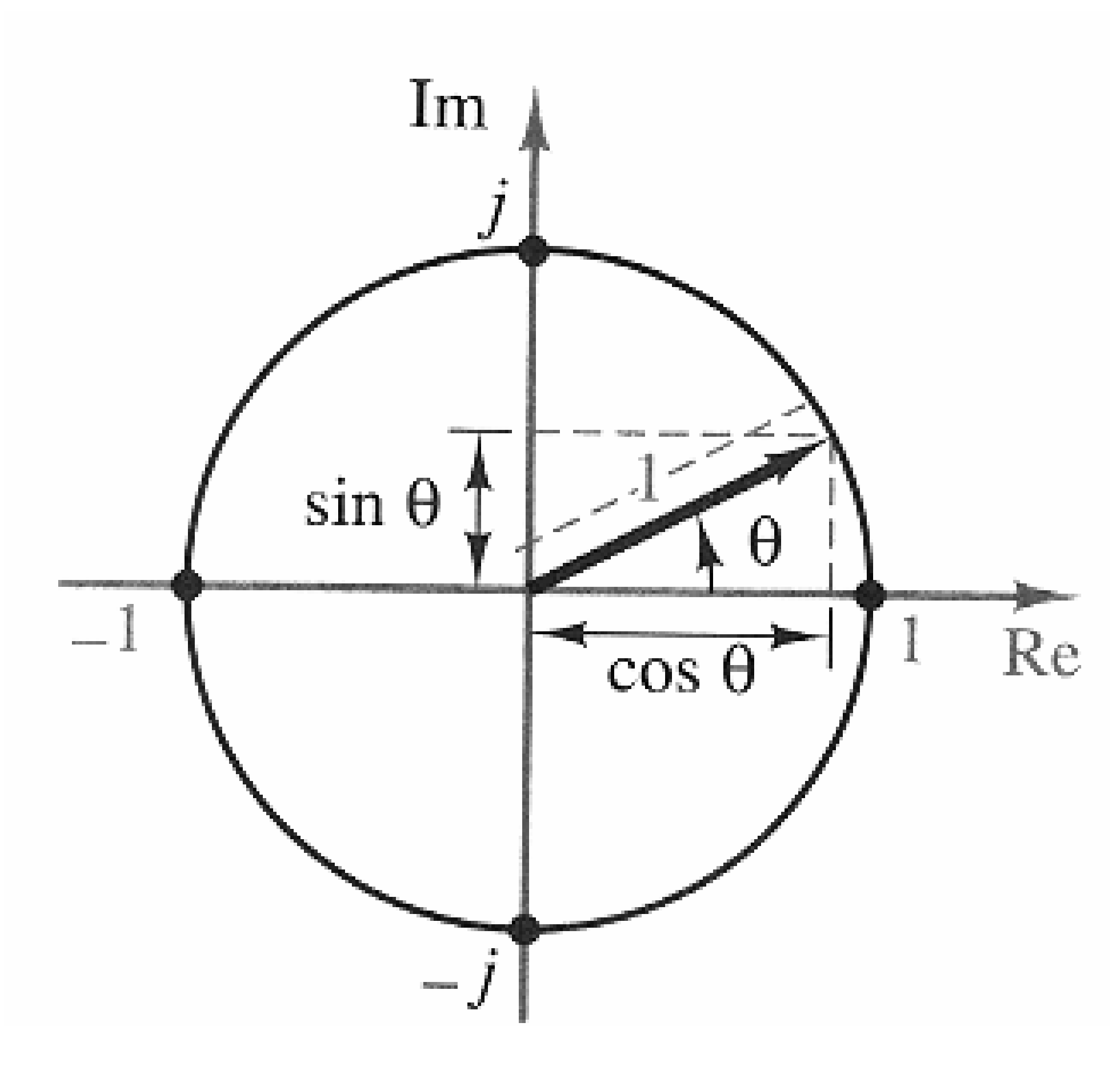
Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$



Euler's Identity

Substituting

$\theta = \pi$ in Euler's Identity,

we obtain :

$$e^{j\pi} = \cos\pi + j\sin\pi$$
$$= -1 + j\,0$$

$$e^{j\pi} + 1 = 0$$

$\Rightarrow$ **Most beautiful relationship in Number theory**

Real Exponentials

$$x(t) = x(t_\infty) + \left[x(t_0) - x(t_\infty)\right] e^{\frac{-(t-t_0)}{\tau}}$$

- A **real exponentials** is **uniquely** identified by 3 parameters:

  τ, $x(t_0)$, and $x(t_\infty)$

- Sum of 2 or more **real exponentials** of the same **time constant** τ results in another **real exponential** with **time constant** τ.

$$\sum_{i=0}^{n} k_i e^{-\frac{t}{\tau}} = \left(\sum_{i=0}^{n} k_i\right) e^{-\frac{t}{\tau}}$$

Complex Exponentials

$$x(t) = Ae^{j(\omega t+\theta)}$$
$$= A\cos(\omega t+\theta) + jA\sin(\omega t+\theta)$$

- A **complex exponentials** is **uniquely** identified by 3 parameters:

  ω, A, and θ

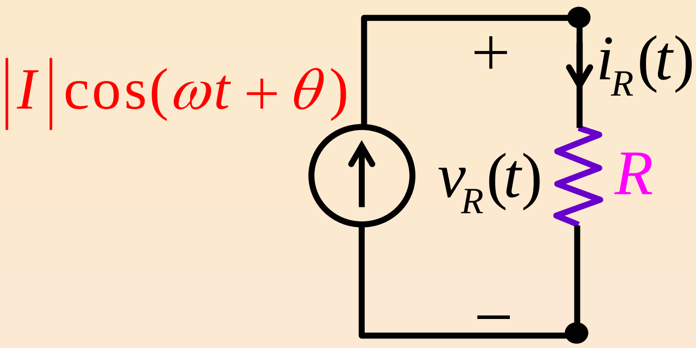
- Sum of 2 or more **complex exponentials** (sinusoids) of the same **frequency** ω results in another **complex exponential** (sinusoids) with **frequency** ω.

$$\sum_{i=0}^{n} A_i e^{j(\omega t+\theta_i)} = \underbrace{\left(\sum_{i=0}^{n} A_i e^{j\theta_i}\right)}_{\left(Ae^{j\theta}\right)} e^{j\omega t}$$

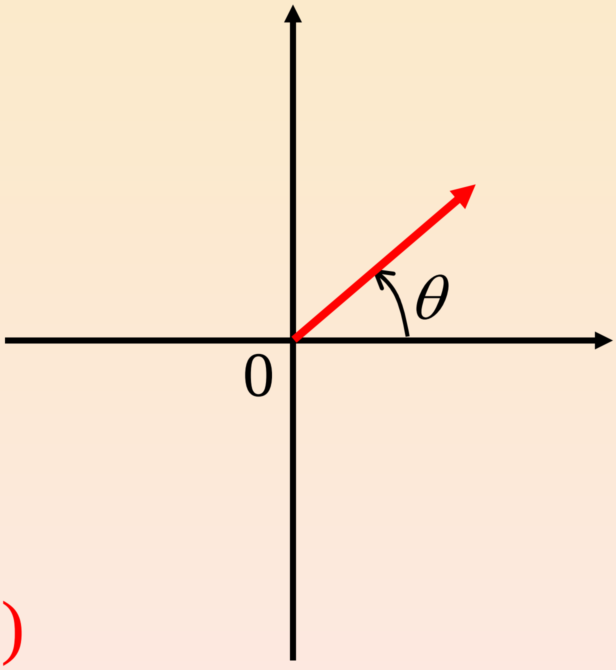
$$\left(Ae^{j\theta}\right)e^{j\omega t}$$

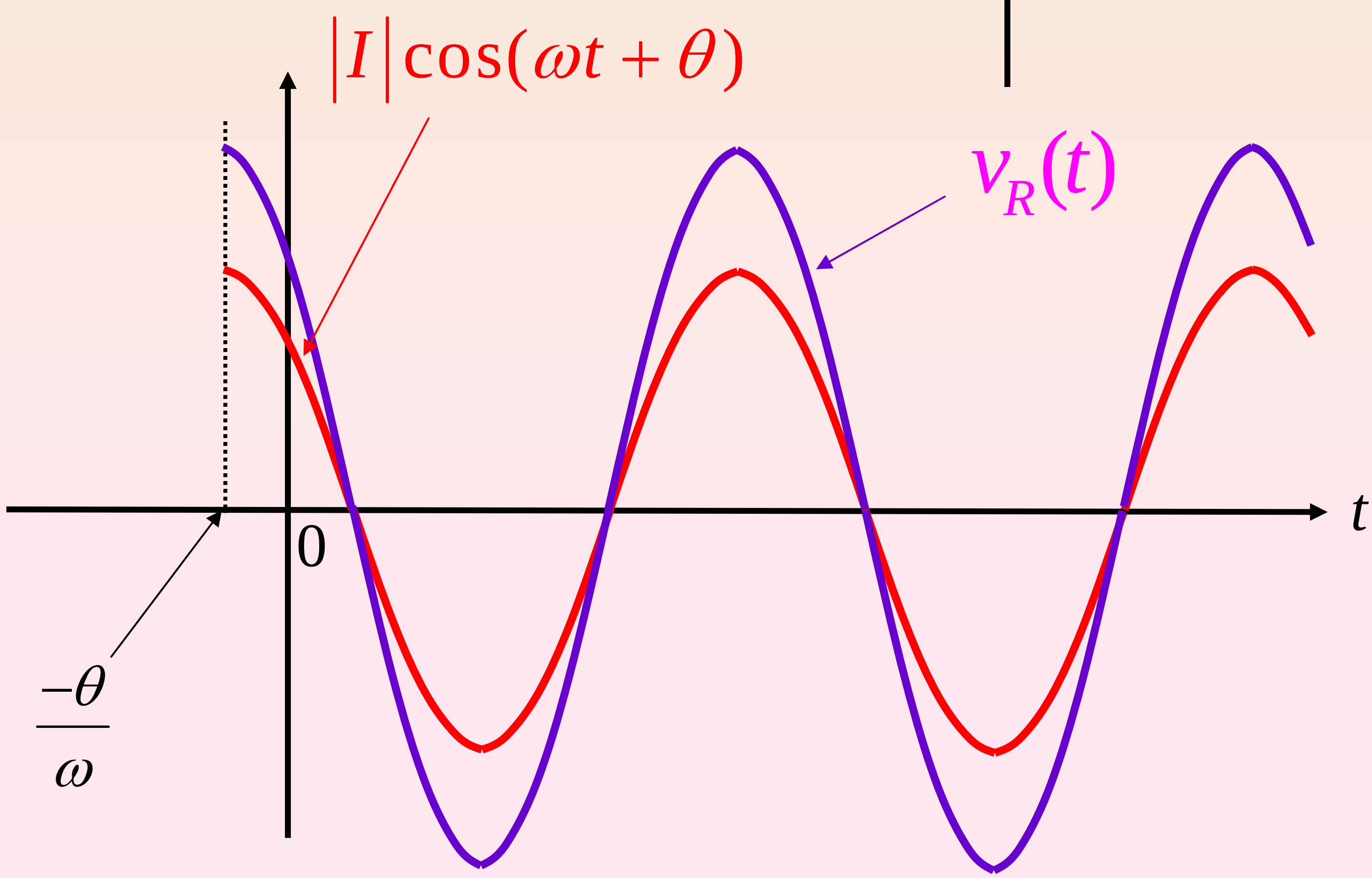
$$= \left[A\cos(\omega t+\theta)\right] + j\left[A\sin(\omega t+\theta)\right]$$

Phasor Diagram



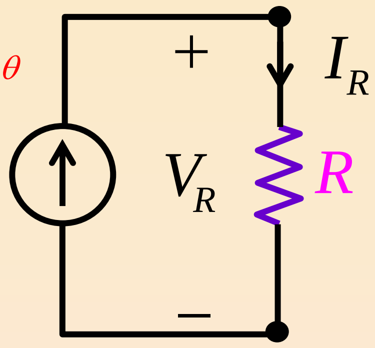
$v_R(t) = R|I|\cos(\omega t+\theta)$

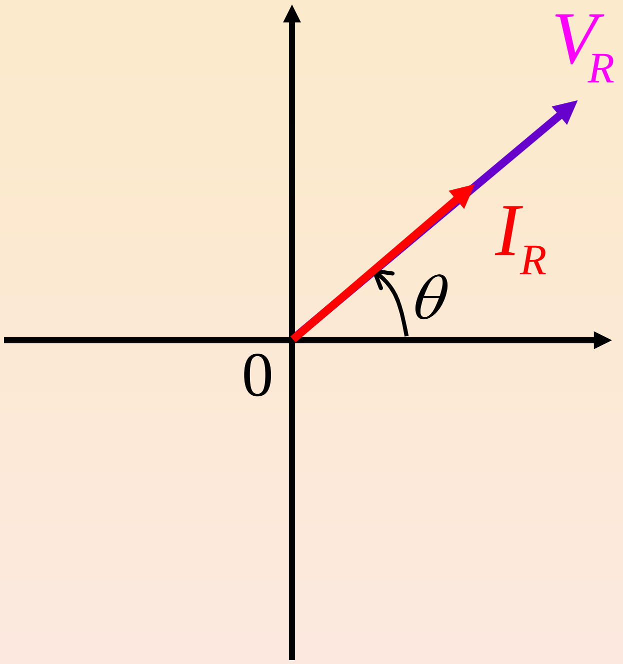




Phasor Diagram

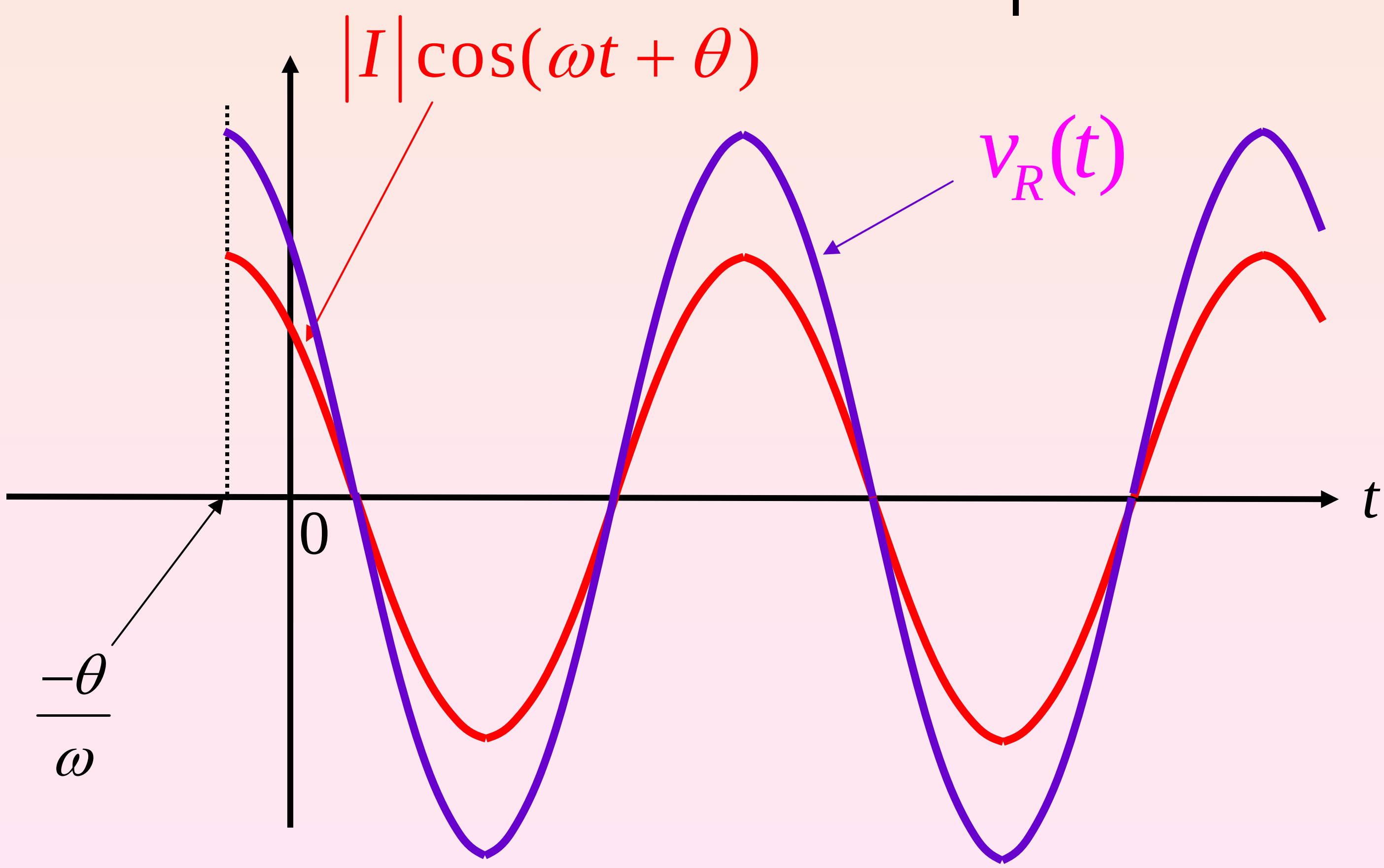
$I = |I| e^{j\theta}$



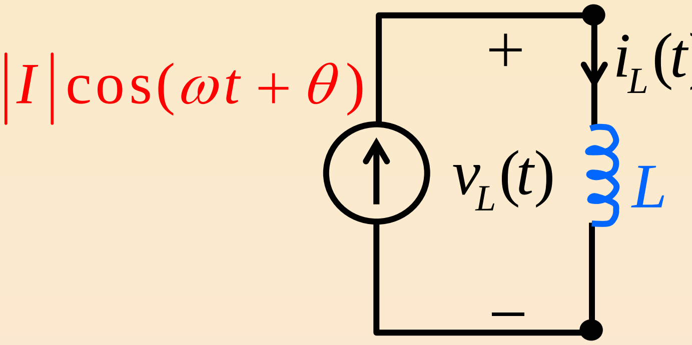


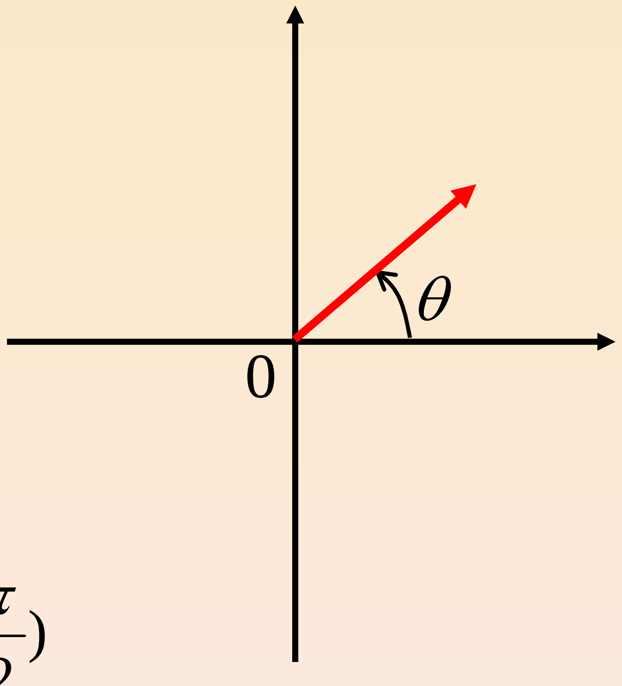
$V_R = RI_R = R|I|e^{j\theta}$

$\therefore$ Resistor Current is **in phase** with Resistor Voltage.



Phasor Diagram

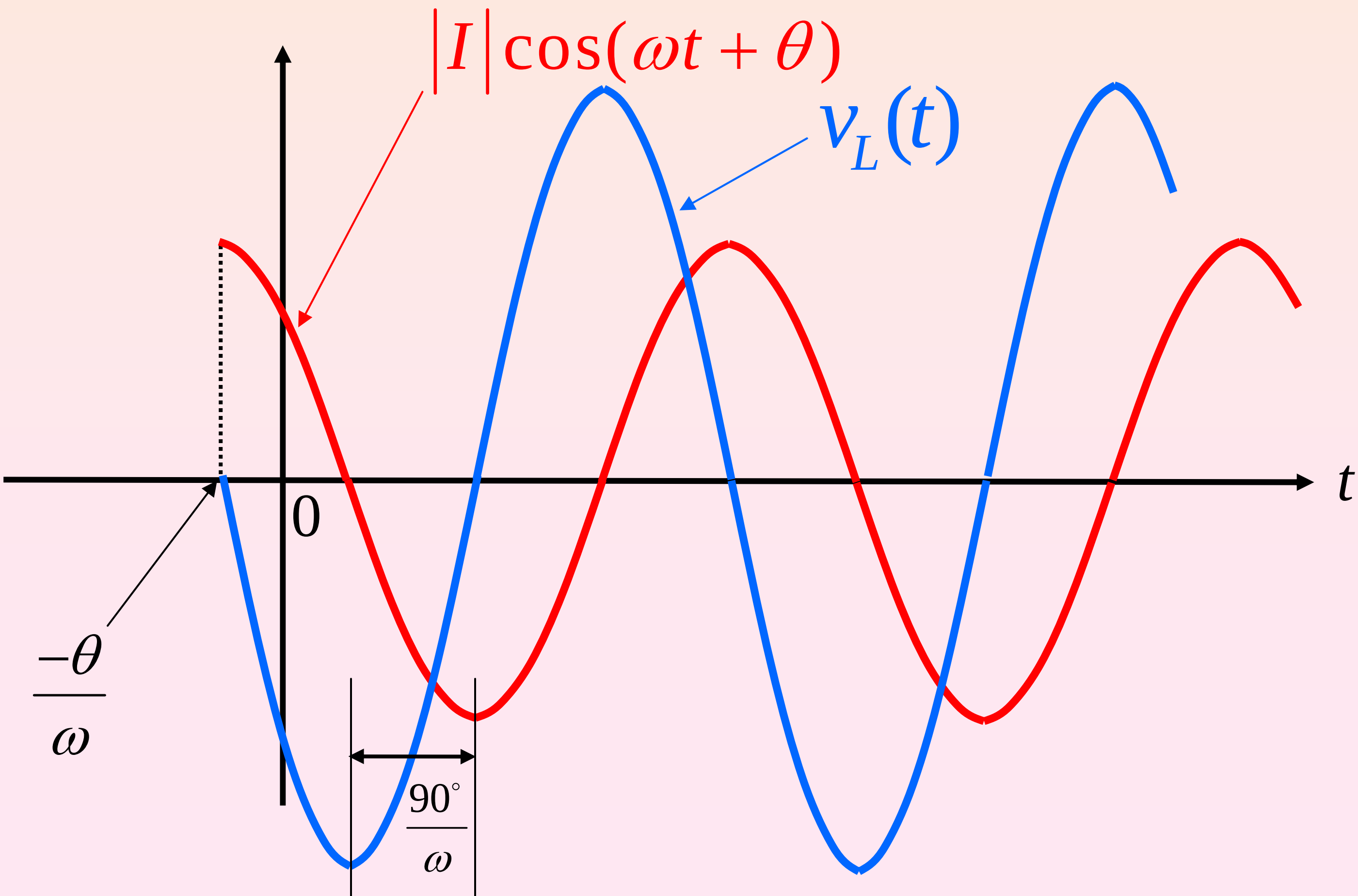




$$v_L(t) = L\frac{di_L(t)}{dt}$$

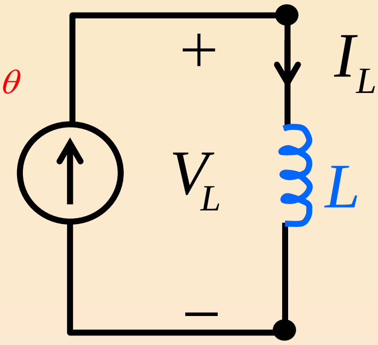
$$= -\omega L|I|\sin(\omega t+\theta)$$

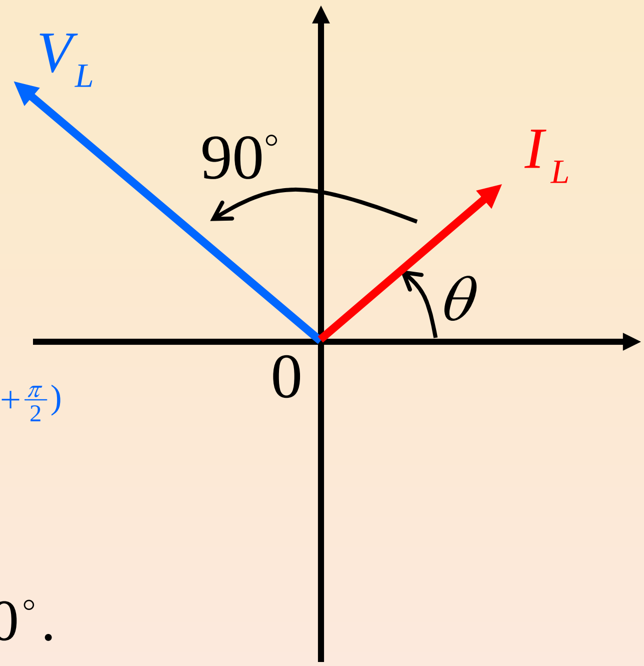
$$= \omega L|I|\cos(\omega t+\theta+\frac{\pi}{2})$$



Phasor Diagram

$I = |I| e^{j\theta}$

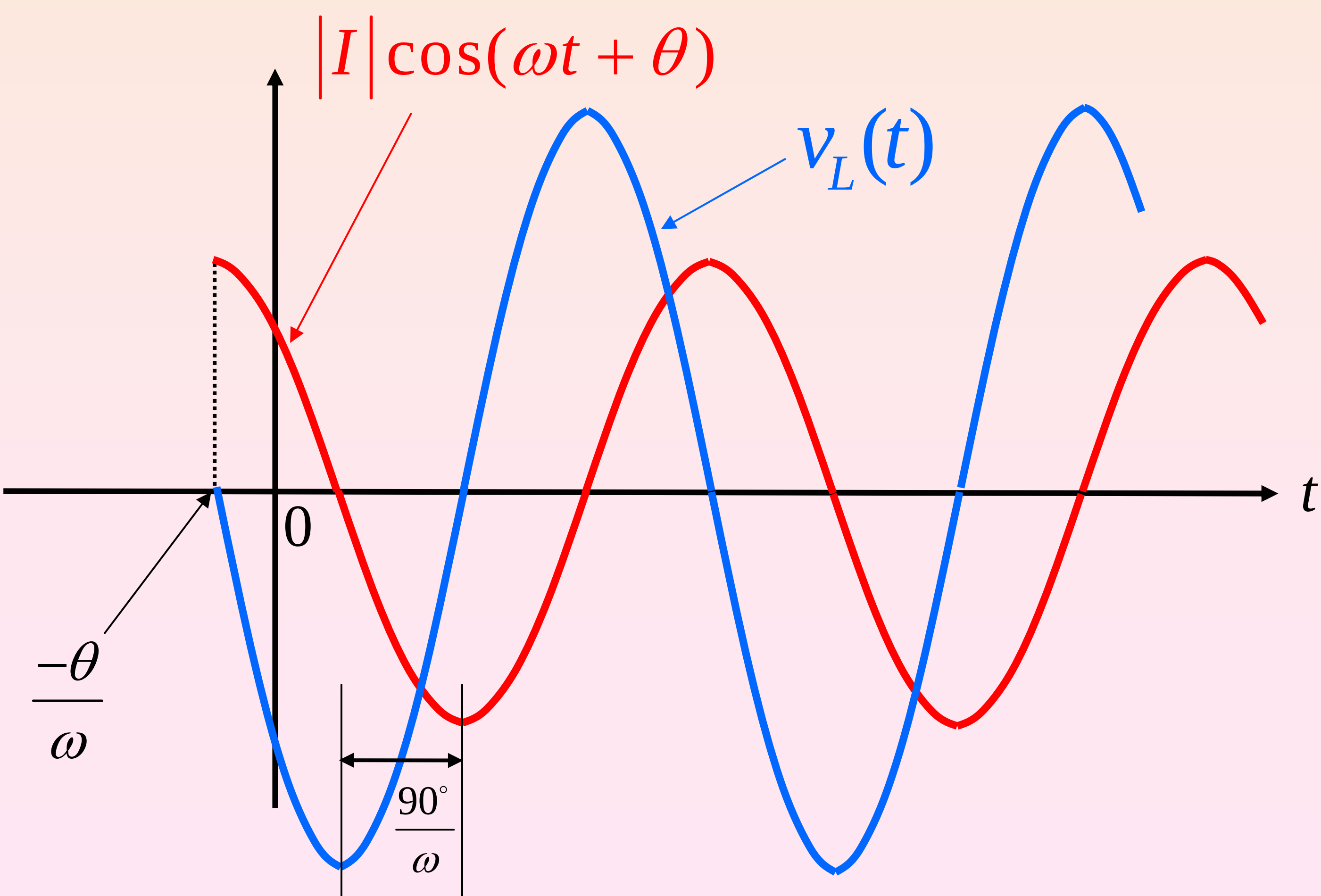




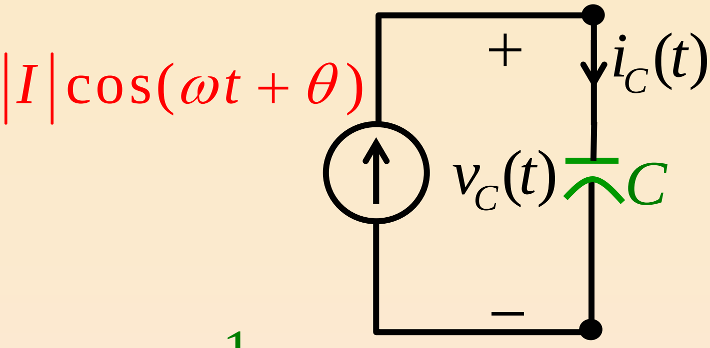
$$V_L = (j\omega L) I_L = \omega L |I| e^{j(\theta + \frac{\pi}{2})}$$

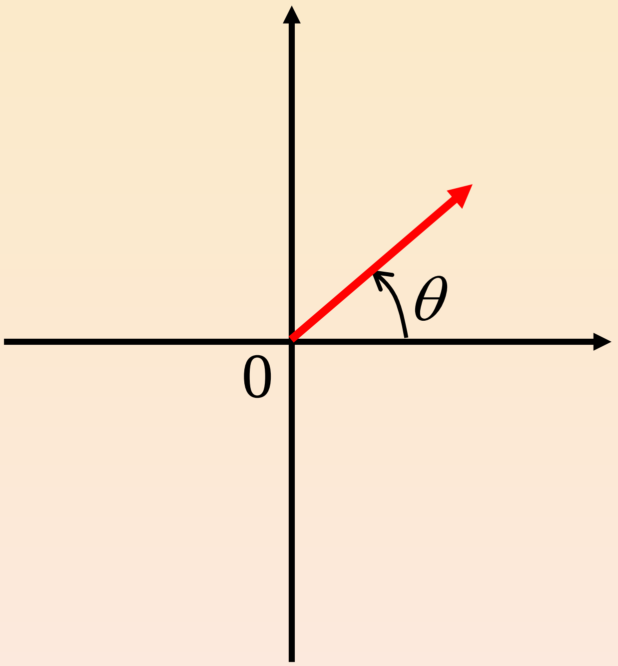
∴ Inductor Current lags

Inductor Voltage by 90°.

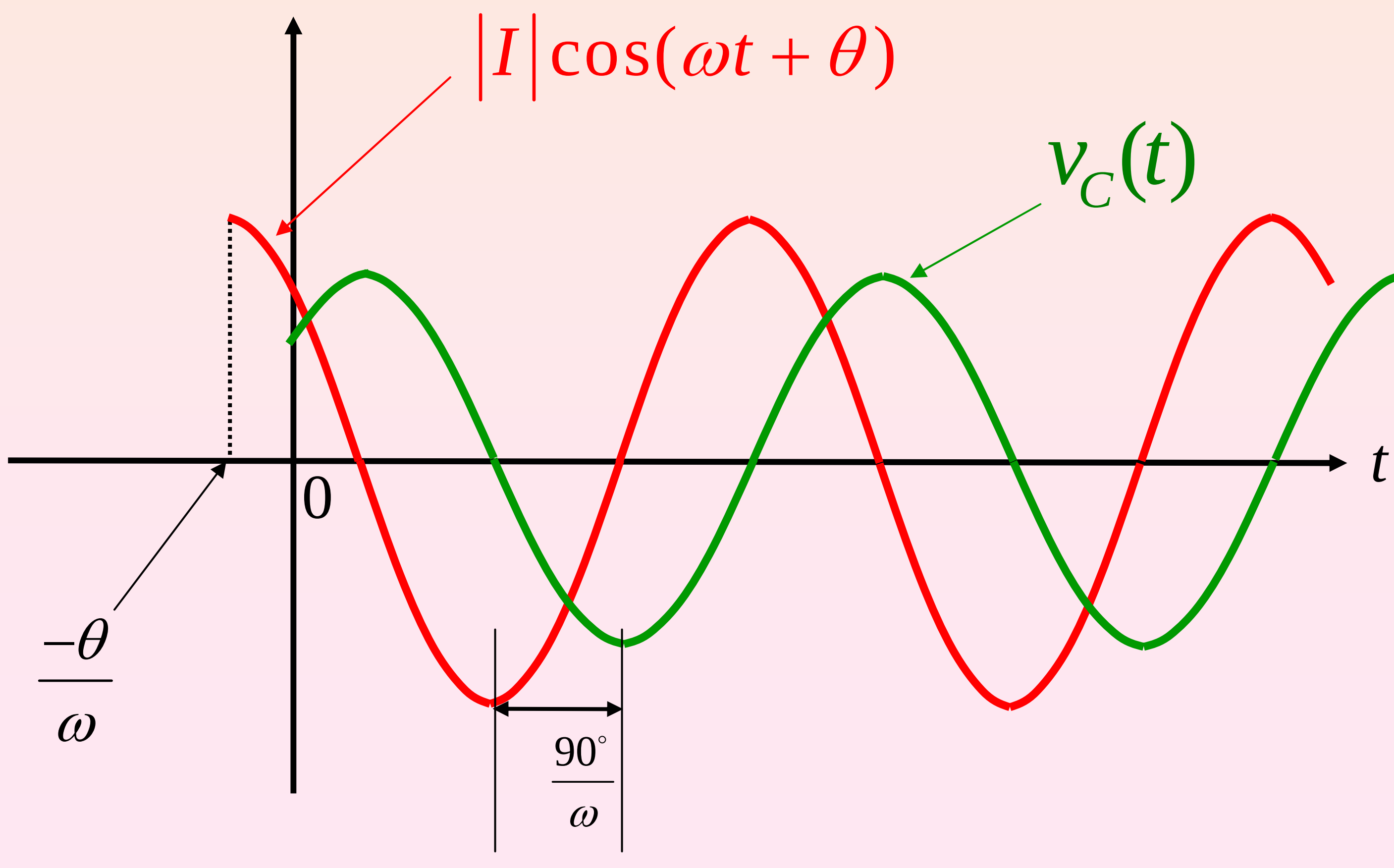


Phasor Diagram



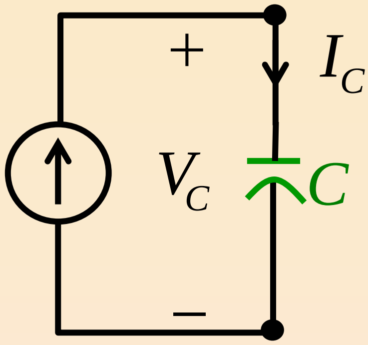


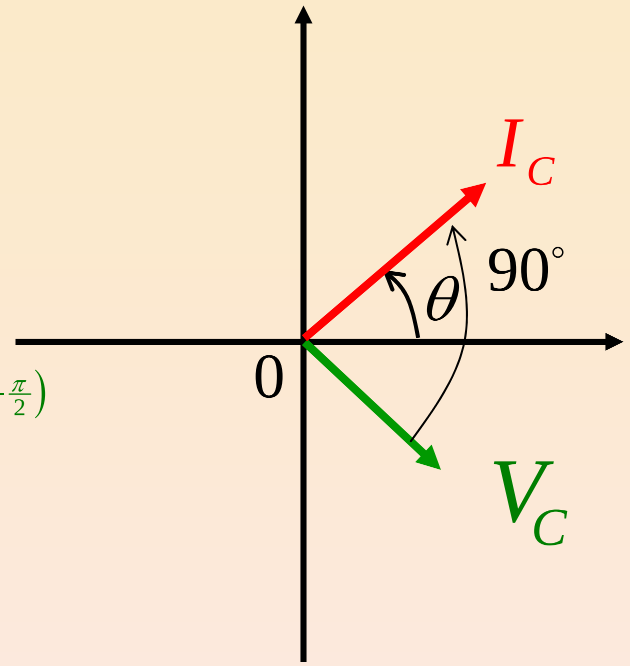
$$v_C(t)=\frac{1}{C}\int i_C(t)dt$$
$$=\frac{|I|}{\omega C}\sin(\omega t+\theta)$$
$$=\frac{|I|}{\omega C}\cos(\omega t+\theta-\frac{\pi}{2})$$



Phasor Diagram

$$I = |I| e^{j\theta}$$

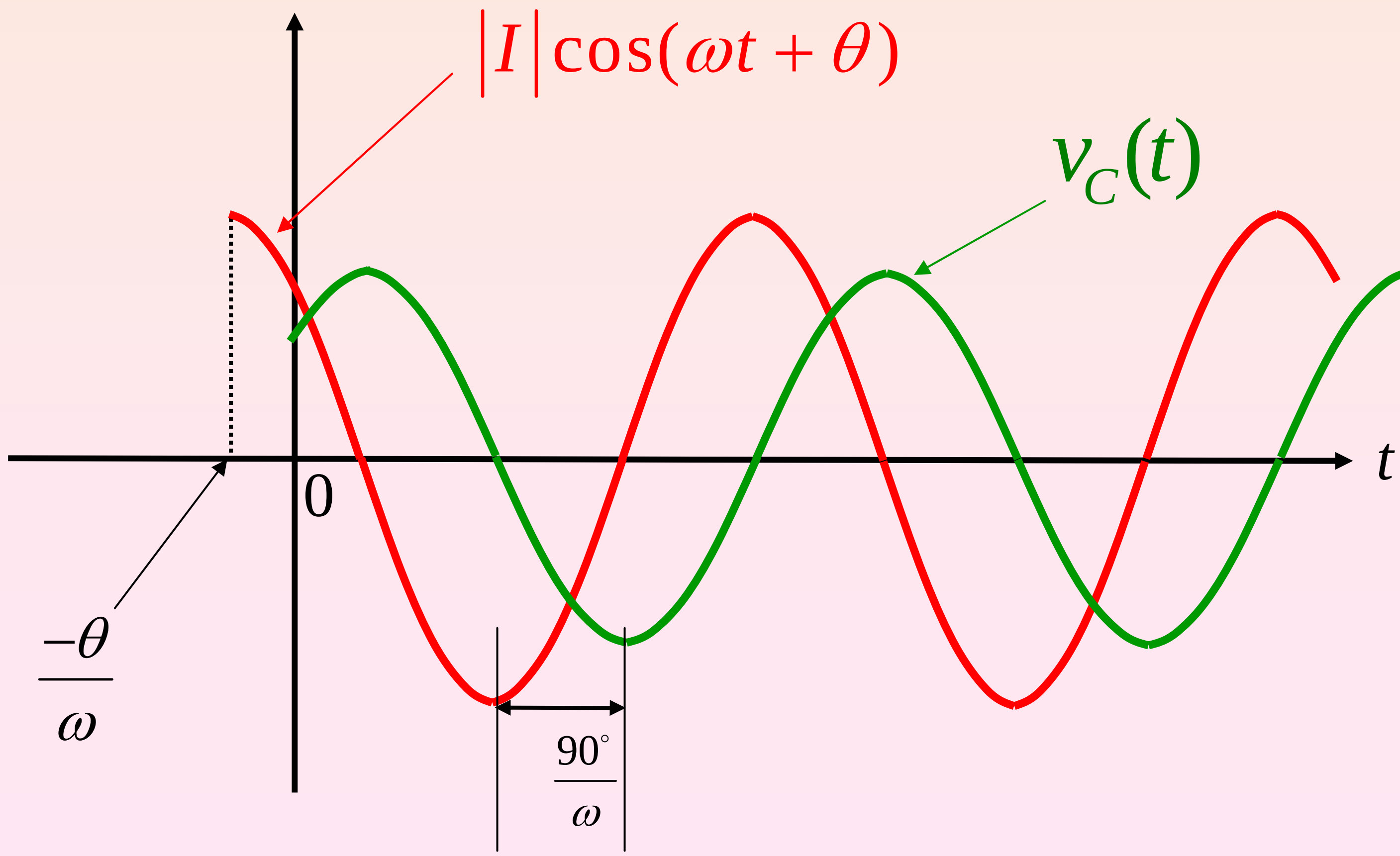




$$V_C = \left(\frac{1}{j\omega C}\right) I_C = \frac{|I|}{\omega C} e^{j\left(\theta - \frac{\pi}{2}\right)}$$

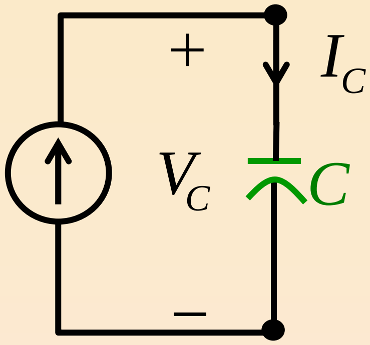
∴ Capacitor Current leads

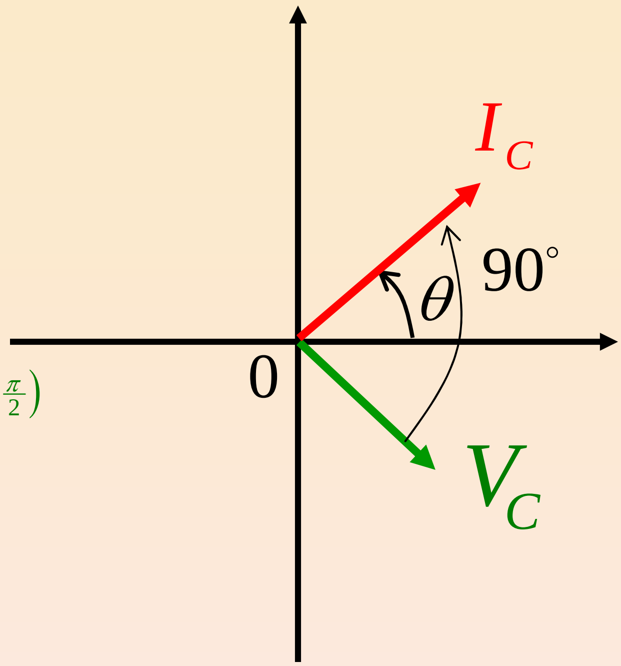
Capacitor Voltage 90°.



Phasor Diagram

$I = |I| e^{j\theta}$



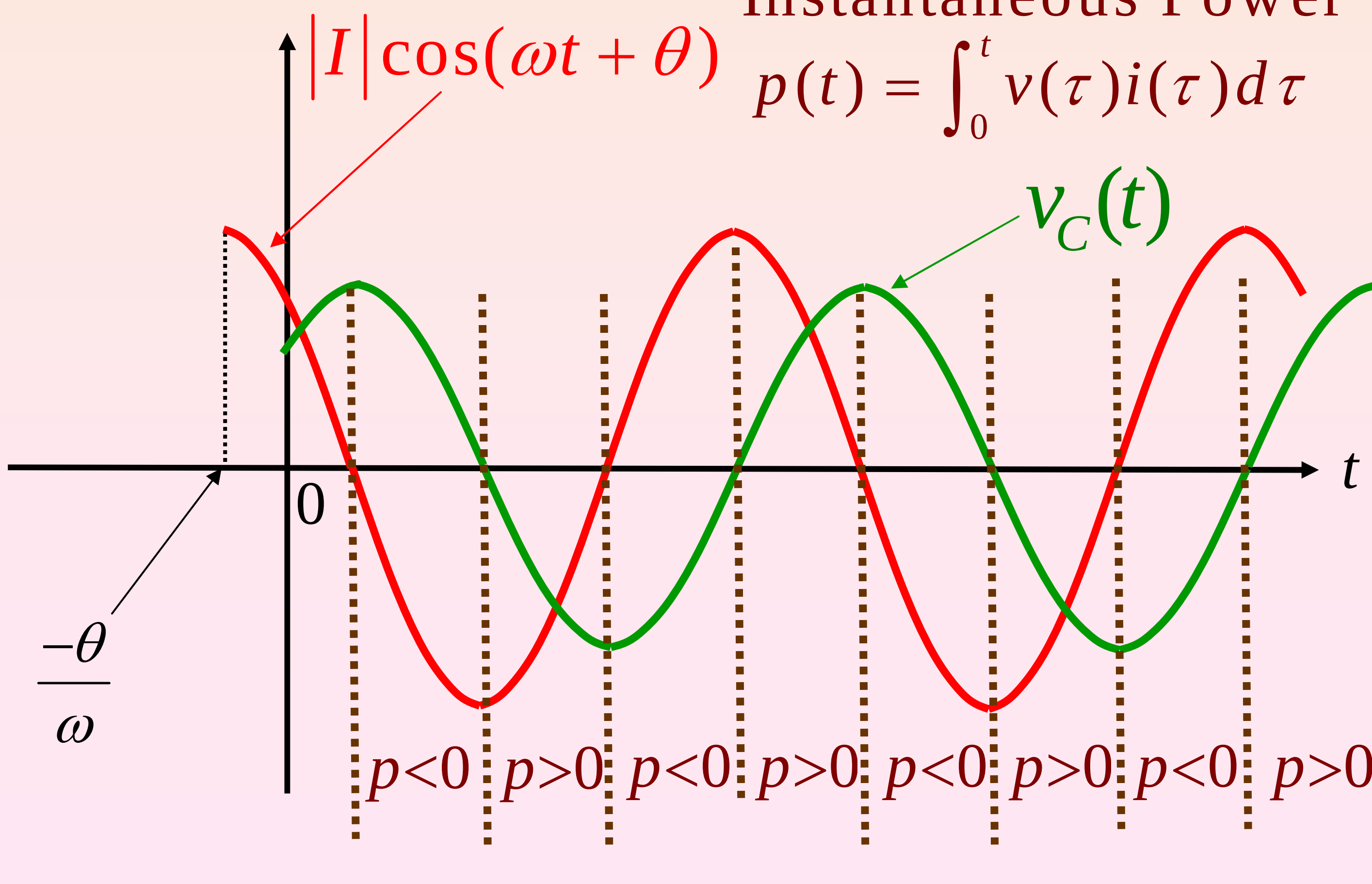


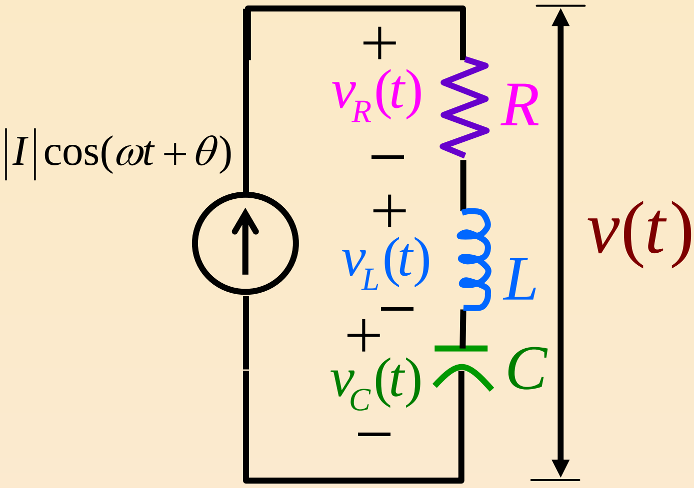
$$V_C = \left(\frac{1}{j\omega C}\right) I_C = \frac{|I|}{\omega C} e^{j\left(\theta - \frac{\pi}{2}\right)}$$

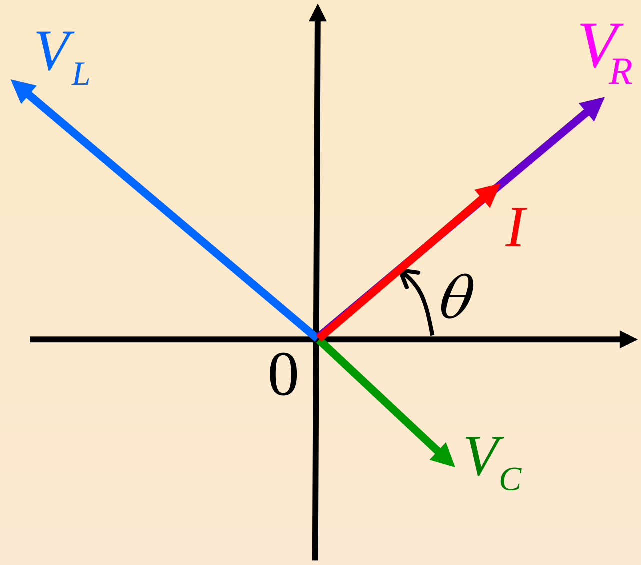
∴ Capacitor Current leads Capacitor Voltage 90°.

Instantaneous Power

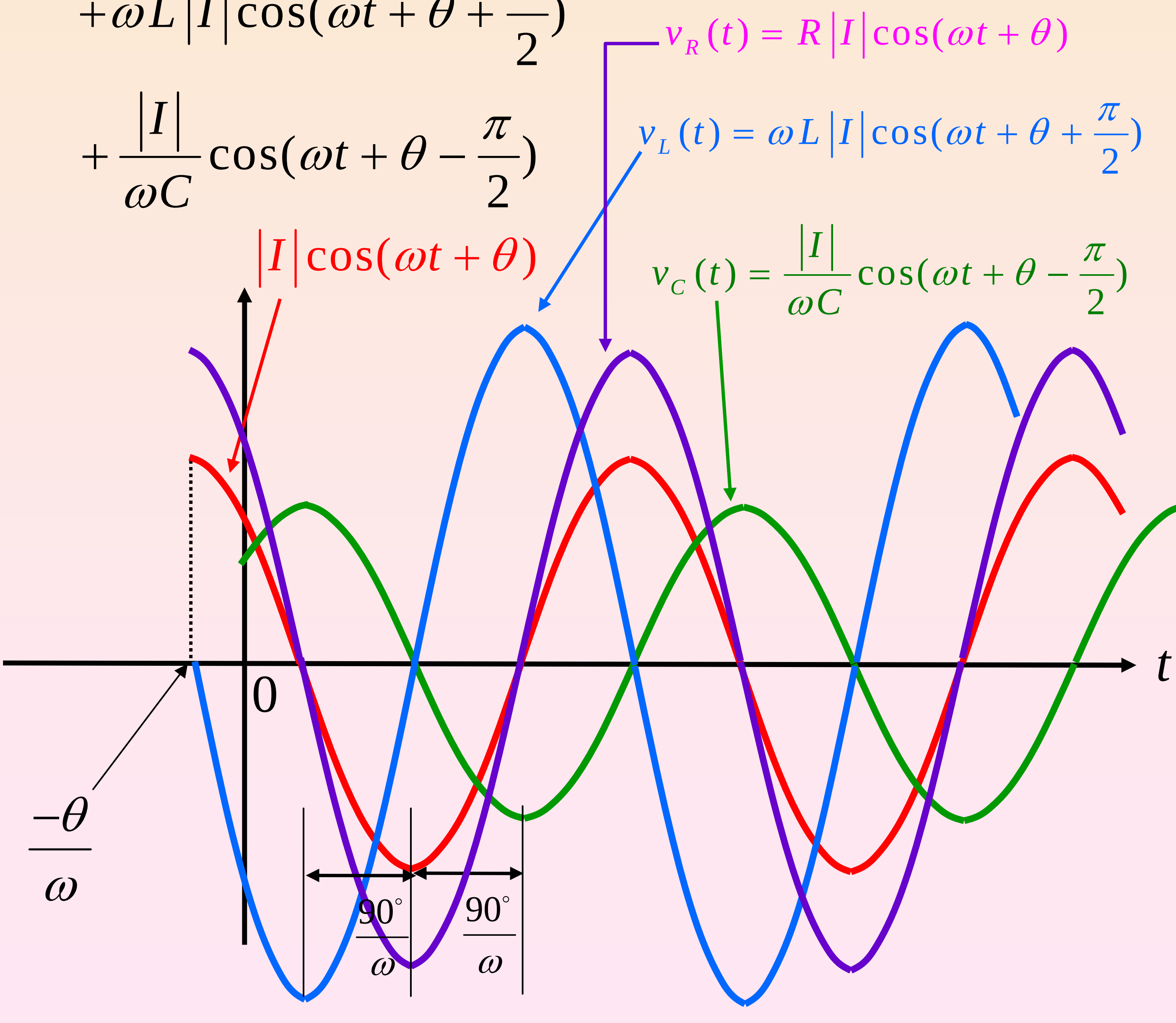
$$p(t) = \int_0^t v(\tau) i(\tau) d\tau$$

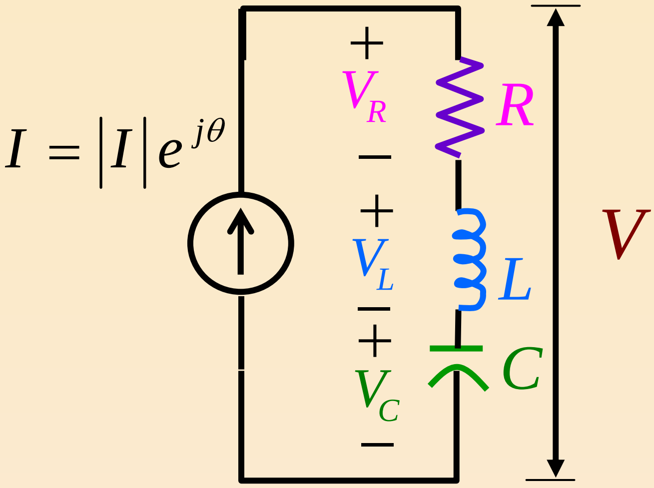


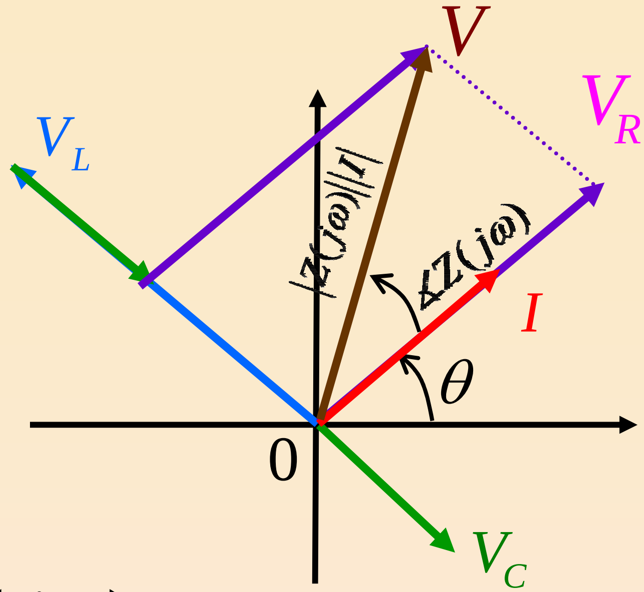




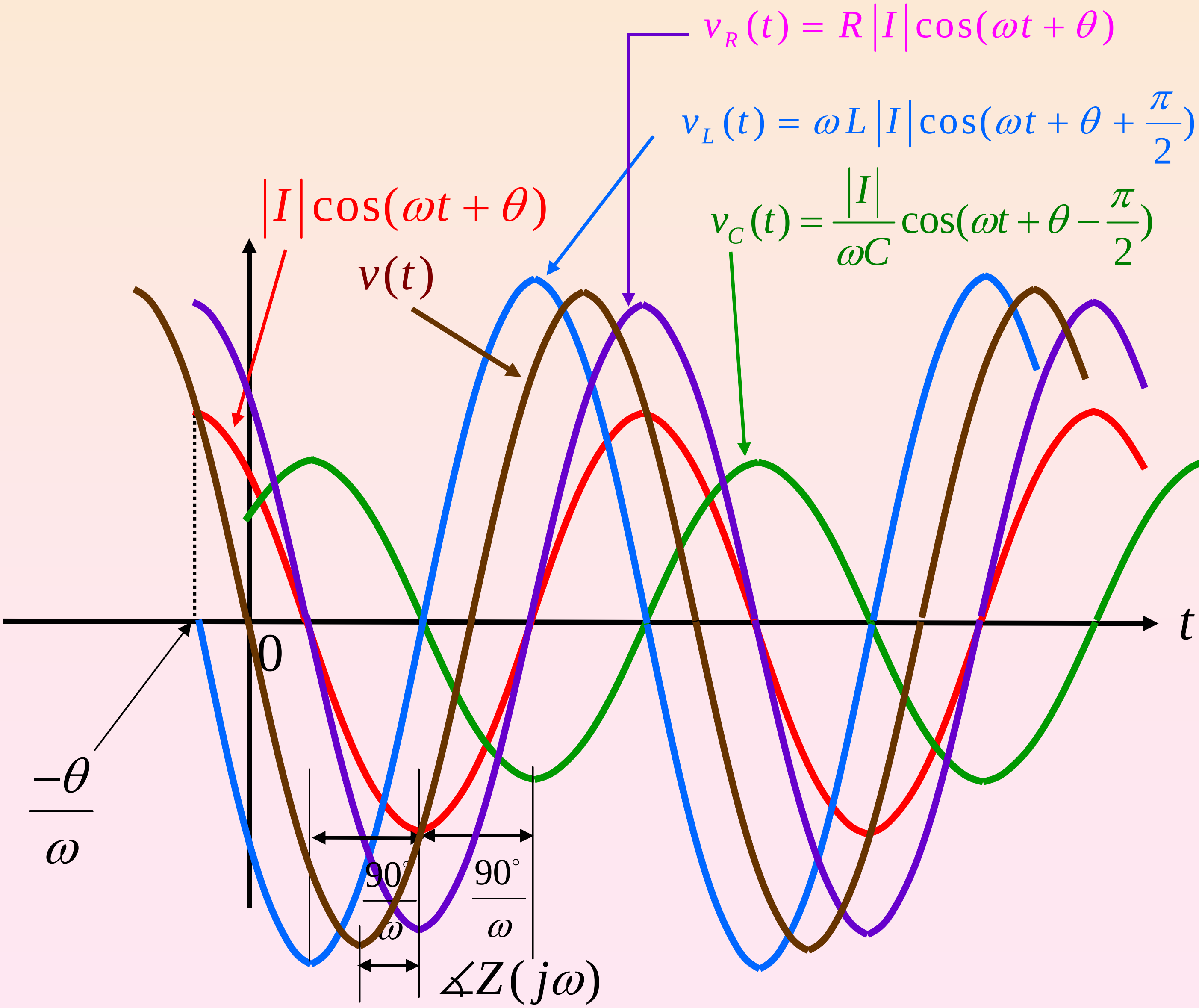
$$v(t) = R\left|I\right|\cos(\omega t+\theta) + \omega L\left|I\right|\cos(\omega t+\theta+\frac{\pi}{2}) + \frac{\left|I\right|}{\omega C}\cos(\omega t+\theta-\frac{\pi}{2})$$

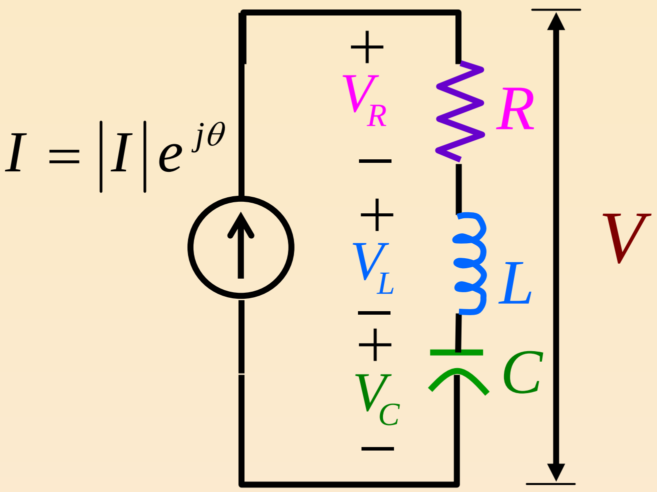


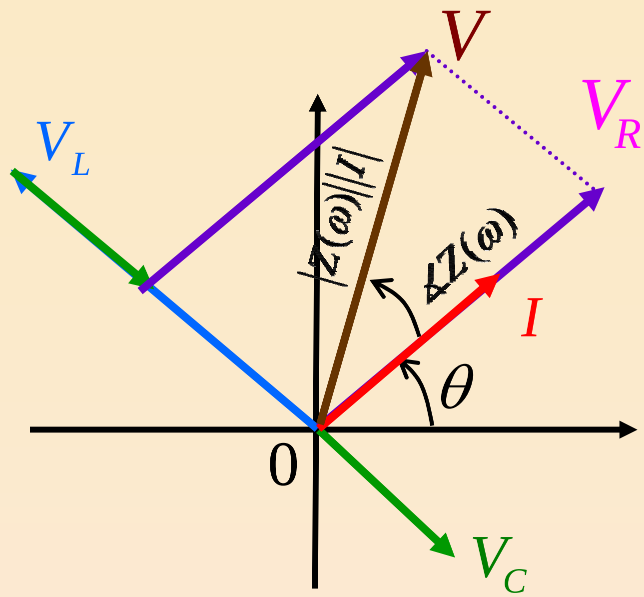




$$V = V_R + V_L + V_C = Z(j\omega)I$$







$$V = V_R + V_L + V_C$$

$$= \left(R + j\omega L + \frac{1}{j\omega C} \right) I$$

$$= \underbrace{Z(j\omega)}_{\textbf{Impedance}} I$$