Chapters 1 and 2

- Outline
  - Electrical quantities
    - Charge, Current, Voltage, Power
  - The ideal basic circuit element
  - Sign conventions
  - Circuit element I-V characteristics
  - Construction of a circuit model
  - Kirchhoff's Current Law
  - Kirchhoff's Voltage Law

Electric Charge

- Electrical effects are due to
  - separation of charge → electric force (voltage)
  - charges in motion → electric flow (current)
- Macroscopically, most matter is electrically neutral most of the time.
  - Exceptions: clouds in a thunderstorm, people on carpets in dry weather, plates of a charged capacitor, etc.
- Microscopically, matter is full of electric charges
  - Electric charge exists in discrete quantities, integral multiples of the electronic charge -1.6×10^{-19} *Coulomb*

Electric Current

Definition: rate of positive charge flow

Symbol: i

Units: Coulombs per second ≡ Amperes (A)

Note: Current has polarity.

$i = dq/dt$ where

q = charge (Coulombs)

t = time (in seconds)

André-Marie Ampère's

1775-1836

Electric Current Examples

1. 10^5 positively charged particles (each with charge 1.6×10^{-19} C) flow to the right ($+x$ direction) every nanosecond

$$I = \frac{Q}{t} = +\frac{10^5\times1.6\times10^{-19}}{10^{-9}} = 1.6\times10^{-5}\ \text{A}$$

2. 10^5 electrons flow to the right ($+x$ direction) every microsecond

$$I = \frac{Q}{t} = -\frac{10^5\times1.6\times10^{-19}}{10^{-9}} = -1.6\times10^{-5} A$$

Electric Potential (Voltage)

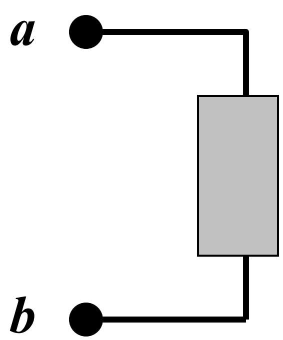
- **<u>Definition</u>:** energy per unit charge
- **<u>Symbol</u>:** v
- **<u>Units</u>:** Joules/Coulomb ≡ Volts (V)

Alessandro Volta
(*1745–1827*)

$$v = dw/dq$$

where w = energy (in Joules), q = charge (in Coulombs)

Note: Potential is always referenced to some point.



<u>Subscript convention</u>:

v_{ab} means the potential at a minus the potential at b.

$$v_{ab} \equiv v_a - v_b$$

Electric Power

- **Definition:** transfer of energy per unit time
- **Symbol:** p
- **Units:** Joules per second ≡ Watts (W)

James Watt
1736 - 1819

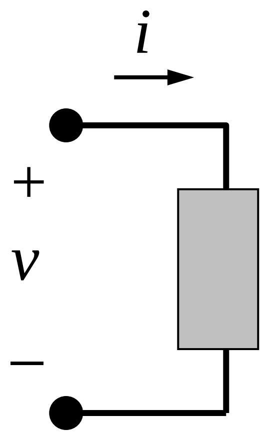
$$p = dw/dt = (dw/dq)(dq/dt) = vi$$

- **Concept:**

  As a positive charge q moves through a drop in voltage v, it loses energy

  - energy change = qv
  - rate is proportional to # charges/sec

The Ideal Basic Circuit Element



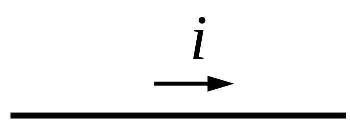
- Polarity reference for voltage can be indicated by plus and minus signs
- Reference direction for the current is indicated by an arrow

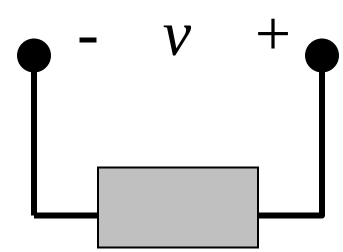
Attributes:

- Two terminals (points of connection)
- Mathematically described in terms of current and/or voltage
- Cannot be subdivided into other elements

A Note about Reference Directions

- A problem like “Find the current” or “Find the voltage” is always accompanied by a definition of the direction:

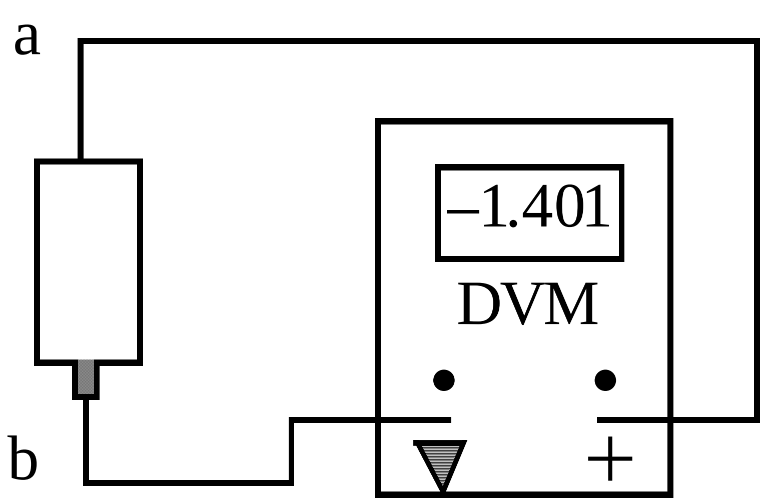




- In this case, if the current turns out to be 1 mA flowing to the left, we would say *i* = -1 mA.
- In order to perform circuit analysis to determine the voltages and currents in an electric circuit, you need to specify reference directions.
- There is no need to guess the reference direction so that the answers come out positive.

Sign Convention Example

Suppose you have an unlabelled battery and you measure its voltage with a digital voltmeter (DVM). It will tell you the **magnitude and sign** of the voltage.



With this circuit, you are measuring v_{ab}.

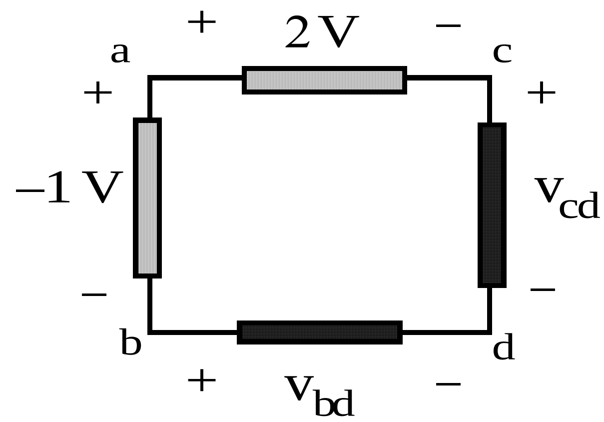
The DVM indicates –1.401, so v_a is lower than v_b by 1.401 V.

Which is the positive battery terminal?

Note that we have used the "ground" symbol (▼) for the reference node on the DVM. Often it is labeled "C" for "common."

Another Example

Find v_{ab}, v_{ca}, v_{cb}

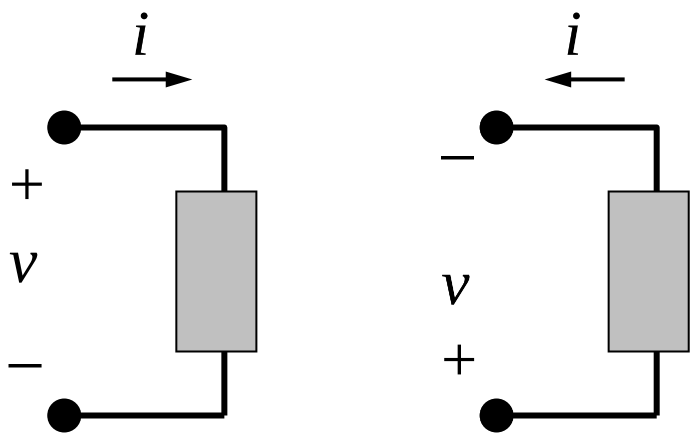


Note that the labeling convention has nothing to do with whether or not v is positive or negative.

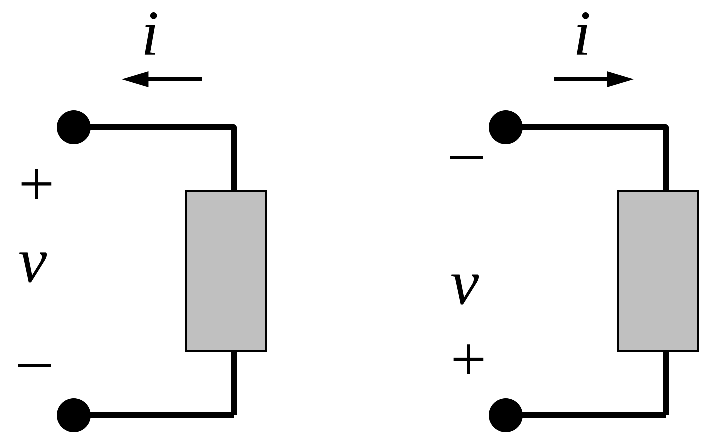
Sign Convention for Power

Passive sign convention

$$p = vi$$



$$p = -vi$$

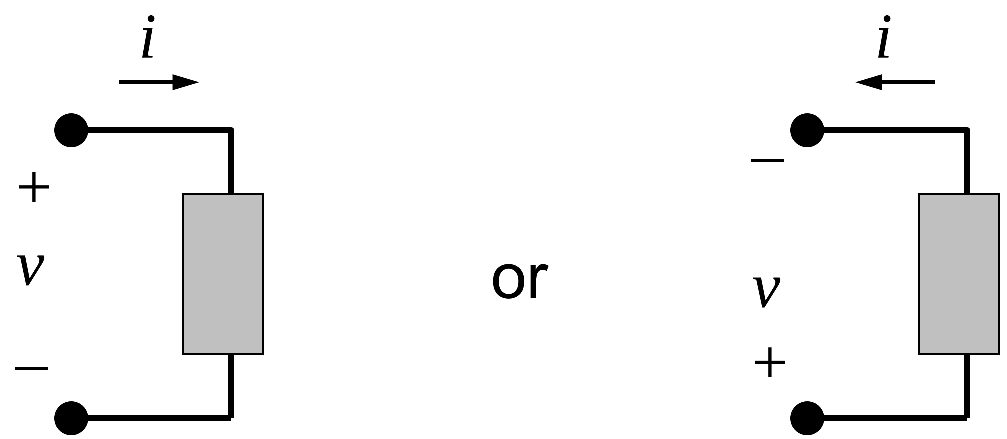


- **If $p > 0$, power is being delivered to the box.**
- **If $p < 0$, power is being extracted from the box.**

Power

If an element is absorbing power (*i.e.* if $p > 0$), positive charge is flowing from higher potential to lower potential.

$p = vi$ if the "passive sign convention" is used:

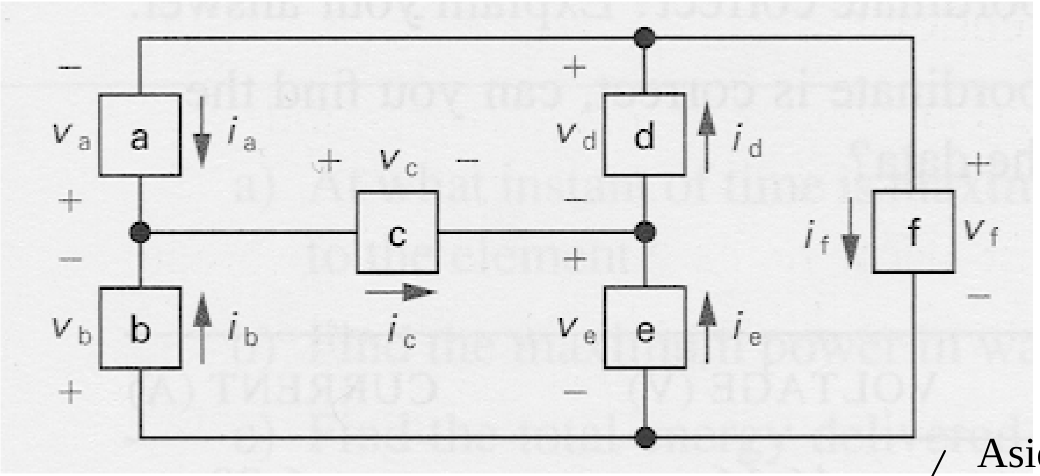


How can a circuit element absorb power?

By converting electrical energy into heat (resistors in toasters), light (light bulbs), or acoustic energy (speakers); by storing energy (charging a battery).

Power Calculation Example

Find the power absorbed by each element:



Conservation of energy
➔ total power delivered equals total power absorbed

Aside: For electronics these are unrealistically large currents – milliamperes or smaller is more typical

ELEMENT	VOLTAGE (V)	CURRENT (A)	***vi* (W)**	***p* (W)**
a	−18	−51	**918**	
b	−18	45	**- 810**	
c	2	−6	**- 12**	
d	20	−20	**- 400**	
e	16	−14	**- 224**	
f	36	31	**1116**	

Circuit Elements

- 5 ideal basic circuit elements:
  - voltage source
  - current source
  - resistor
  - inductor
  - capacitor

voltage source and current source: ***active elements***, capable of generating electric energy

resistor, inductor and capacitor: ***passive elements***, incapable of generating electric energy

- Many practical systems can be modeled with just sources and resistors
- The basic analytical techniques for solving circuits with inductors and capacitors are similar to those for resistive circuits

Electrical Sources

- An ***electrical source*** is a device that is capable of converting non-electric energy to electric energy and *vice versa*.

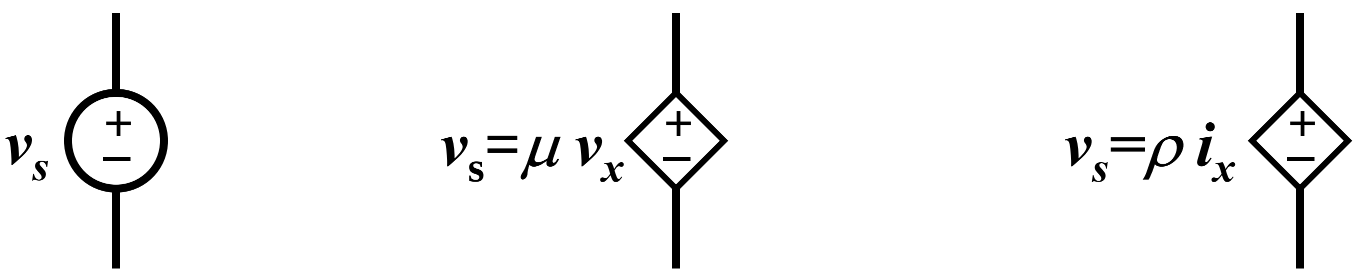
  Examples:
  - battery: chemical ⟺ electric
  - dynamo (generator/motor): mechanical ⟺ electric
    (Ex. gasoline-powered generator, Bonneville dam)

→ Electrical sources can either deliver or absorb power

Ideal Voltage Source

- Circuit element that maintains a prescribed voltage across its terminals, **regardless of the current flowing in those terminals**.
  - Voltage is known, but current is determined by the circuit to which the source is connected.
- The voltage can be either **independent** or **dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

<u>Device symbols</u>:

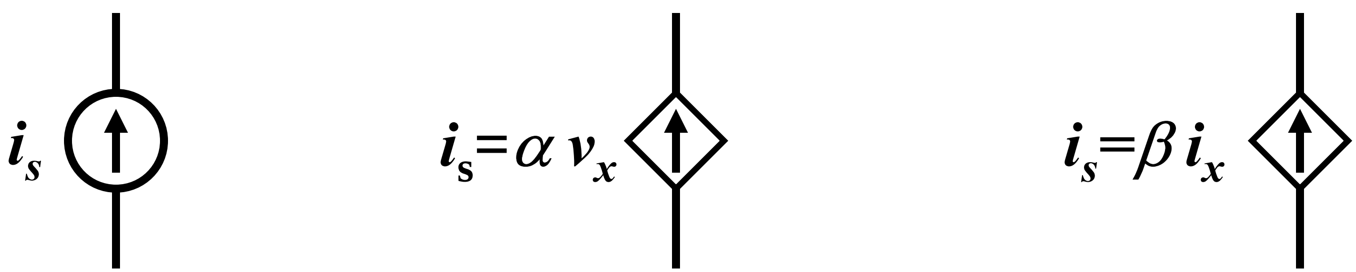


independent **voltage-controlled** **current-controlled**

Ideal Current Source

- Circuit element that maintains a prescribed current through its terminals, **regardless of the voltage across those terminals**.
  - Current is known, but voltage is determined by the circuit to which the source is connected.
- The current can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

Device symbols:

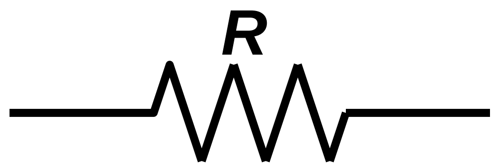


independent **voltage-controlled** **current-controlled**

Electrical Resistance

- ***Resistance:*** the ratio of voltage drop and current. The circuit element used to model this behavior is the ***resistor***.

<u>Circuit symbol</u>:



<u>Units</u>: Volts per Ampere ≡ ohms (Ω)

- The current flowing in the resistor is proportional to the voltage across the resistor:

Georg Simon Ohm
1789-1854

$$v = i\,R$$ (Ohm's Law)

where v = voltage (V), i = current (A), and R = resistance (Ω)

Electrical Conductance

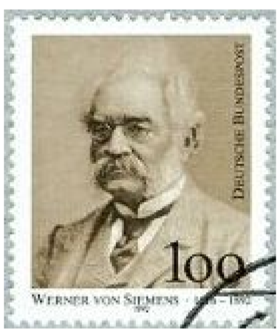
- **Conductance** is the reciprocal of resistance.

<u>Symbol</u>: G

<u>Units</u>: siemens (S) or mhos (℧)

<u>Example</u>:

Consider an 8 Ω resistor. *What is its conductance?*



Werner von Siemens
1816-1892

Short Circuit and Open Circuit

- Short circuit
  - R = 0 → no voltage difference exists
  - all points on the wire are at the same potential.
  - Current can flow, as determined by the circuit
- Open circuit
  - R = ∞ → no current flows
  - Voltage difference can exist, as determined by the circuit

Example: Power Absorbed by a Resistor

$$p = vi = (iR)i = i^2R$$

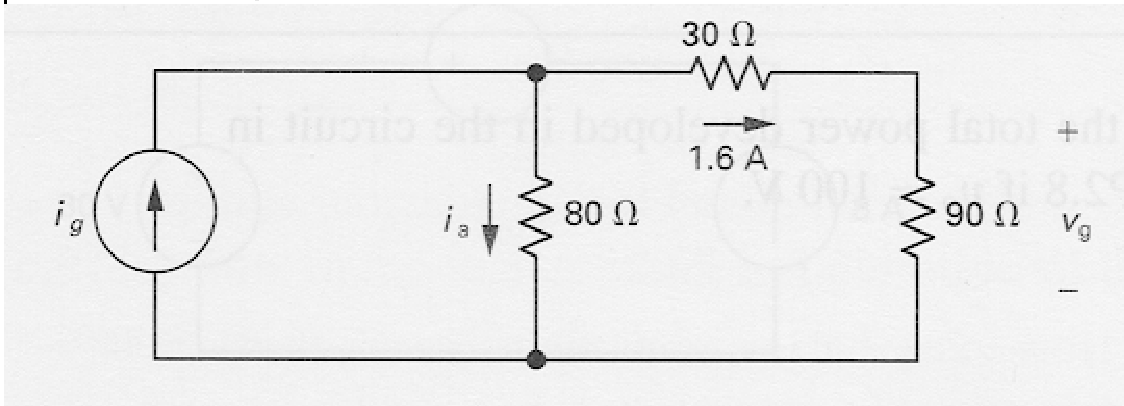
$$p = vi = v(v/R) = v^2/R$$

Note that $p > 0$ always, for a resistor → a resistor dissipates electric energy

Example:

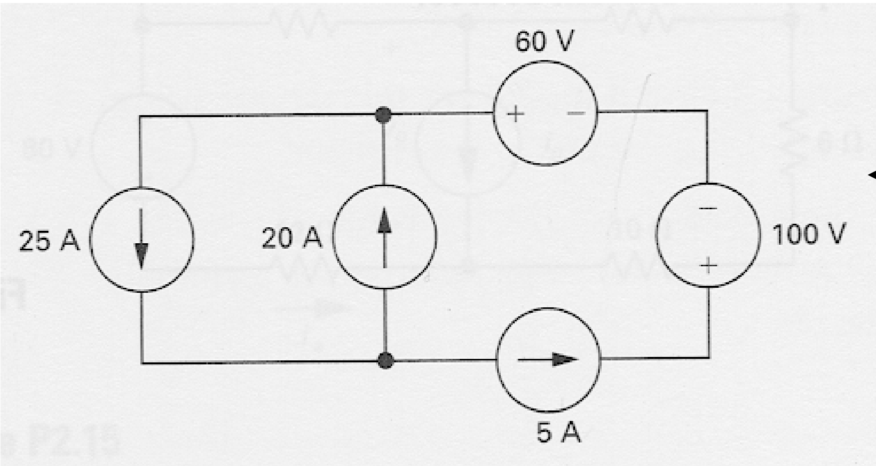
a) Calculate the voltage v_g and current i_a.

b) Determine the power dissipated in the 80Ω resistor.



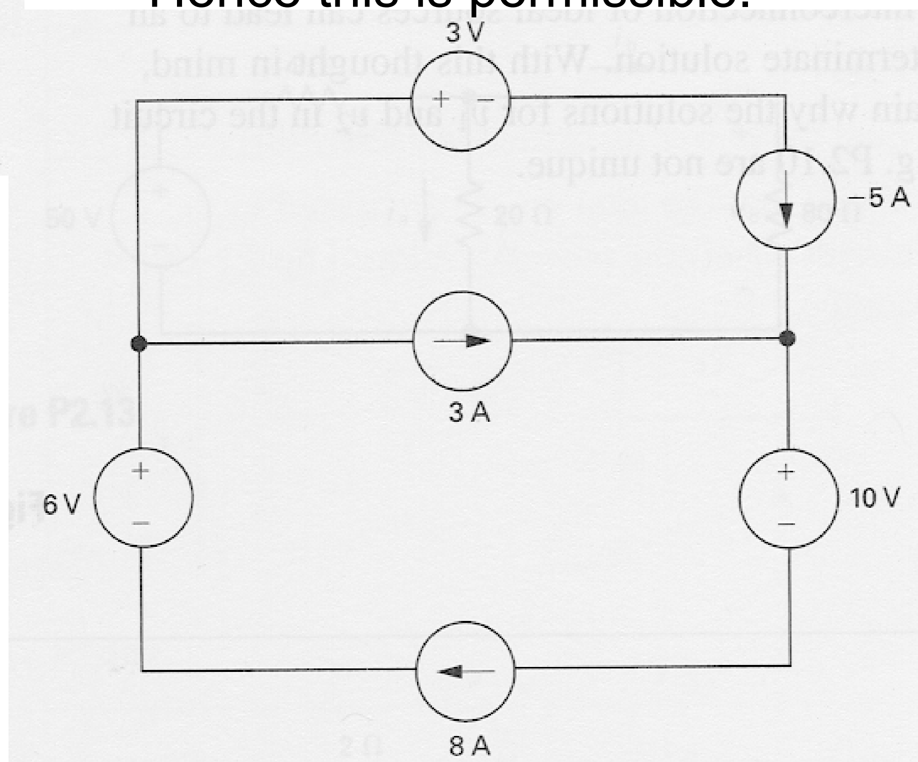
More Examples

- Are these interconnections permissible?



This circuit connection is permissible. This is because the current sources can sustain any voltage across; Hence this is permissible.

This circuit connection is NOT permissible. It violates the KCL.



Summary

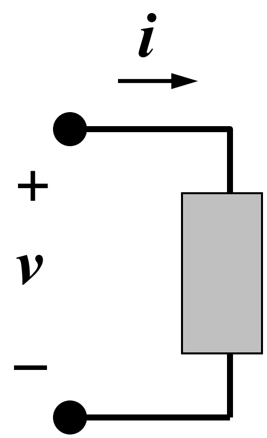
- ***Current*** = rate of charge flow *i = dq/dt*
- ***Voltage*** = energy per unit charge created by charge separation
- ***Power*** = energy per unit time
- ***Ideal Basic Circuit Elements***
  – two-terminal component that cannot be sub-divided
  – described mathematically in terms of its terminal voltage and current
  – An ***ideal voltage source*** maintains a prescribed voltage regardless of the current in the device.
  – An ***ideal current source*** maintains a prescribed current regardless of the voltage across the device.
  – A ***resistor*** constrains its voltage and current to be proportional to each other: $v = iR$ (Ohm's law)

Summary (cont'd)

- ***Passive sign convention***
  - For a passive device, the reference direction for current through the element is in the direction of the reference voltage drop across the element

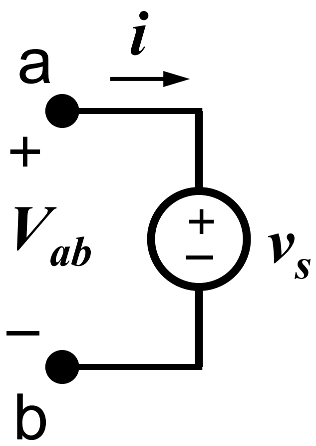
Current *vs.* Voltage (*I-V*) Characteristic

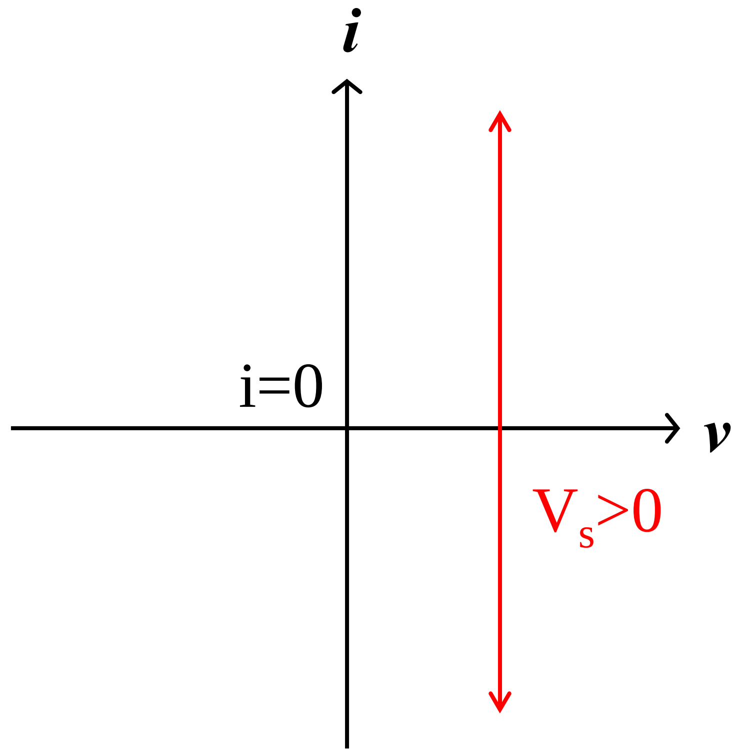
- Voltage sources, current sources, and resistors can be described by plotting the current (*i*) as a function of the voltage (*v*)



Passive? Active?

I-V Characteristic of Ideal Voltage Source

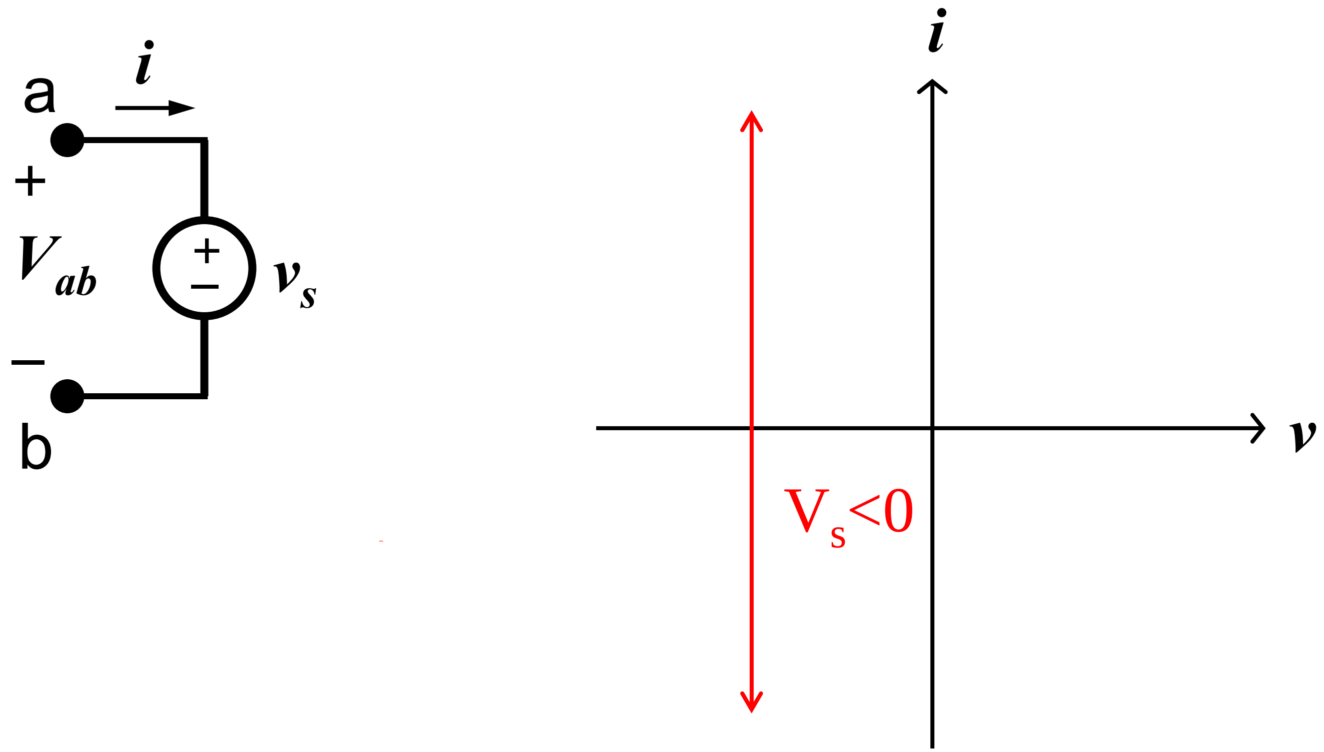




1. Plot the *I-V* characteristic for $v_s > 0$. For what values of i does the source absorb power? For what values of i does the source release power?

V_s>0 → i<0 release power; i>0 absorb power

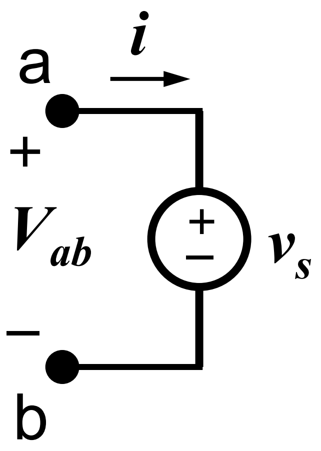
I-V Characteristic of Ideal Voltage Source

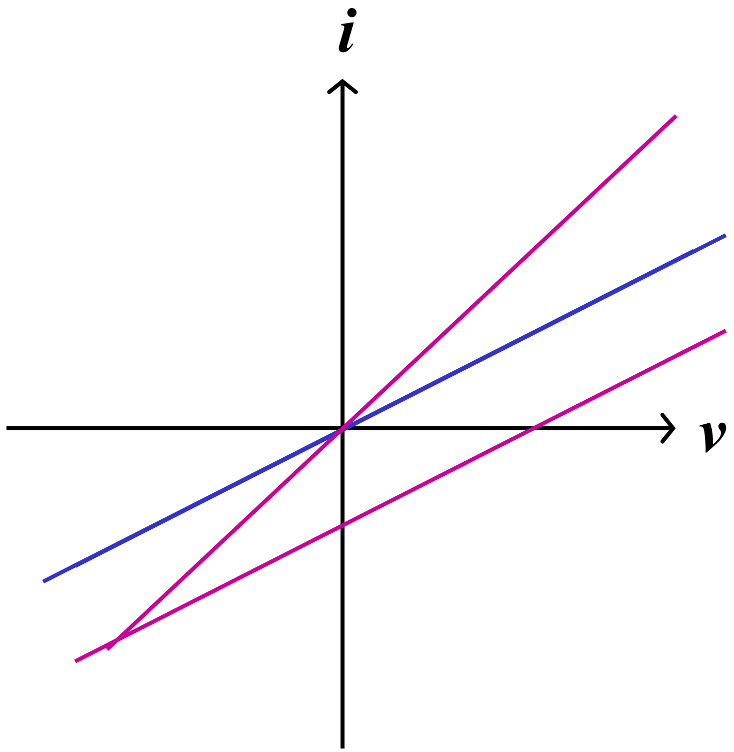


2. **Plot the *I-V* characteristic for $v_s < 0$. For what values of i does the source absorb power? For what values of i does the source release power?**

$V_s<0$ → i>0 release power; i<0 absorb power

I-V Characteristic of Ideal Voltage Source

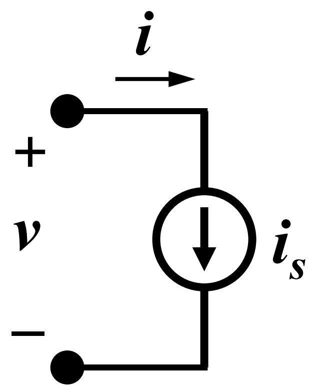


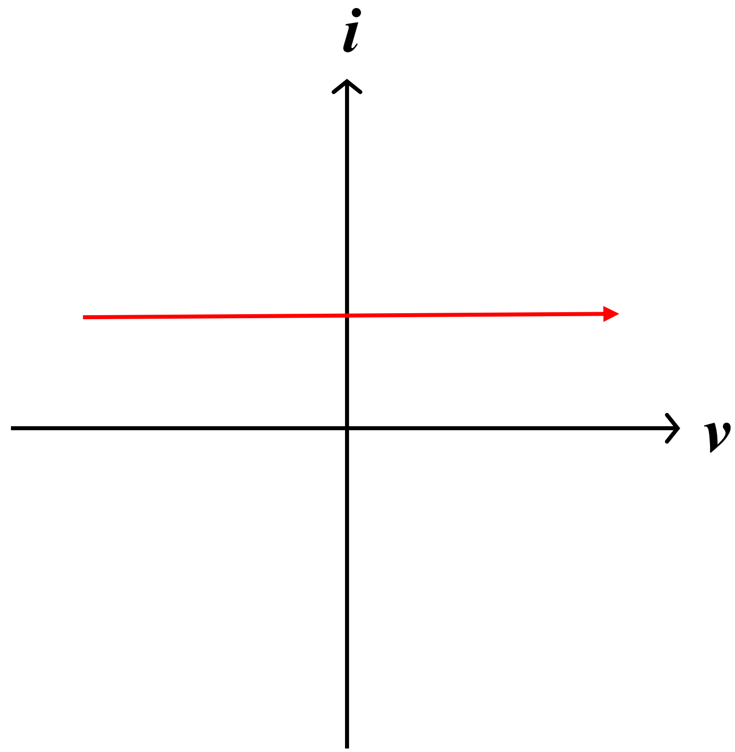


3. What is the *I-V* characteristic for an ideal wire?

Do not forget V_{ab}=-V_{ba}

I-V Characteristic of Ideal Current Source





1. **Plot the *I-V* characteristic for $i_s > 0$. For what values of v does the source absorb power? For what values of v does the source release power?**

V>0 absorb power; V<0 release power

Short Circuit and Open Circuit

Wire ("short circuit"):

- $R = 0$ → **no voltage difference exists**
  (all points on the wire are at the same potential)
- Current can flow, as determined by the circuit

Air ("open circuit"):

- $R = \infty$ → **no current flows**
- Voltage difference can exist, as determined by the circuit

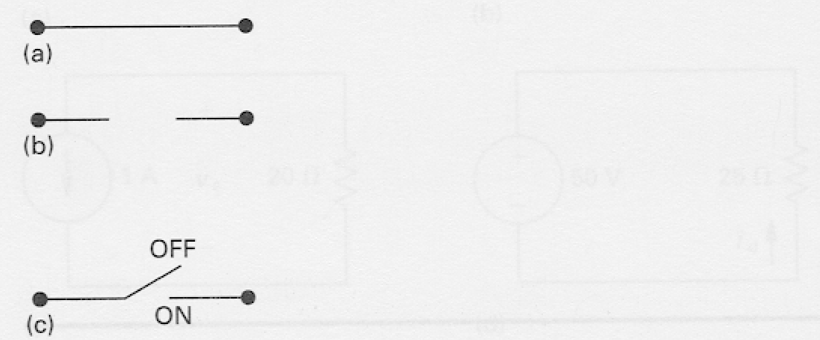
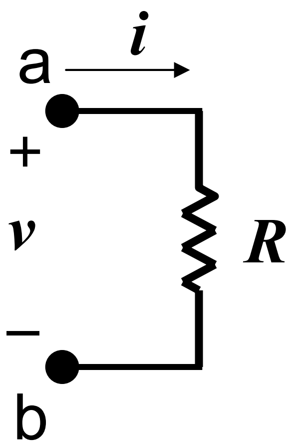
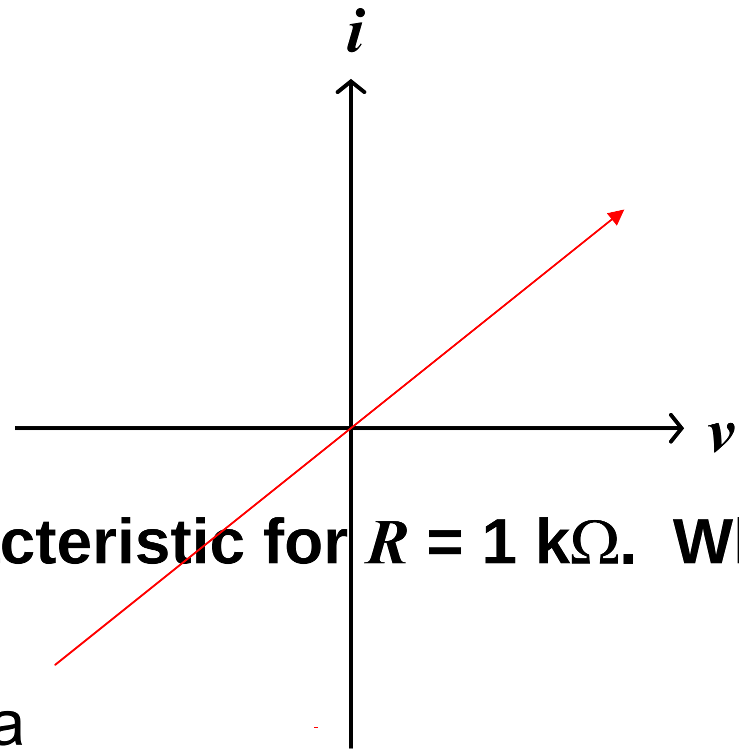


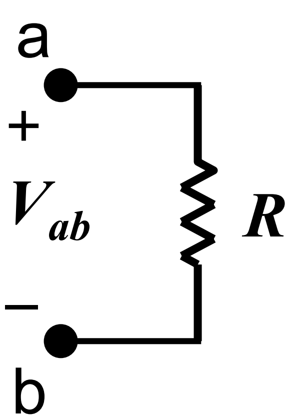
Figure 2.10 Circuit symbols. (a) Short circuit. (b) Open circuit. (c) Switch.

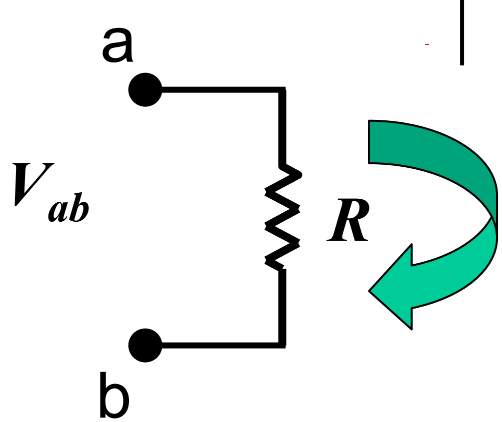
I-V Characteristic of Ideal Resistor





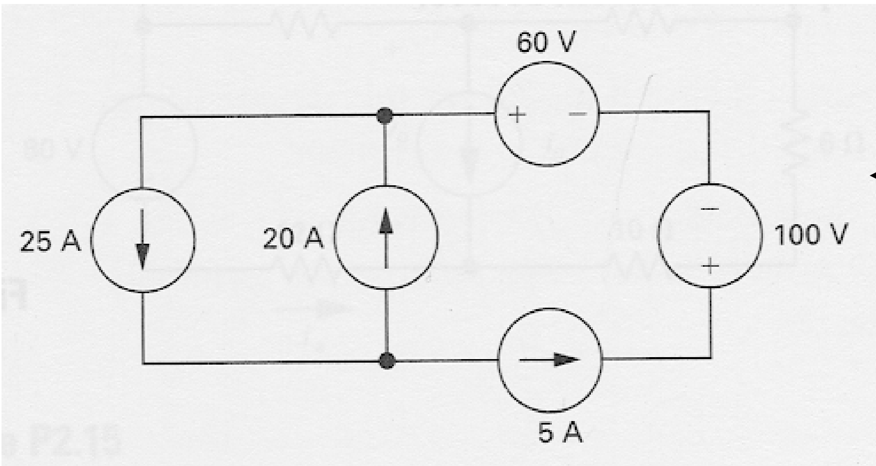
1. **Plot the *I-V* characteristic for R = 1 kΩ. What is the slope?**





More Examples: Correction from last Lec.

- Are these interconnections permissible?



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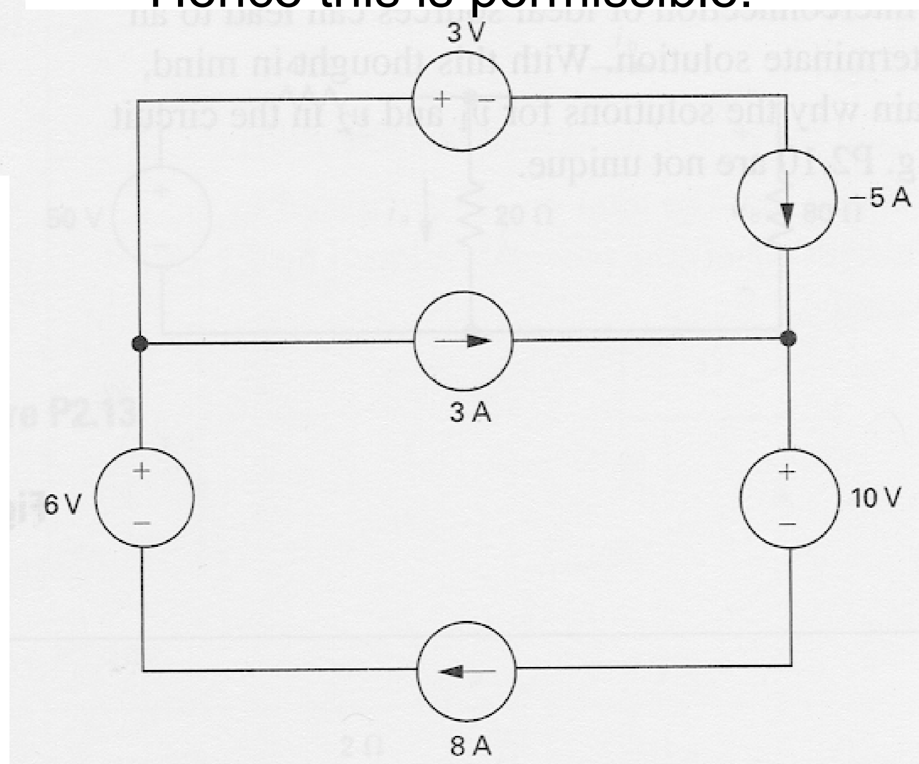


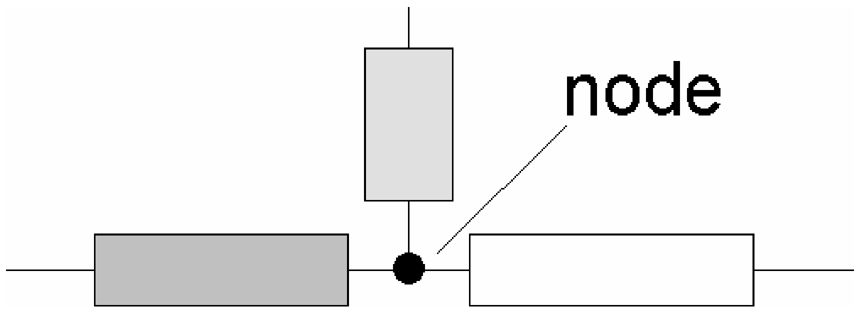
This circuit connection is NOT permissible. It violates the KCL.

Construction of a Circuit Model

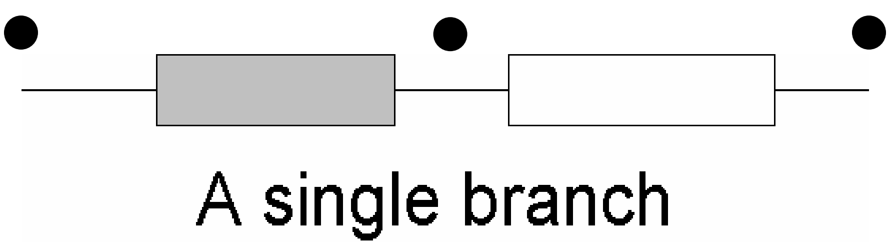
- The electrical behavior of each physical component is of primary interest.
- We need to account for undesired as well as desired electrical effects.
- Simplifying assumptions should be made wherever reasonable.

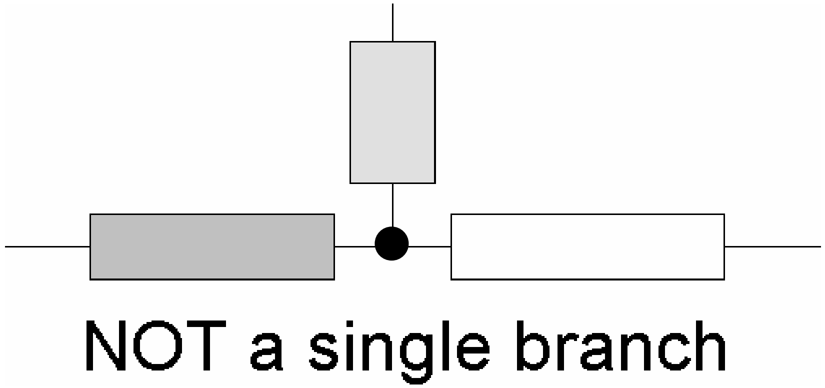
Terminology: Nodes and Branches

***Node*:** A point where two or more circuit elements are connected



***Branch*:** A path that connects two nodes





Circuit Nodes and Loops

- A ***node*** is a point where two or more circuit elements are connected.
- A ***loop*** is formed by tracing a closed path in a circuit through selected basic circuit elements without passing through any intermediate node more than once

Kirchhoff’s Laws

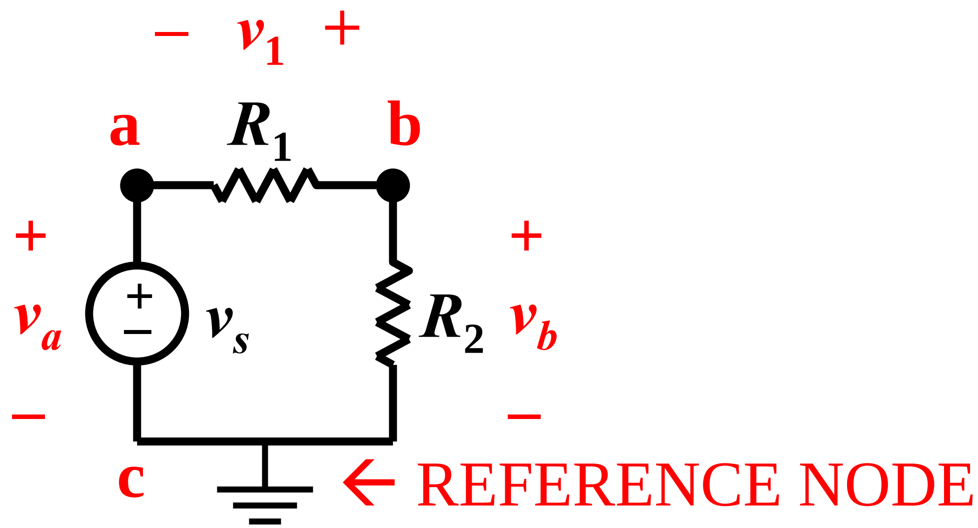
- **<u>Kirchhoff’s Current Law (KCL)</u>:**
  - The algebraic sum of all the **currents** entering any **node** in a circuit equals zero.
- **<u>Kirchhoff’s Voltage Law (KVL)</u>:**
  - The algebraic sum of all the **voltages** around any **loop** in a circuit equals zero.

Gustav Robert Kirchhoff
1824-1887

Notation: Node and Branch Voltages

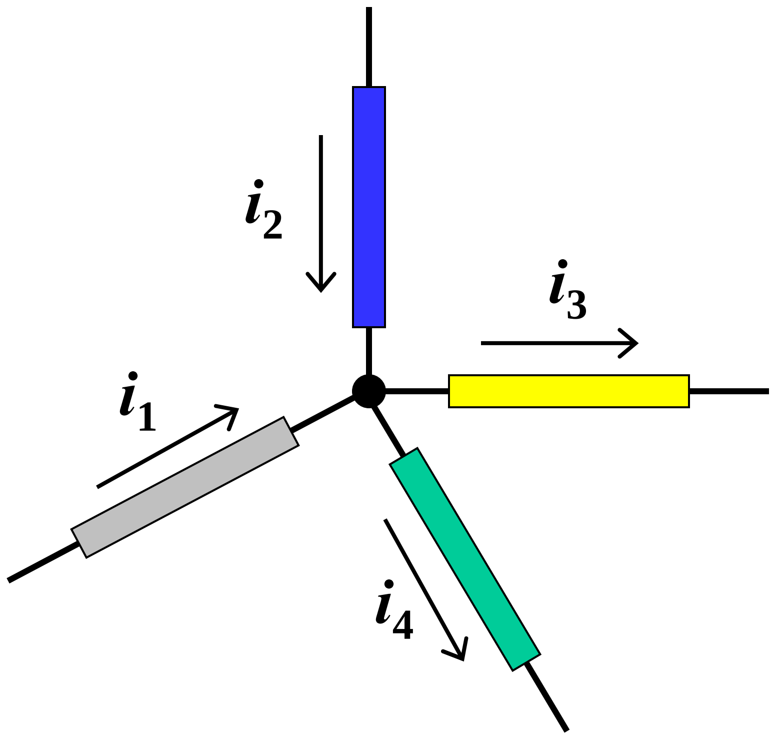
- Use one node as the reference (the "common" or "ground" node) – label it with a symbol
- The voltage drop from node **x** to the reference node is called the **node voltage** $\mathbf{v_x}$.
- The voltage across a circuit element is defined as the difference between the node voltages at its terminals

Example:



Using Kirchhoff's Current Law (KCL)

Consider a node connecting several branches:



- Use **reference directions** to determine whether currents are "entering" or "leaving" the node – with no concern about actual current directions

Formulations of Kirchhoff's Current Law

(Charge stored **in node** is zero.)

Formulation 1:

Sum of currents entering node
= sum of currents leaving node

Formulation 2:

Algebraic sum of currents entering node = 0

- Currents leaving are included with a minus sign.

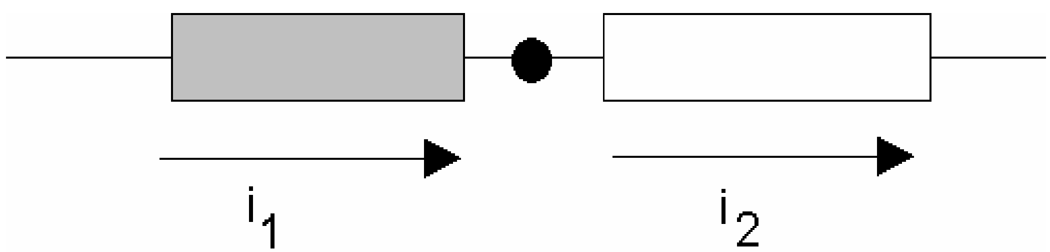
Formulation 3:

Algebraic sum of currents leaving node = 0

- Currents entering are included with a minus sign.

A Major Implication of KCL

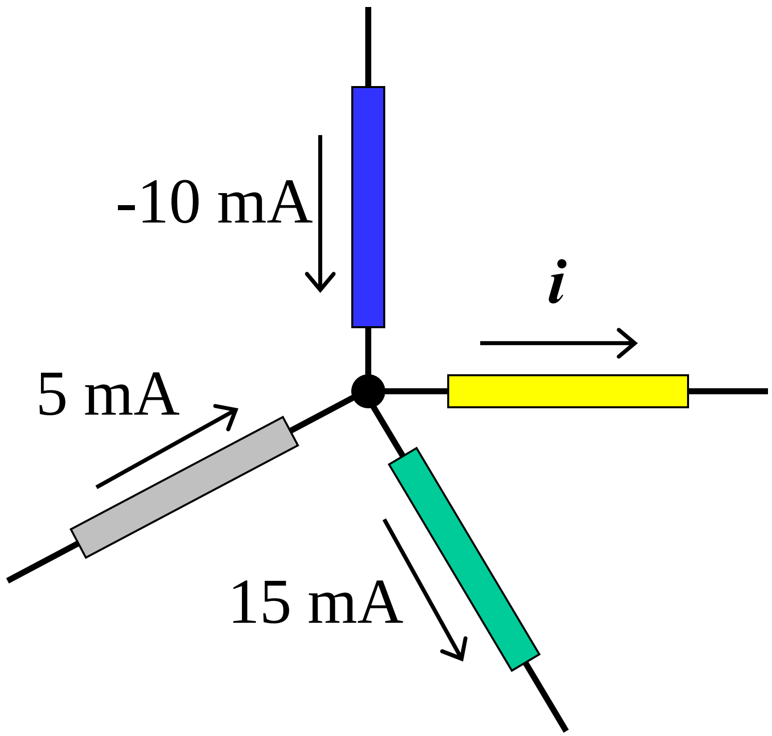
- KCL tells us that **all of the elements in a single branch carry the same current**.
- We say these elements are connected ***in series***.



Current entering node = Current leaving node

$$i_1 = i_2$$

KCL Example



Currents entering the node:

Currents leaving the node:

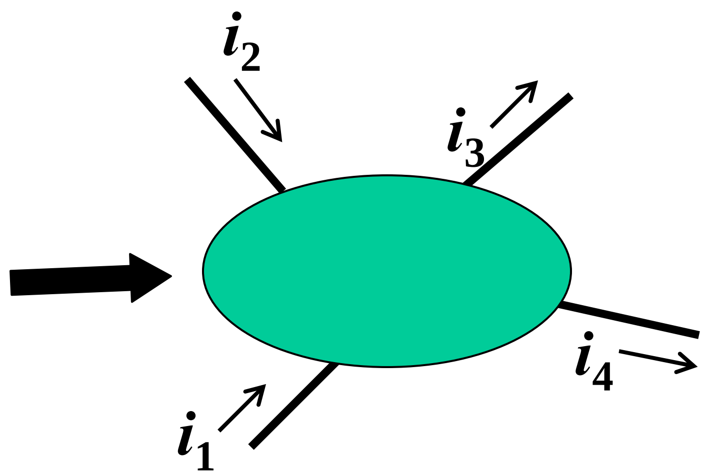
<u>3 formulations of KCL</u>:

1.
2.
3.

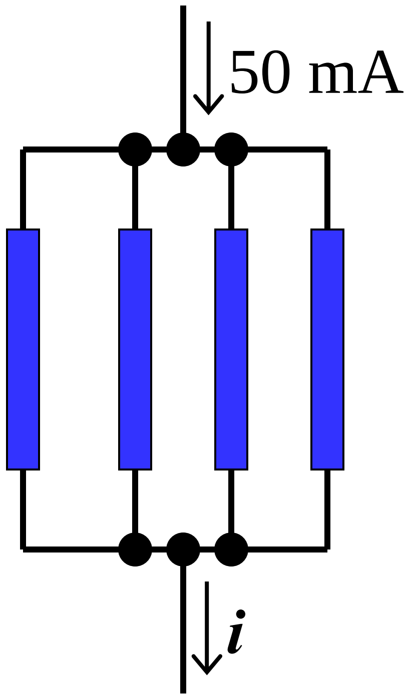
Generalization of KCL

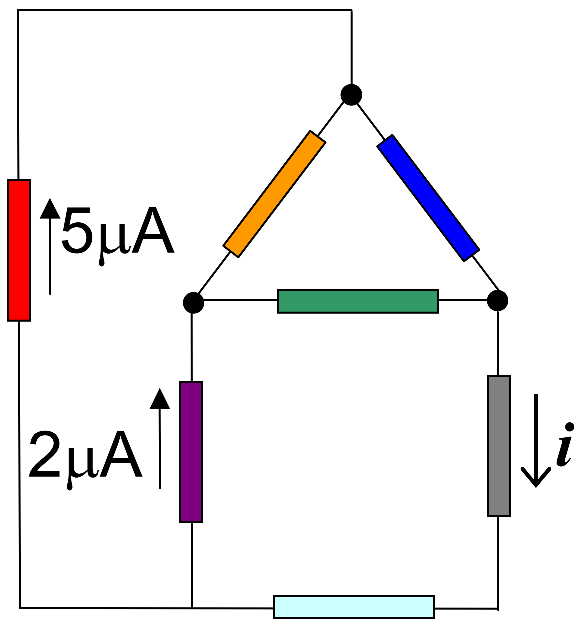
- The sum of currents entering/leaving a **closed surface** is zero. Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.* a “black box”



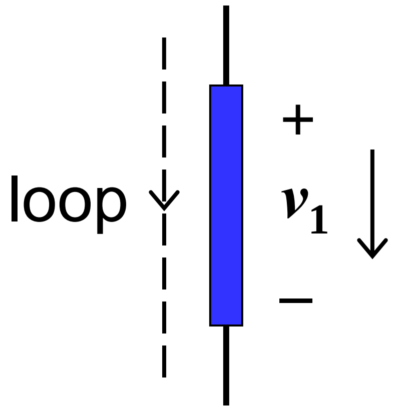
Generalized KCL Examples



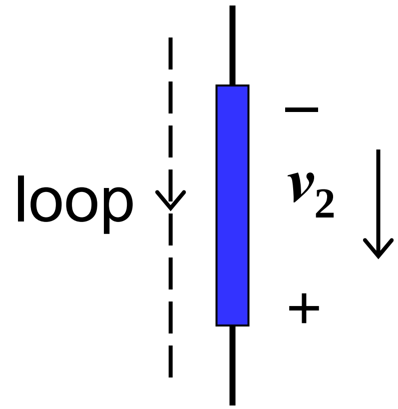


Using Kirchhoff's Voltage Law (KVL)

Consider a branch which forms part of a loop:



Moving from + to -
We add V_1



Moving from - to +
We subtract V_1

- Use **reference polarities** to determine whether a voltage is dropped
- No concern about actual voltage polarities

Formulations of Kirchhoff's Voltage Law

(Conservation of energy)

Formulation 1:

Sum of voltage drops around loop
= sum of voltage rises around loop

Formulation 2:

Algebraic sum of voltage drops around loop = 0

- Voltage rises are included with a minus sign.

(Handy trick: Look at the first sign you encounter on each element when tracing the loop.)

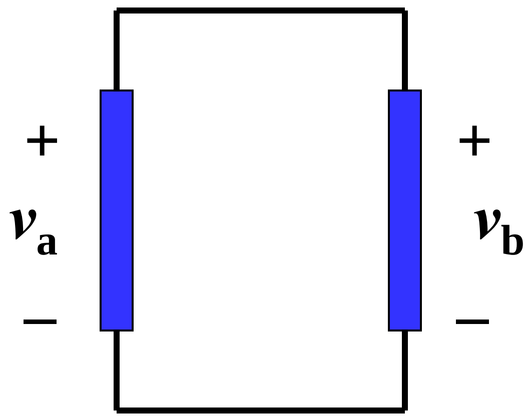
Formulation 3:

Algebraic sum of voltage rises around loop = 0

- Voltage drops are included with a minus sign.

A Major Implication of KVL

- KVL tells us that **any set of elements which are connected at both ends carry the same voltage**.
- We say these elements are connected **in parallel**.

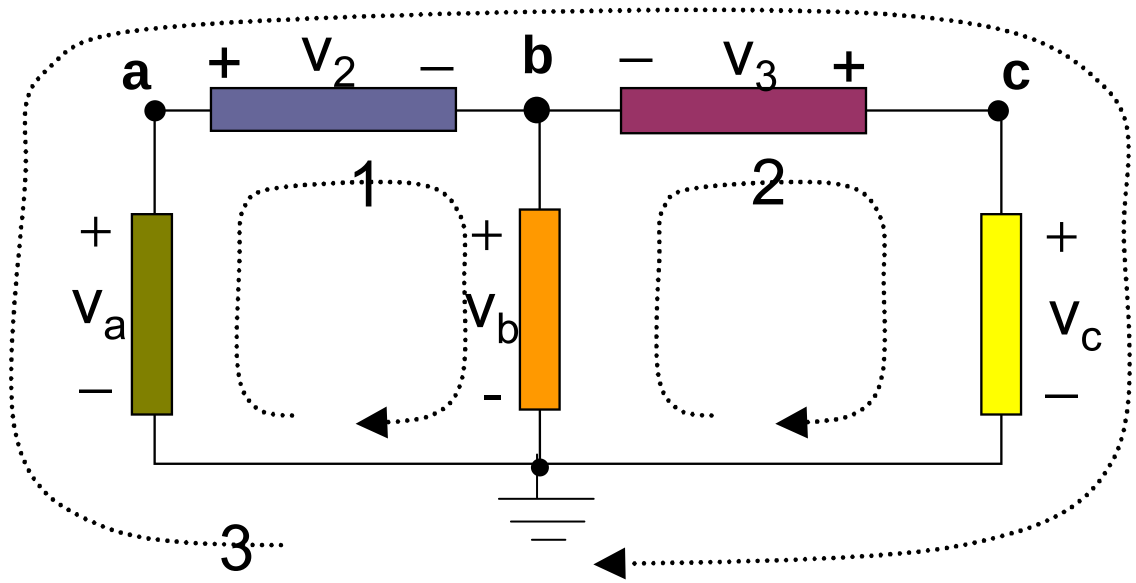


Applying KVL in the clockwise direction, starting at the top:

$$v_b - v_a = 0 \rightarrow v_b = v_a$$

KVL Example

Three closed paths:



<u>Path 1</u>:

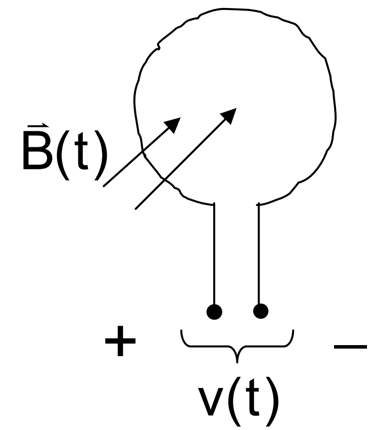
<u>Path 2</u>:

<u>Path 3</u>:

An Underlying Assumption of KVL

- No time-varying magnetic flux through the loop
  Otherwise, there would be an induced voltage (Faraday's Law)

- Note: Antennas are designed to "pick up" electromagnetic waves; "regular circuits" often do so undesirably.



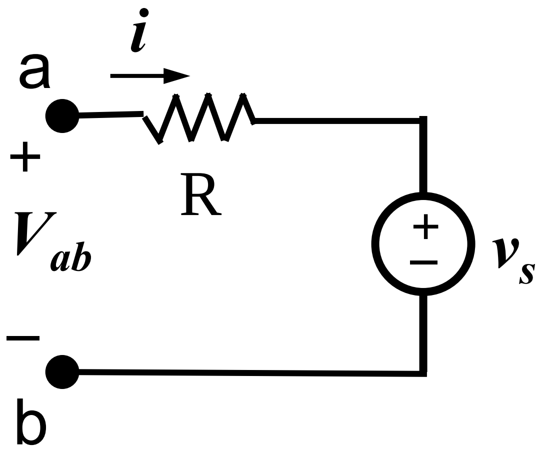
Avoid these loops!

How do we deal with antennas (EECS 117A)?

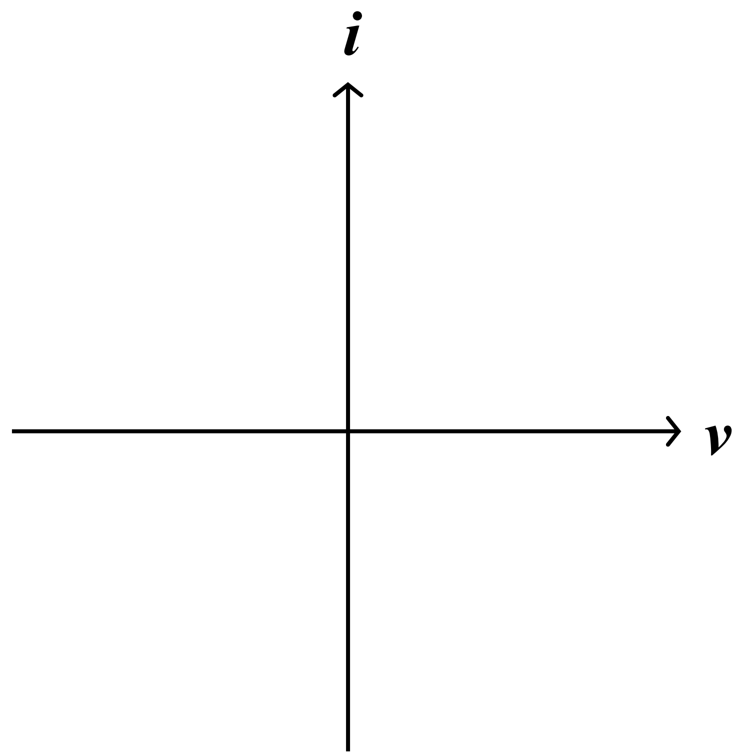
Include a voltage source as the circuit representation of the induced voltage or "noise".

(Use a lumped model rather than a distributed (wave) model.)

I-V Characteristic of Elements



Find the *I-V* characteristic.



Summary

- An electrical system can be modeled by an **electric circuit** (***combination of paths***, each containing 1 or more ***circuit elements***)
  - Lumped model
- The ***Current versus voltage characteristics (I-V plot)*** is a universal means of describing a circuit element.
- ***Kirchhoff's current law (KCL)*** states that the algebraic sum of all currents at any node in a circuit equals zero.
  - Comes from conservation of charge
- ***Kirchhoff's voltage law (KVL)*** states that the algebraic sum of all voltages around any closed path in a circuit equals zero.
  - Comes from conservation of potential energy