Circuit N

\[ \begin{align*}
\text{Diode } D_2 & \quad \text{Node 3} \\
2A & \quad \text{Source} v_4 \\
i_4 & \quad \text{Current} \\
+ & \quad \text{Voltage} v_2 \\
- & \quad \text{Ground} \\
\end{align*} \]

Reduced Incidence Matrix \( A \)

\[
\begin{bmatrix}
1 & 1 & 0 & -1 \\
-1 & 0 & 1 & 0 \\
\end{bmatrix}
\]

Digraph G

\[ \begin{align*}
\text{Node 1} & \quad \text{Node 2} \\
\text{Node 3} & \quad \text{Node 4} \\
1 & \quad \text{Branch 1} \\
2 & \quad \text{Branch 2} \\
3 & \quad \text{Branch 3} \\
4 & \quad \text{Branch 4} \\
\end{align*} \]
**Circuit N**

**Digraph G**

**Reduced Incidence Matrix A**

<table>
<thead>
<tr>
<th>branch number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>node number 1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>node number 2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Circuit N

\[ e_1 i_1 + v_1 - e_2 \]
\[ i_2 L_1 i_3 \]
\[ v_3 v_4 \]
\[ 2A \]

Digraph G

\[ 1 \rightarrow 2 \]
\[ 2 \rightarrow 1 \]
\[ 1 \rightarrow 3 \]
\[ 3 \rightarrow 2 \]

Reduced Incidence Matrix A

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & 1 & 0 & -1 \\
-1 & 0 & 1 & 0
\end{bmatrix}
\]
**Circuit N**

![Circuit N Diagram]

**Digraph G**

![Digraph G Diagram]

**Reduced Incidence Matrix A**

\[
\begin{bmatrix}
1 & 1 & 0 & -1 \\
-1 & 0 & 1 & 0
\end{bmatrix}
\]
Element Constitutive Relations

Element 1: Resistor
   Described by Ohm’s Law: $v_1 = 4 \ i_1$

Element 2: Resistor
   Described by Ohm’s Law: $v_2 = 3 \ i_2$

Element 3: Voltage source
   Described by: $v_3 = 6$

Element 4: Current source
   Described by: $i_4 = 2$

Rearranging these equations so that circuit variables appear on the left-hand side, we obtain

Observe we have obtained 4 additional independent equations.

Equations obtained from the element constitutive relations are guaranteed to be independent because different elements involved different circuit variables.
Rearranging these equations so that circuit variables appear on the left-hand side, we obtain

\[
\begin{align*}
\text{Element 1: Linear Inductor} & \quad v_1 = L_1 \frac{di_1}{dt} \\
\text{Described by} & \\
\text{Element 2: Nonlinear Resistor} & \quad i_2 = I_0 \left( e^{\frac{v_2}{\nu T}} - 1 \right) \\
\text{Described by} & \\
\text{Element 3: Voltage source} & \quad \nu_3 = 6 \\
\text{Described by} & \\
\text{Element 4: Current source} & \quad i_4 = 2 \\
\end{align*}
\]

Observe we have obtained 4 additional independent equations.

Equations obtained from the element constitutive relations are guaranteed to be independent because different elements involved different circuit variables.
Element Constitutive Relations

Element 1: Capacitor
Described by
\[ i_1 = C_1 \frac{dv_1}{dt} \]

Element 2: Nonlinear Resistor
Described by
\[ i_2 = I_0 \left( e^{\frac{v_2}{V_T}} - 1 \right) \]

Element 3: Voltage source
Described by
\[ v_3 = 6 \]

Element 4: Current source
Described by
\[ i_4 = 2 \]

Rearranging these equations so that circuit variables appear on the left-hand side, we obtain

\[
\begin{align*}
    i_1 - C_1 \frac{dv_1}{dt} &= 0 \\
    i_2 - I_0 \left( e^{\frac{v_2}{V_T}} - 1 \right) &= 0 \\
    v_3 &= 6 \\
    i_4 &= 2
\end{align*}
\]

Observe we have obtained 4 additional independent equations.

Equations obtained from the element constitutive relations are guaranteed to be independent because different elements involved different circuit variables.
Element Constitutive Relations

Element 1: Inductor
Described by
\[ v_1 = L_1 \frac{di_1}{dt} \]

Element 2: Capacitor
Described by
\[ i_2 = C_2 \frac{dv_2}{dt} \]

Element 3: Voltage source
Described by
\[ v_3 = 6 \sin \omega t \]

Element 4: Current source
Described by
\[ i_4 = 2 \cos \omega t \]

Rearranging these equations so that circuit variables appear on the left-hand side, we obtain

Observe we have obtained 4 additional independent equations.

Equations obtained from the element constitutive relations are guaranteed to be independent because different elements involved different circuit variables.
We can always recast any system of linear constitutive equations into the following standard matrix form

\[
\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4 \\
i_1 \\
i_2 \\
i_3 \\
i_4
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
6 \\
2
\end{bmatrix}
\]

\[
H_v v + H_i i = u
\]