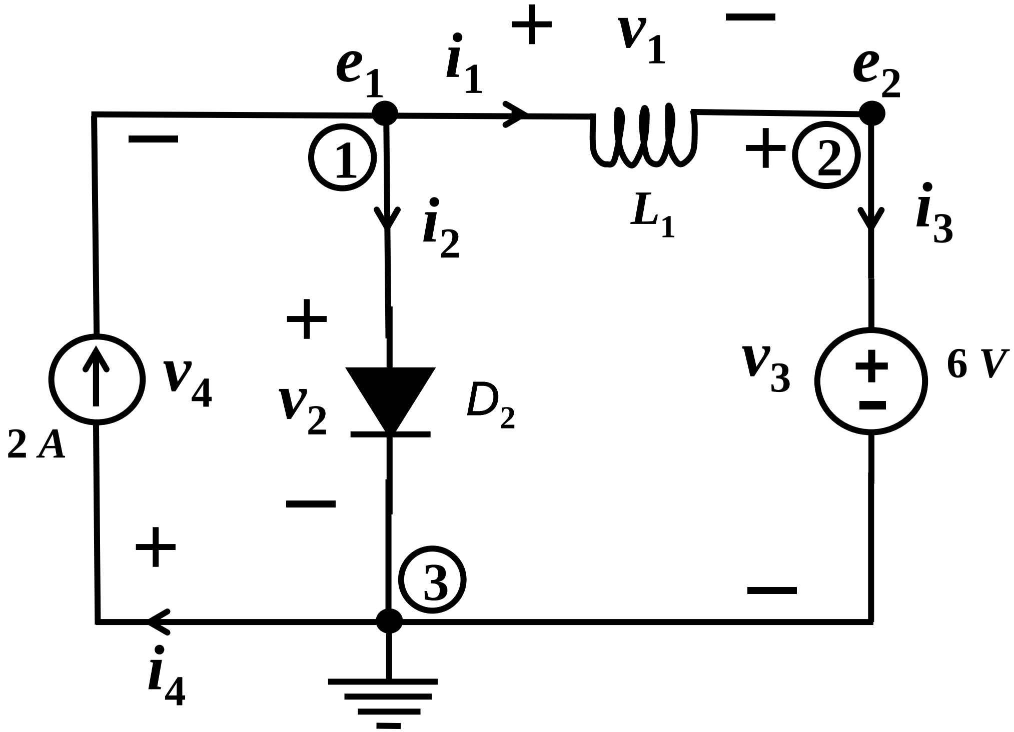
branch number

node number

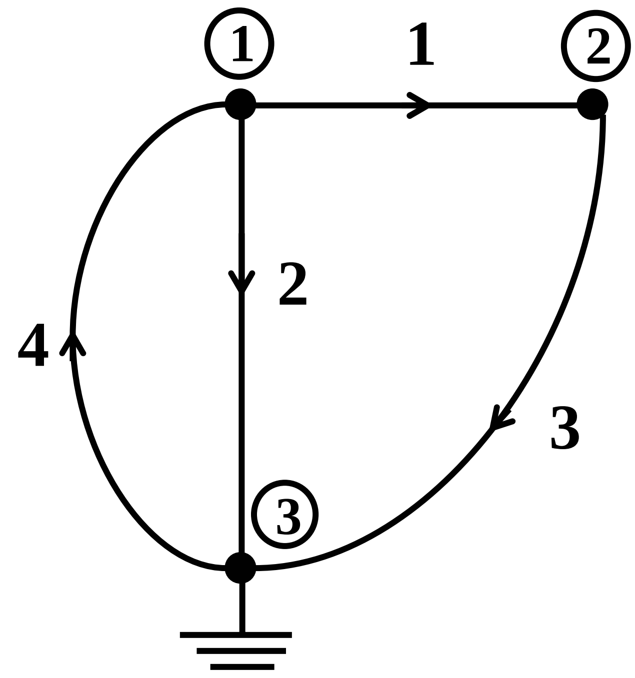
$$
\begin{array}{c} \\ ① \\ ② \end{array}
\begin{array}{cccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \end{array}
$$

$$
\begin{array}{c} ① \\ ② \end{array}
\begin{bmatrix} 1 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix}
$$

Circuit N



Digraph G



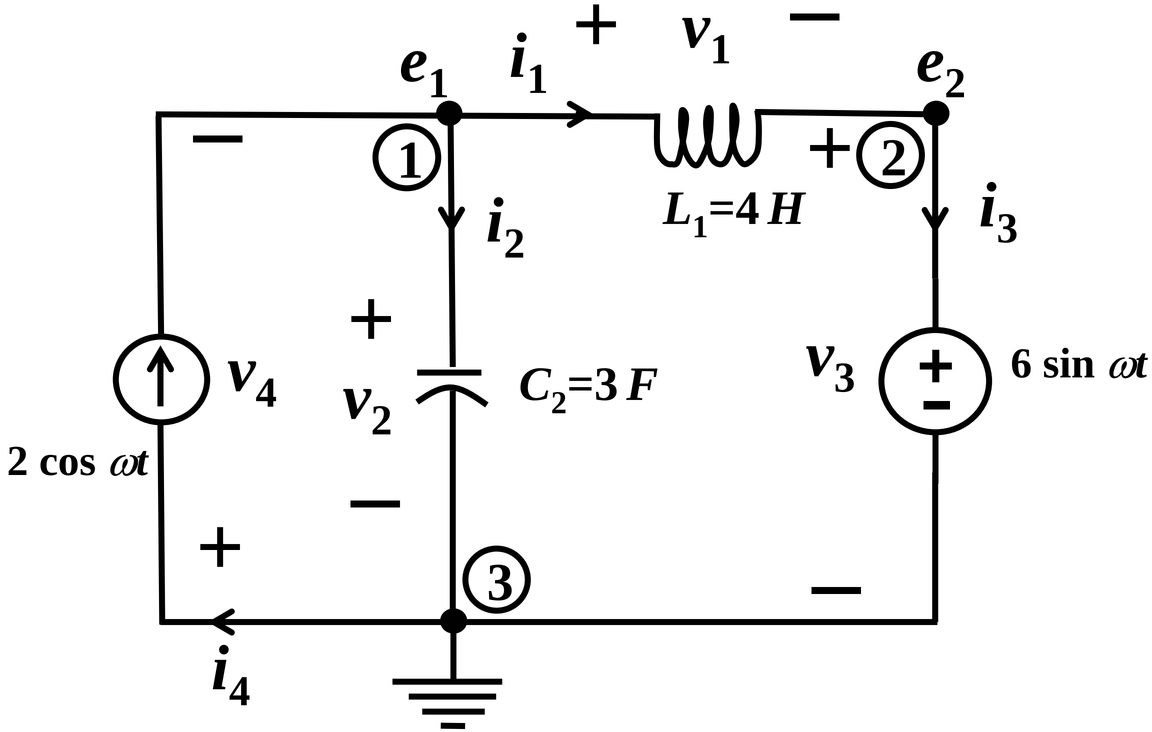
Reduced Incidence Matrix A

branch number

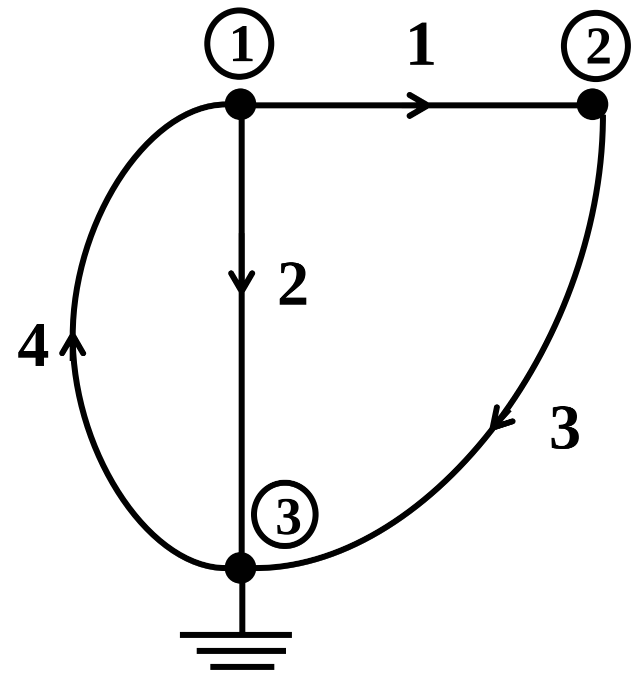
node number

$$\begin{array}{c} \\ ① \\ ② \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Circuit N



Digraph G



Reduced Incidence Matrix A

branch number

	1	2	3	4
①	1	1	0	–1
②	–1	0	1	0

node number

Element Constitutive Relations

Element 1: Resistor
Described by Ohm's Law : $v_1 = 4\, i_1$

Element 2: Resistor
Described by Ohm's Law : $v_2 = 3\, i_2$

Element 3: Voltage source
Described by : $v_3 = 6$

Element 4: Current source
Described by : $i_4 = 2$

Rearranging these equations so that circuit variables appear on the left-hand side, we obtain

Observe we have obtained 4 additional independent equations.

Element Equations

$$
\begin{aligned}
-v_1 + 4\, i_1 &= 0 \\
-v_2 + 3\, i_2 &= 0 \\
v_3 &= 6 \\
i_4 &= 2
\end{aligned}
$$

Equations obtained from the element constitutive relations are guaranteed to be **independent** because different elements involved different circuit variables.

Element Constitutive Relations

Element 1: Linear Inductor
Described by $$v_1 = L_1 \frac{di_1}{dt}$$

Element 2: Nonlinear Resistor
Described by $$i_2 = I_0 \left(e^{v_2/V_T} - 1 \right)$$

Element 3: Voltage source
Described by $$v_3 = 6$$

Element 4: Current source
Described by $$i_4 = 2$$

Rearranging these equations so that circuit variables appear on the left-hand side, we obtain

Observe we have obtained 4 additional independent equations.

Element Equations

$$\begin{aligned} v_1 - L_1 \frac{di_1}{dt} &= 0 \\ i_2 - I_0 \left(e^{v_2/V_T} - 1 \right) &= 0 \\ v_3 &= 6 \\ i_4 &= 2 \end{aligned}$$

Equations obtained from the element constitutive relations are guaranteed to be **independent** because different elements involved different circuit variables.

Element Constitutive Relations

Element 1: Capacitor
Described by $$i_1 = C_1 \frac{dv_1}{dt}$$

Element 2: Nonlinear Resistor
Described by $$i_2 = I_0 \left(e^{v_2 / V_T} - 1 \right)$$

Element 3: Voltage source
Described by $$v_3 = 6$$

Element 4: Current source
Described by $$i_4 = 2$$

Rearranging these equations so that circuit variables appear on the left-hand side, we obtain

Observe we have obtained 4 additional independent equations.

Element Equations

$$\begin{aligned} i_1 - C_1 \frac{dv_1}{dt} &= 0 \\ i_2 - I_0 \left(e^{v_2 / V_T} - 1 \right) &= 0 \\ v_3 &= 6 \\ i_4 &= 2 \end{aligned}$$

Equations obtained from the element constitutive relations are guaranteed to be **independent** because different elements involved different circuit variables.

Element Constitutive Relations

Element 1: Inductor

Described by $$v_1 = L_1 \frac{di_1}{dt}$$

Element 2: Capacitor

Described by $$i_2 = C_2 \frac{dv_2}{dt}$$

Element 3: Voltage source

Described by $$v_3 = 6 \sin \omega t$$

Element 4: Current source

Described by $$i_4 = 2 \cos \omega t$$

Rearranging these equations so that circuit variables appear on the left-hand side, we obtain

Observe we have obtained 4 additional independent equations.

Element Equations

$$-v_1 + L_1 \frac{di_1}{dt} = 0$$

$$C_2 \frac{dv_2}{dt} - i_2 = 0$$

$$v_3 = 6 \sin \omega t$$

$$i_4 = 2 \cos \omega t$$

Equations obtained from the element constitutive relations are guaranteed to be **independent** because different elements involved different circuit variables.

We can always recast **any** system of **linear** constitutive equations into the following standard matrix form

$$\underbrace{\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{H}_v} \underbrace{\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{H}_i} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ \hline i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}}_{\begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 6 \\ 2 \end{bmatrix}}_{\mathbf{u}}$$

independent source vector

$$\Downarrow$$

$$\boxed{\mathbf{H}_v \, \mathbf{v} + \mathbf{H}_i \, \mathbf{i} = \mathbf{u}}$$