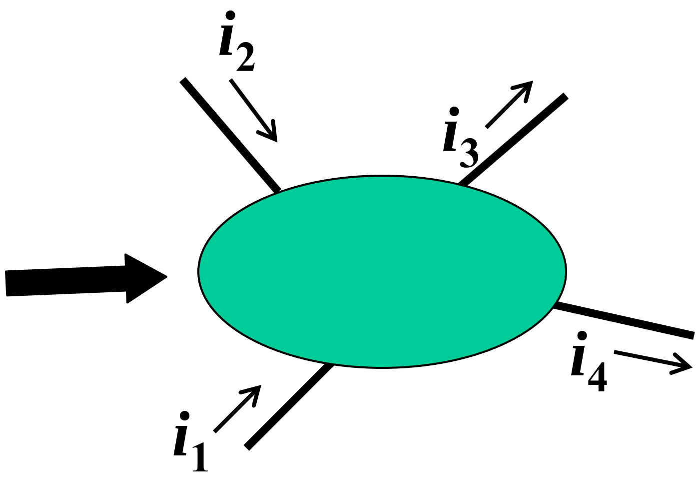
EE100 Su08 Lecture #3 (June 27th 2008)

- Administrivia
  - Videos for lectures 1 and 2 are up (WMV format). Quality is pretty good ☺.
- For today:
  - **Questions?**
  - **Wrap up chapter 2.**
  - Start chapter 3. Refer to pdf slides for week 2.
  - An outline of Labs #1 and #2.

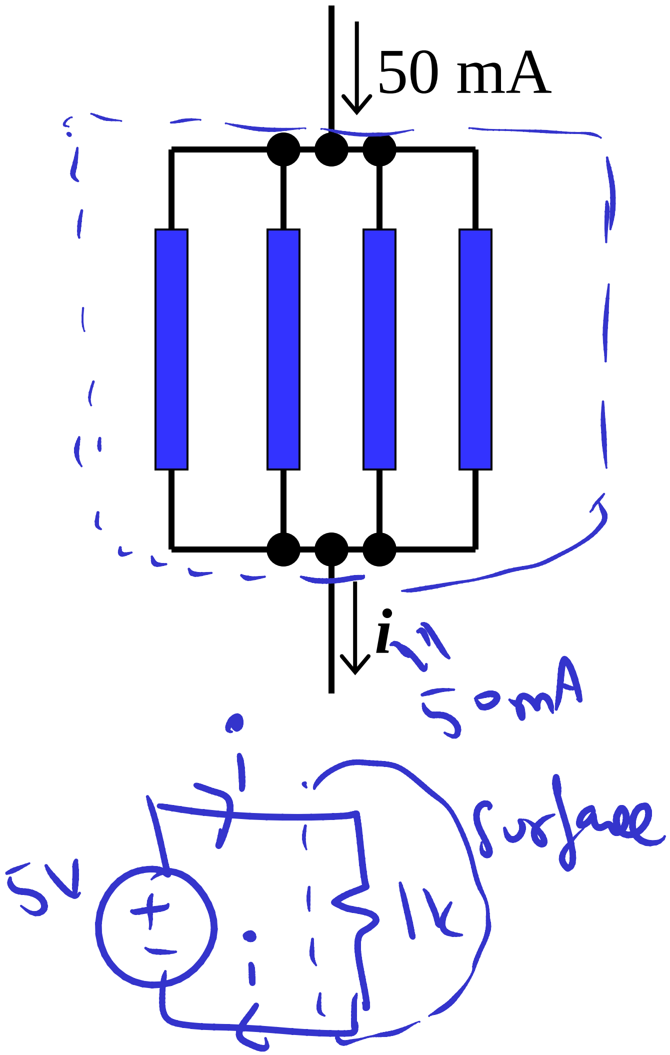
Generalization of KCL

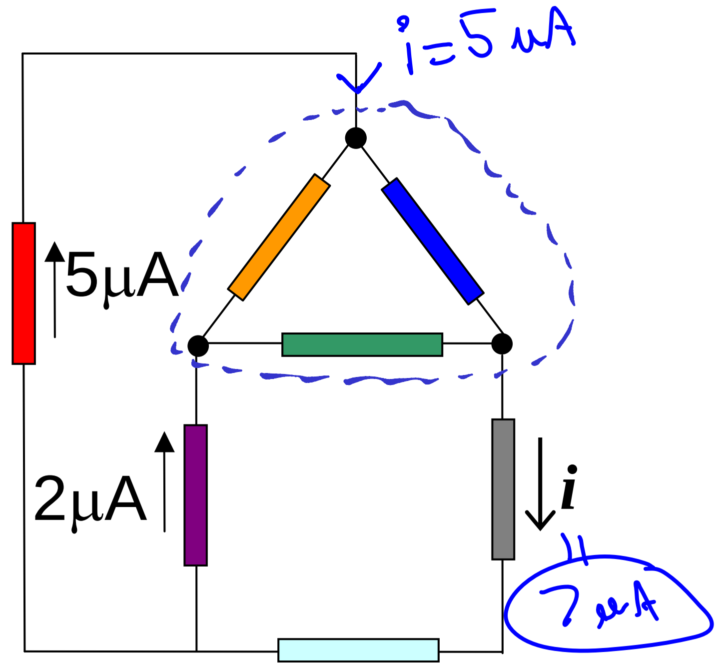
- The sum of currents entering/leaving a **closed surface** is zero. Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.* a "black box"



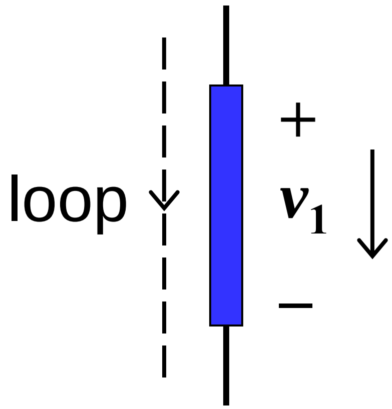
Generalized KCL Examples



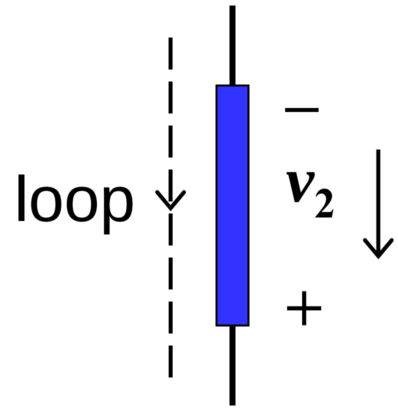


Using Kirchhoff's Voltage Law (KVL)

Consider a branch which forms part of a loop.
One possibility for sign convention:



Moving from + to -
We add V_1

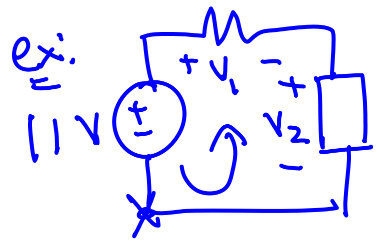


Moving from - to +
We subtract V_2

- Use **reference polarities** to determine whether a voltage is dropped
- No concern about actual voltage polarities

Formulations of Kirchhoff's Voltage Law

(Conservation of energy) ex:



Formulation 1:

Sum of voltage drops around loop

= sum of voltage rises around loop

$11 = V_1 + V_2$

Formulation 2: $-V_2 - V_1 + 11 = 0$

Algebraic sum of voltage drops around loop = 0

- Voltage rises are included with a minus sign.

(Handy trick: Look at the first sign you encounter on each element when tracing the loop.)

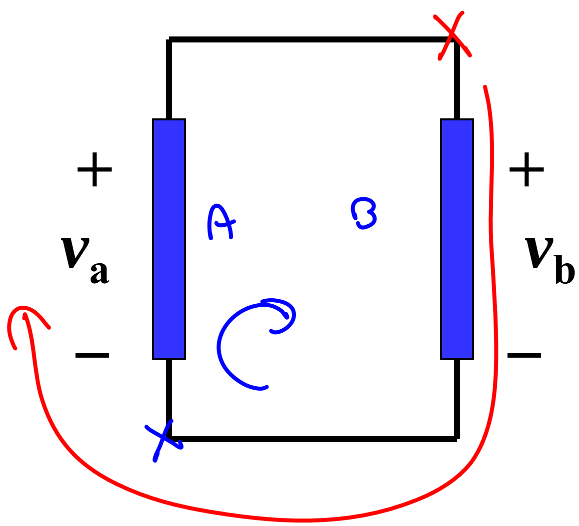
Formulation 3: $V_1 + V_2 - 11 = 0$

Algebraic sum of voltage rises around loop = 0

- Voltage drops are included with a minus sign.

A Major Implication of KVL

- KVL tells us that **any set of elements which are connected at both ends carry the same voltage**.
- We say these elements are connected **in parallel**.



Element A & B are in $||^{lel}$ (parallel)

because $v_a = v_b$
$[V_a - V_b = 0]$

Applying KVL in the clockwise direction, starting at the top:

$+V_b - V_a = 0$

$$v_b - v_a = 0 \;\; \rightarrow \;\; v_b = v_a$$

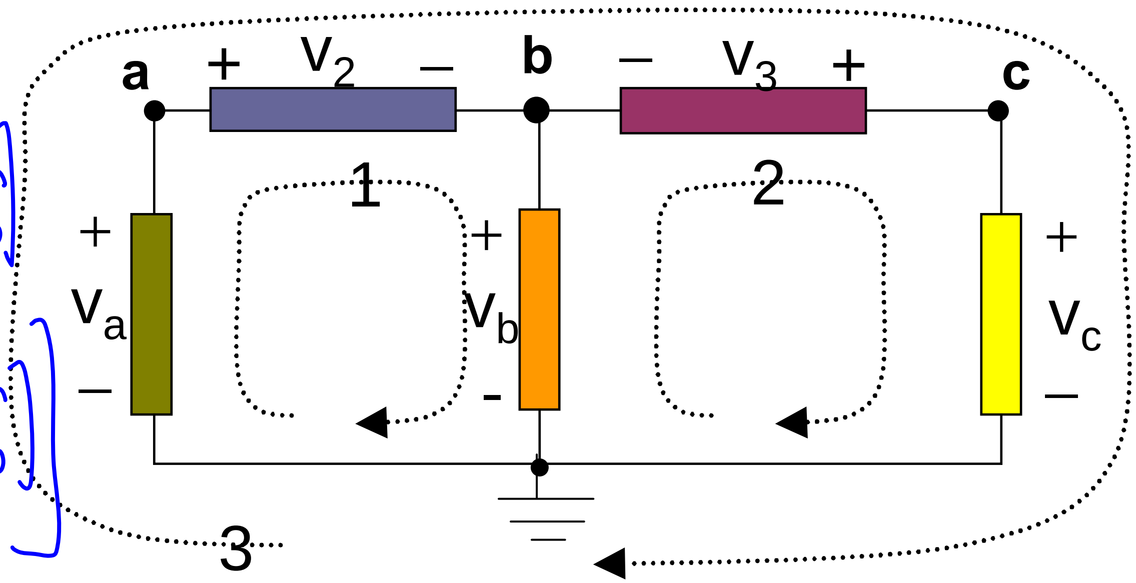
KVL Example

Three closed paths:

Note: I use Physics:

$+V_a$ [rise in energy]

$-V_d$ [drop in energy]



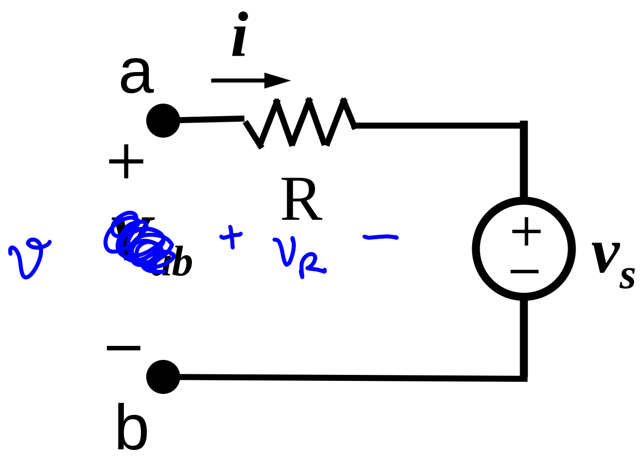
Path 1: $V_a - V_2 - V_b = 0\ V$

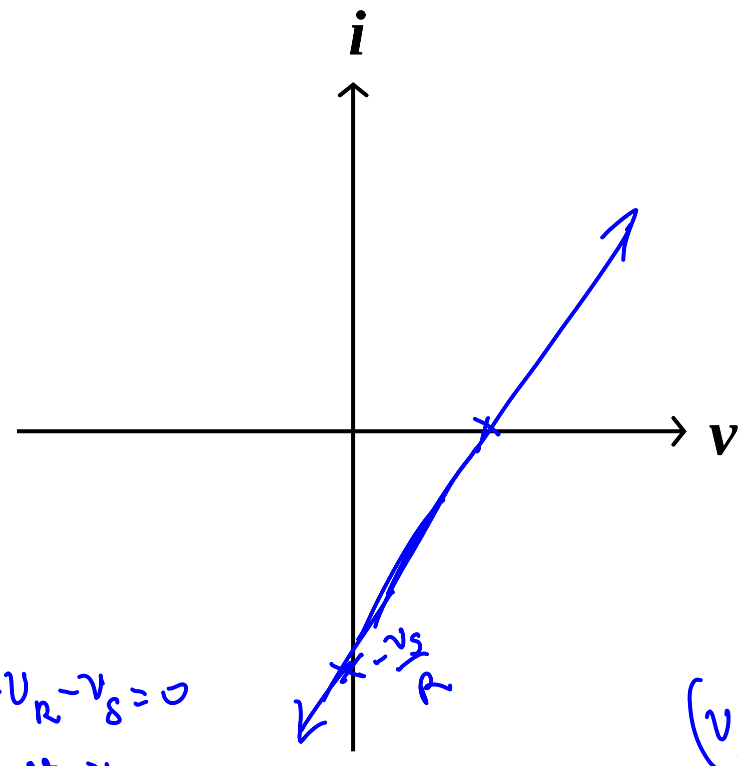
Path 2: $V_b + V_3 - V_c = 0\ V$

Path 3: $V_a - V_2 + V_3 - V_c = 0\ V$

I-V Characteristic of Elements

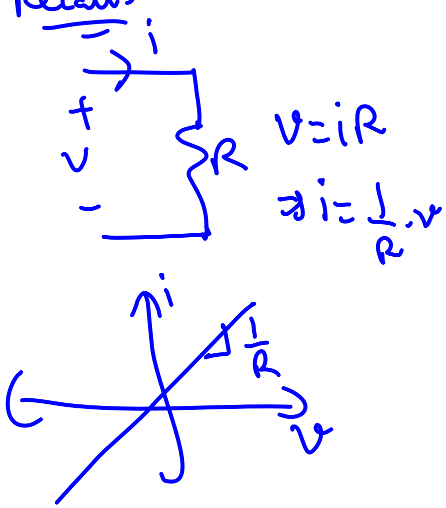
Find the *I-V* characteristic.





$(v_s > 0)$

Recall:



$v = iR$

$\Rightarrow i = \frac{1}{R} \cdot v$

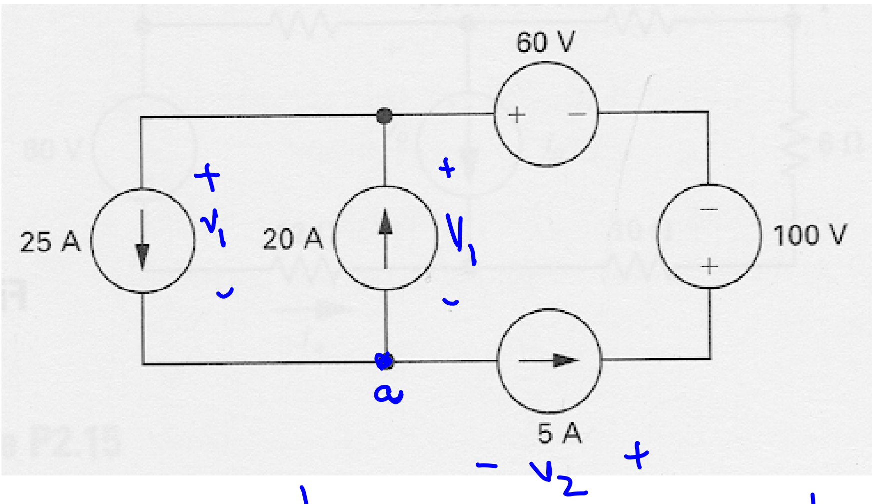
KVL: $v - v_R - v_s = 0$

$\Rightarrow v_R = v - v_s$

$\therefore$ Ohm's law: $i = \frac{v_R}{R} = \frac{v - v_s}{R} \Rightarrow \boxed{i = \frac{1}{R} v - \frac{v_s}{R}}$

More Examples

- Are these interconnections permissible?



KCL @ "a": $25 = 20 + 5$ ✓

KVL: $V_2 - 100 + 60 - V_1 = 0$

$\Rightarrow V_2 - V_1 = 40$

Power: 25A: $p_1 = 25v_1$ 5A: $p_3 = -5v_2$

20A: $p_2 = -20v_1$ 100V: $p_4 = +500\,W$

60V: $p_5 = -300\,W$

Power: $\sum_i p_i = 0$

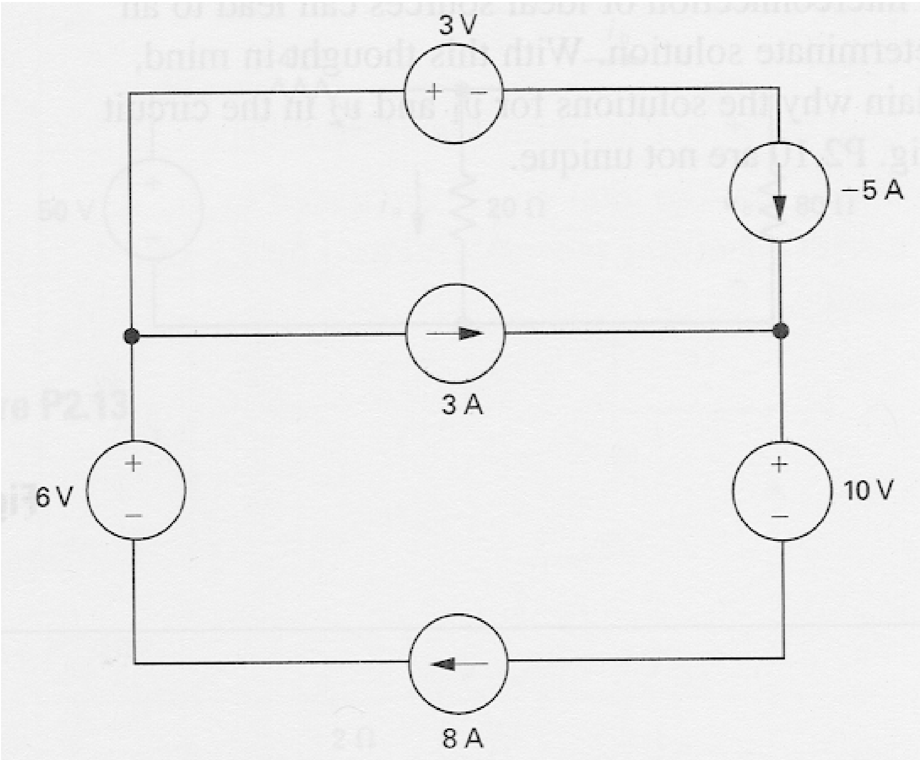
$\Rightarrow p_1 + p_2 + p_3 + p_4 + p_5 = 0$

$\Rightarrow 25v_1 - 20v_1 - 5v_2 + 500 - 300 = 0$

$\Rightarrow 5v_1 - 5v_2 + 200 = 0$

$\Rightarrow 5v_2 - 5v_1 = 200$

$\Rightarrow v_2 - v_1 = 40$



← Please try yourself, post on bspace if you have questions.

Summary

- An electrical system can be modeled by an **electric circuit** (***combination of paths***, each containing 1 or more ***circuit elements***)
  - Lumped model
- The ***Current versus voltage characteristics (I-V plot)*** is a universal means of describing a circuit element.
- ***Kirchhoff's current law (KCL)*** states that the algebraic sum of all currents at any node in a circuit equals zero.
  - Comes from conservation of charge
- ***Kirchhoff's voltage law (KVL)*** states that the algebraic sum of all voltages around any closed path in a circuit equals zero.
  - Comes from conservation of potential energy

Chapters 3 and 4

- Outline
  - Resistors in Series – Voltage Divider
  - Conductances in Parallel – Current Divider

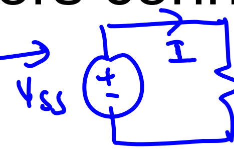
Chap. 3

  - Node-Voltage Analysis
  - Mesh-Current Analysis
  - Superposition
  - Thévenin equivalent circuits
  - Norton equivalent circuits
  - Maximum Power Transfer

Chap. 4

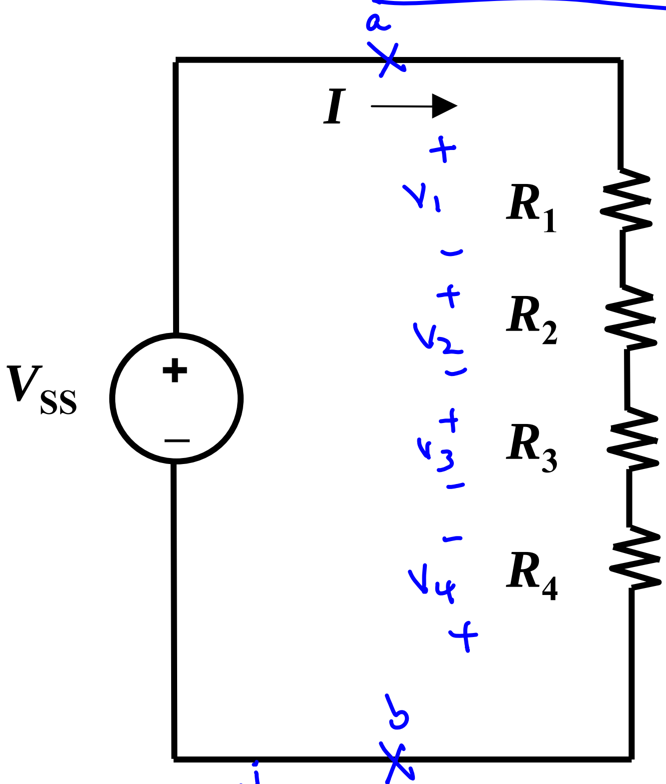
Resistors in **Series**

Consider a circuit with multiple resistors connected in series. Find their "equivalent resistance".



$\Rightarrow V_{SS} = I R_{eq}$ — (1)

Find R_{eq}



- KCL tells us that the same current (***I***) flows through every resistor
- KVL tells us:

$+V_{SS} - V_1 - V_2 - V_3 + V_4 = 0$

Ohm's law:

$V_1 = IR_1$

$V_2 = IR_2$

$V_3 = IR_3$

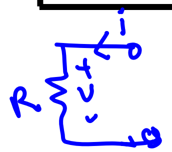
Recall: $\Leftarrow$ $V_4 = -IR_4$

$\therefore V_{SS} = V_1 + V_2 + V_3 - V_4$ (from KVL)

$= IR_1 + IR_2 + IR_3 - (-IR_4)$

$V_{SS} = I(R_1 + R_2 + R_3 + R_4)$ — (2)

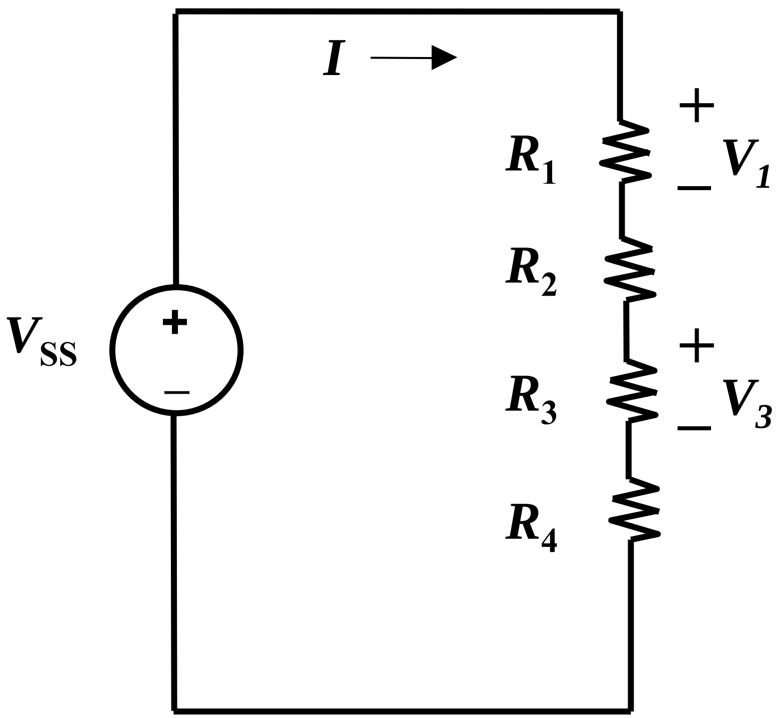
(1) & (2) $\Rightarrow$ $\boxed{R_{eq} = \sum_{j=1}^{4} R_j}$



$\Leftrightarrow$ $v = iR$: Ohm's law

Equivalent resistance of resistors in series is the sum

Voltage Divider



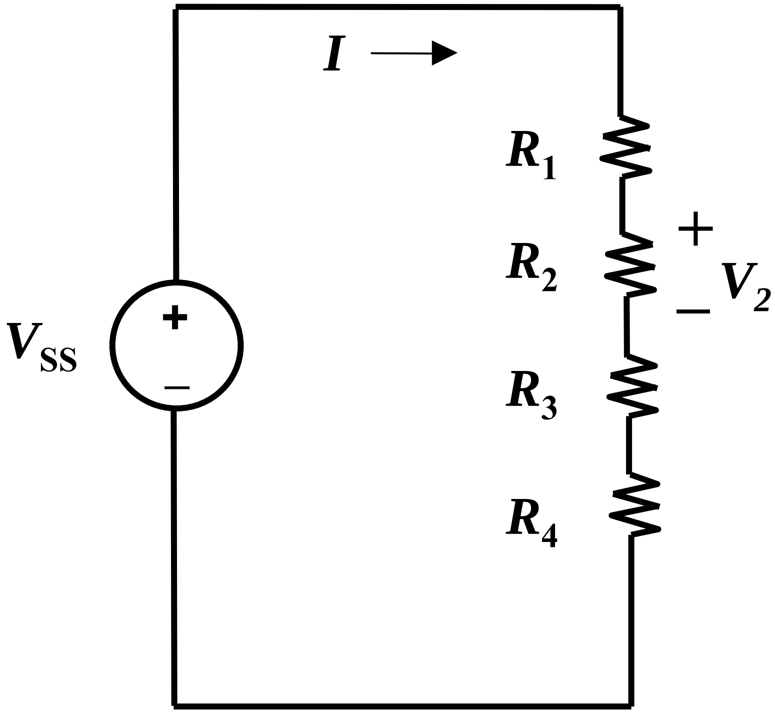
$$I = V_{SS} / (R_1 + R_2 + R_3 + R_4)$$

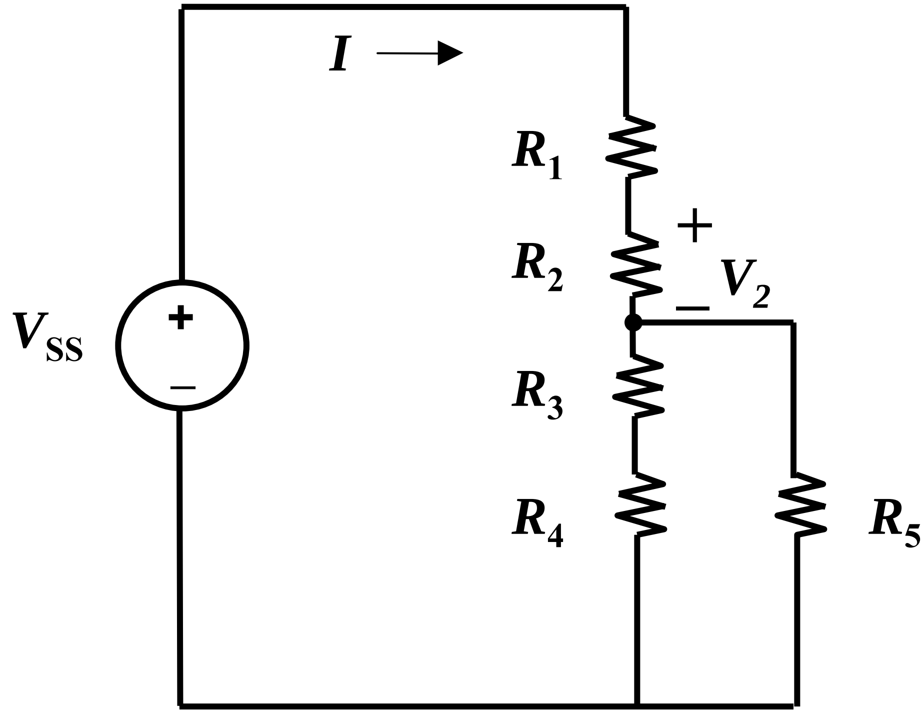
(Q.) Find V_1 & V_2 in terms of V_{ss}, R_1, R_2, R_3 and R_4.

$$V_1 = I R_1$$

$$\Rightarrow \boxed{V_1 = V_{ss} \cdot \frac{R_1}{\sum_{j=1}^{4} R_j}}$$

When can the Voltage Divider Formula be Used?





$$V_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

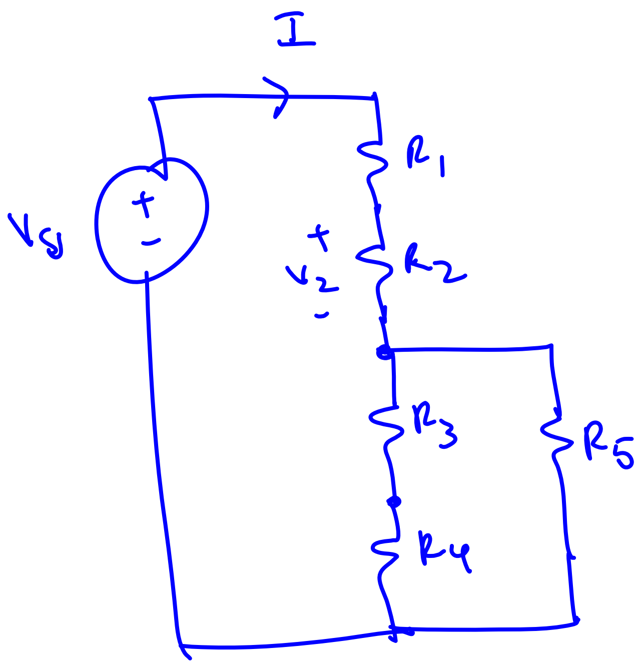
Correct, if nothing else is connected to nodes

$$V_2 \neq \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

Why? What is V_2? $V_2 = R_2 \cdot I$

Because $R_1 + R_2$ not in series with R_3 & R_4

CAUTION: Voltage Divider Formula must be used with care.

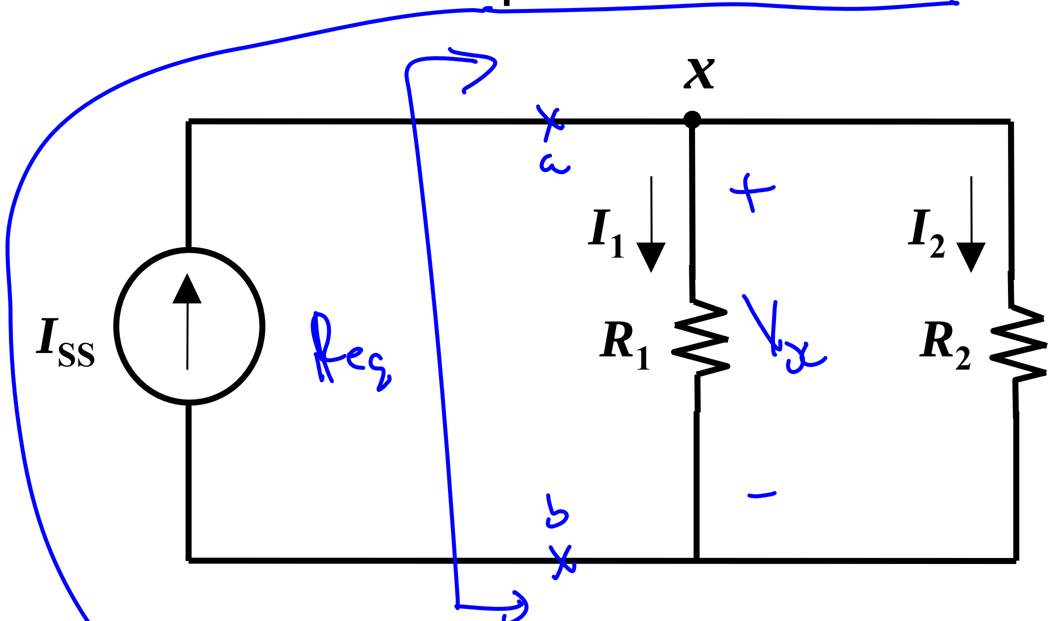


$$V_2 = I R_2$$

$$= \left\{ \frac{V_{SS}}{R_1 + R_2 + \left\{ (R_3 + R_4) \,\|\, R_5 \right\}} \right\} R_2$$

Resistors in Parallel

Consider a circuit with two resistors connected in parallel. Find their "equivalent resistance".

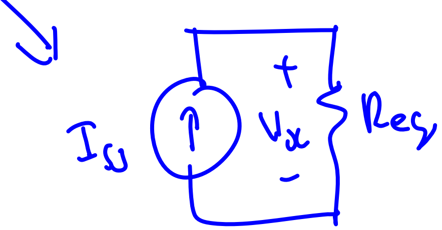


- KVL tells us that the same voltage is dropped across each resistor

$$V_x = I_1 R_1 = I_2 R_2$$

- KCL tells us:

$$I_{SS} = I_1 + I_2$$



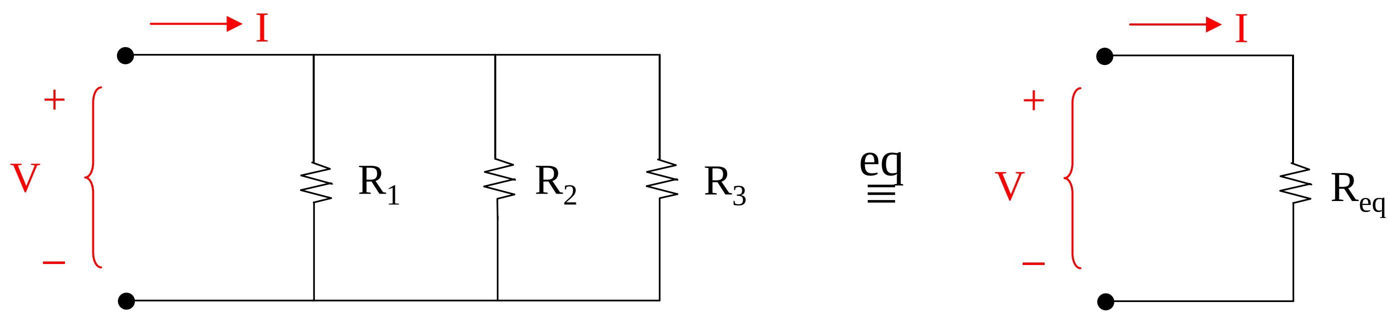
$V_x = I_{SS} R_{eq}$

$$\Rightarrow \frac{V_x}{R_{eq}} = \frac{V_x}{R_1} + \frac{V_x}{R_2}$$

$$\Rightarrow \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

General Formula for Parallel Resistors

What single resistance R_{eq} is equivalent to three resistors in parallel?



In general:

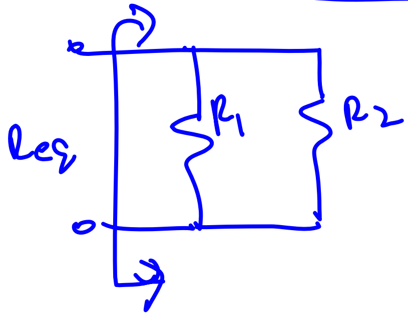
$$\boxed{\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

Easy to derive yourself.

Equivalent <u>conductance</u> of resistors in parallel is the sum

Some important observations about $\|^{lel}$ resistors

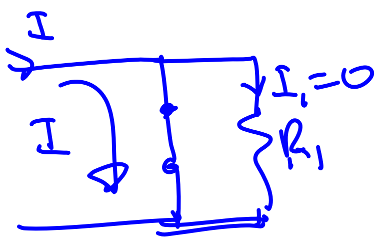
(1) Two resistors in $\|^{lel}$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

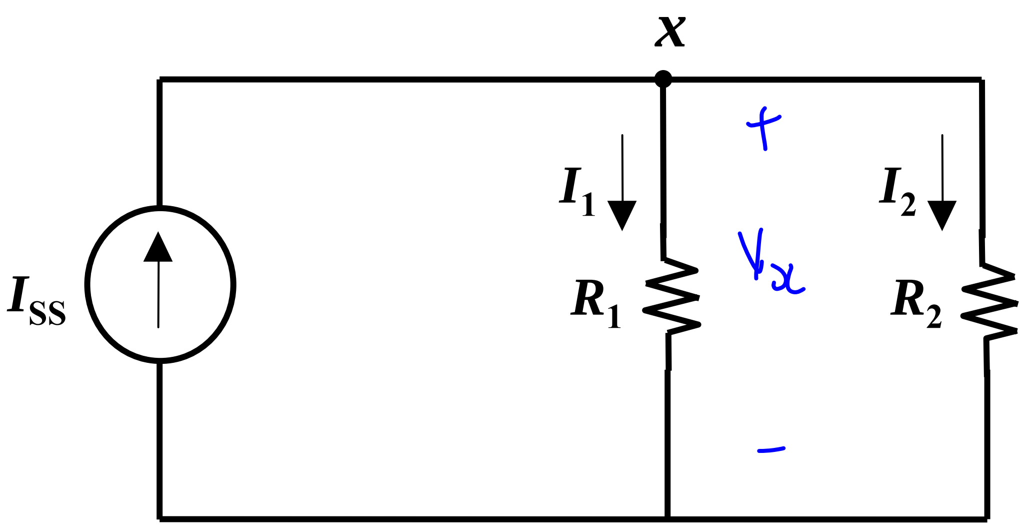
$$\Rightarrow \frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

As $R_1 \to 0$, $R_{eq} = \dfrac{0 \cdot R_2}{0 + R_2} = 0$



Short circuits "short-out" resistors in $\|^{lel}$

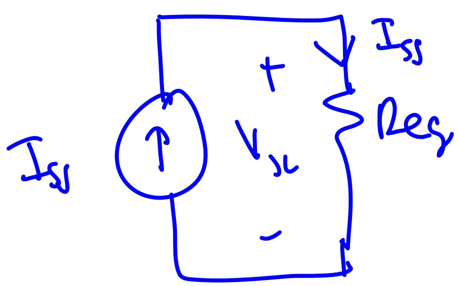
Current Divider



(Q:) Find I_1 & I_2 in terms of I_{SS}, R_1 & R_2?

$$V_x = I_1 R_1 = I_{SS} R_{eq}$$

But, $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$



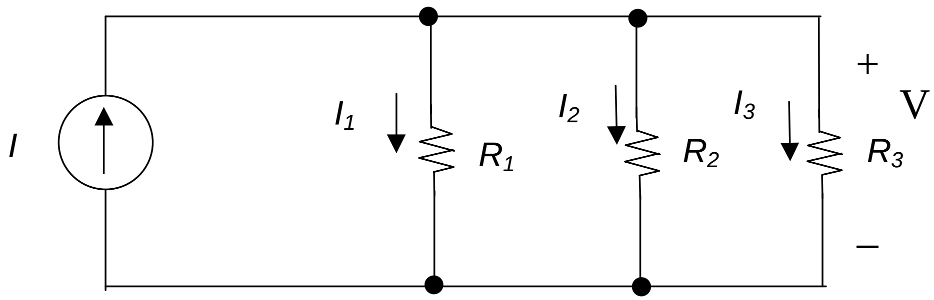
∴ $I_1 R_1 = I_{SS} \cdot \frac{R_1 R_2}{R_1 + R_2}$

$\Rightarrow \boxed{I_1 = \frac{R_2}{R_1 + R_2} \cdot I_{SS}}$

$\boxed{I_2 = \frac{R_1}{R_1 + R_2} I_{SS}}$

Generalized Current Divider Formula

Consider a current divider circuit with >2 resistors in parallel:



$$V = \frac{I}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right)}$$

Note: $V = I \cdot R_{eq}$

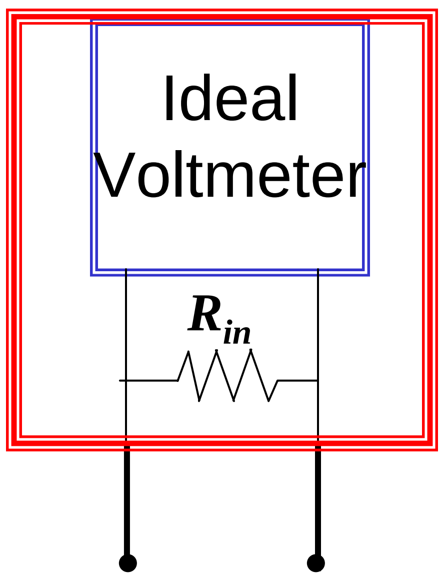
But, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$$I_3 = \frac{V}{R_3} = I\left[\frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3}\right]$$

Measuring Voltage

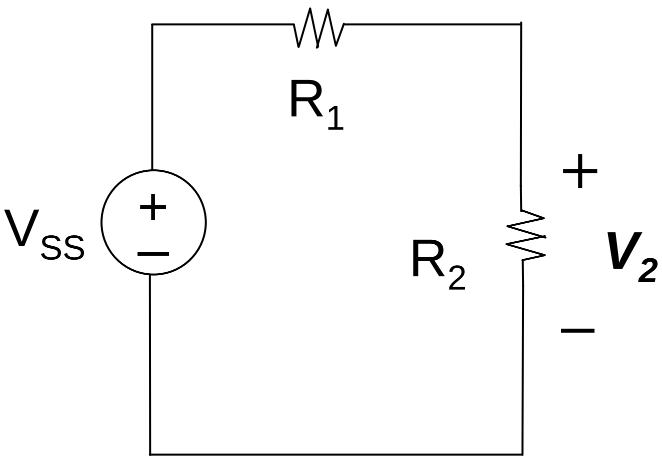
To measure the voltage drop across an element in a real circuit, insert a voltmeter (digital multimeter in voltage mode) **in parallel** with the element.

Voltmeters are characterized by their "voltmeter input resistance" (R_{in}). Ideally, this should be very high (typical value 10 MΩ)

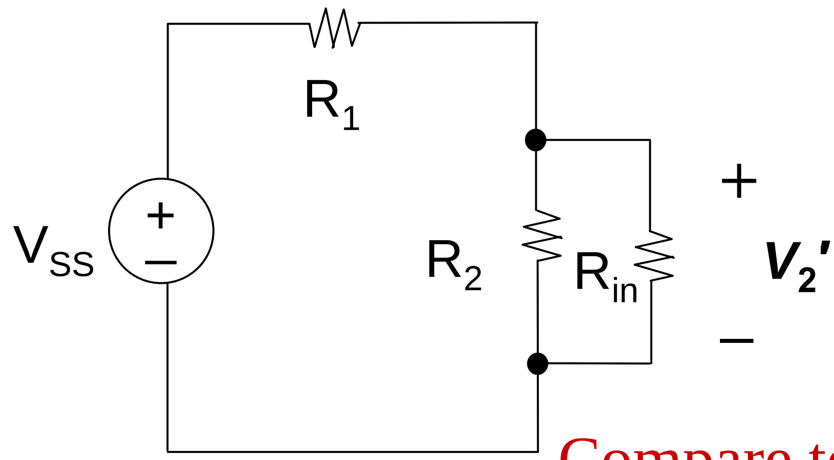


Effect of Voltmeter

undisturbed circuit



circuit with voltmeter inserted



Compare to R_2

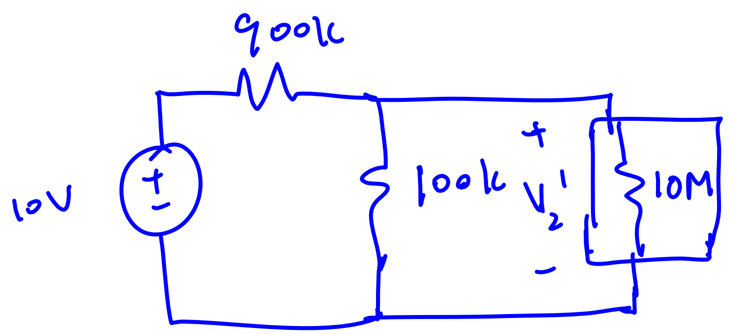
$$V_2 = V_{SS}\left[\frac{R_2}{R_1 + R_2}\right]$$

$$V_2' = V_{SS}\left[\frac{R_2 \parallel R_{in}}{R_2 \parallel R_{in} + R_1}\right]$$

Example: $V_{SS} = 10\,V, R_2 = 100\,K, R_1 = 900\,K \Rightarrow V_2 = 1V$

$$R_{in} = 10M,\ V_2' = ?$$

Effect of Voltmeter



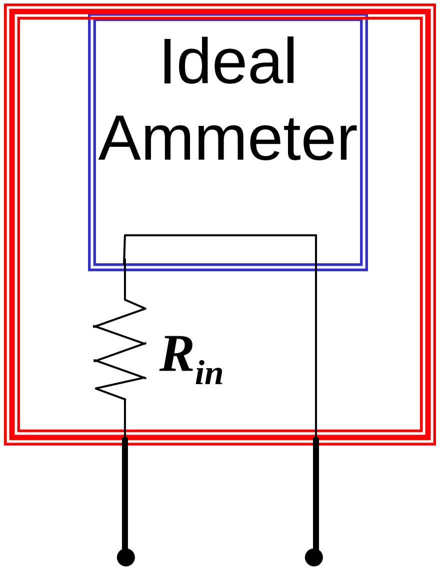
Step (1): $10M \| 100k = \dfrac{10M \cdot 100k}{10M + 100k} = \dfrac{1000 \times 10^6 \times 10^3}{10M + 0.1M} \approx 100k$

Moral: Internal Resistance of a voltmeter is <u>really</u> big.

Measuring Current

To measure the current flowing through an element in a real circuit, insert an ammeter (digital multimeter in current mode) **in series** with the element.

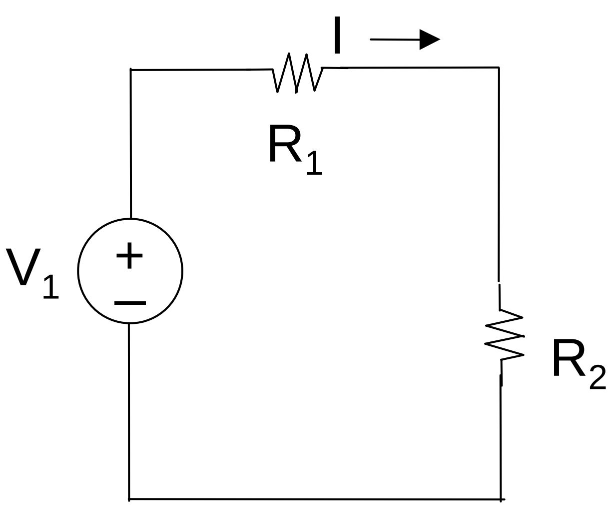
Ammeters are characterized by their "ammeter input resistance" (R_{in}). Ideally, this should be very low (typical value 1Ω).



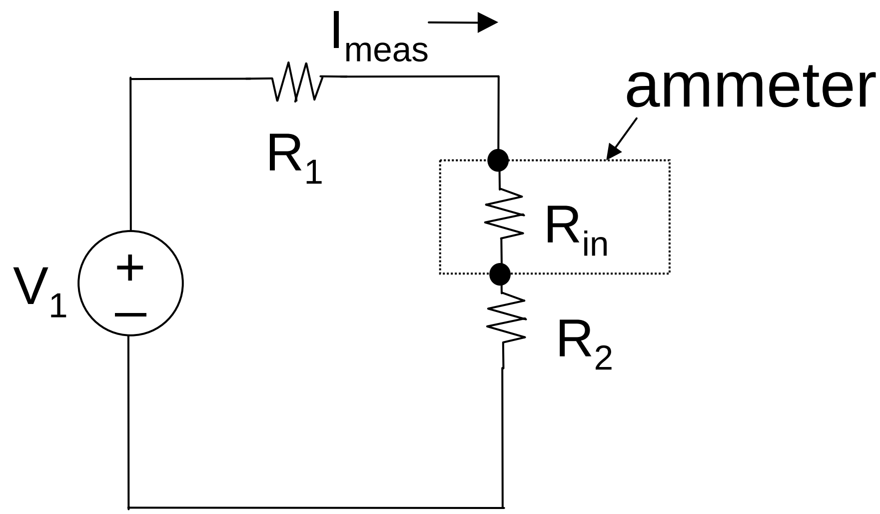
Effect of Ammeter

Measurement error due to non-zero input resistance:

undisturbed circuit



circuit with ammeter inserted



$$I = \frac{V_1}{R_1 + R_2}$$

$$I_{meas} = \frac{V_1}{R_1 + R_2 + R_{in}}$$

Example: $V_1 = 1$ V, $R_1 = R_2 = 500\ \Omega$, $R_{in} = 1\Omega$

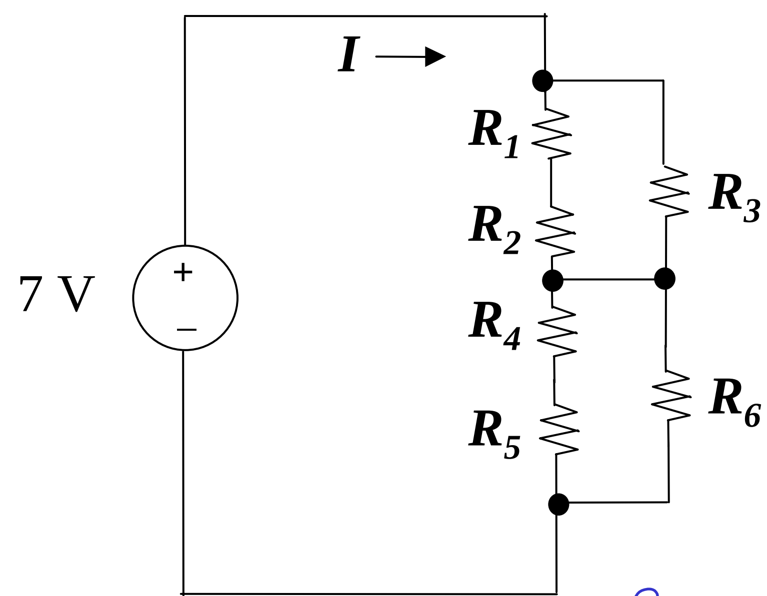
$$I = \frac{1V}{500\Omega + 500\Omega} = 1mA, \quad I_{meas} = ?$$

Compare to R_2

Using Equivalent Resistances

Simplify a circuit before applying KCL and/or KVL:

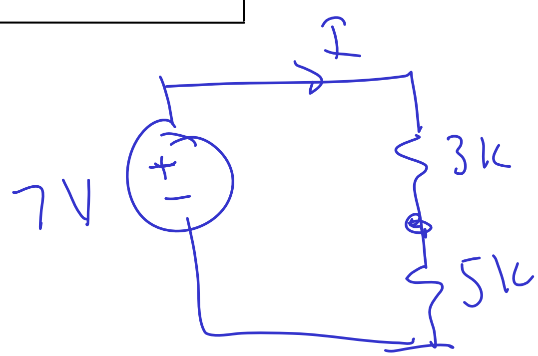
<u>Example</u>: Find ***I***



$R_1 = R_2 = 3\ \text{k}\Omega$
$R_3 = 6\ \text{k}\Omega$

$R_4 = R_5 = 5\ \text{k}\Omega$
$R_6 = 10\ \text{k}\Omega$

<u>Simplify Circuit:</u>



$\boxed{I = \frac{7}{8}\ \text{mA}}$

Wheatstone's Bridge (Section 3.6)

(1) Read it → You will need it for lab #7.

(2) Skip 3.7 (No Δ-Y)

Labs #1 and #2

- COME ON TIME FOR THE LABS!
- UNDERSTAND how to use the breadboard!
- You need to get familiar with the instruments: feel free to use TA office hours for extra help.
- You will be given a kit next week with all components for the lab. Thus you could “pre-wire” your circuit before coming to lab!
- Lab #1: Instruments
- Lab #2: Circuits. Lab #2 depends on chapter 4, especially the Thevenin equivalents. I will cover Thevenin equivalents by July 2nd (Wednesday) lecture, but please READ chapter 4 this weekend!