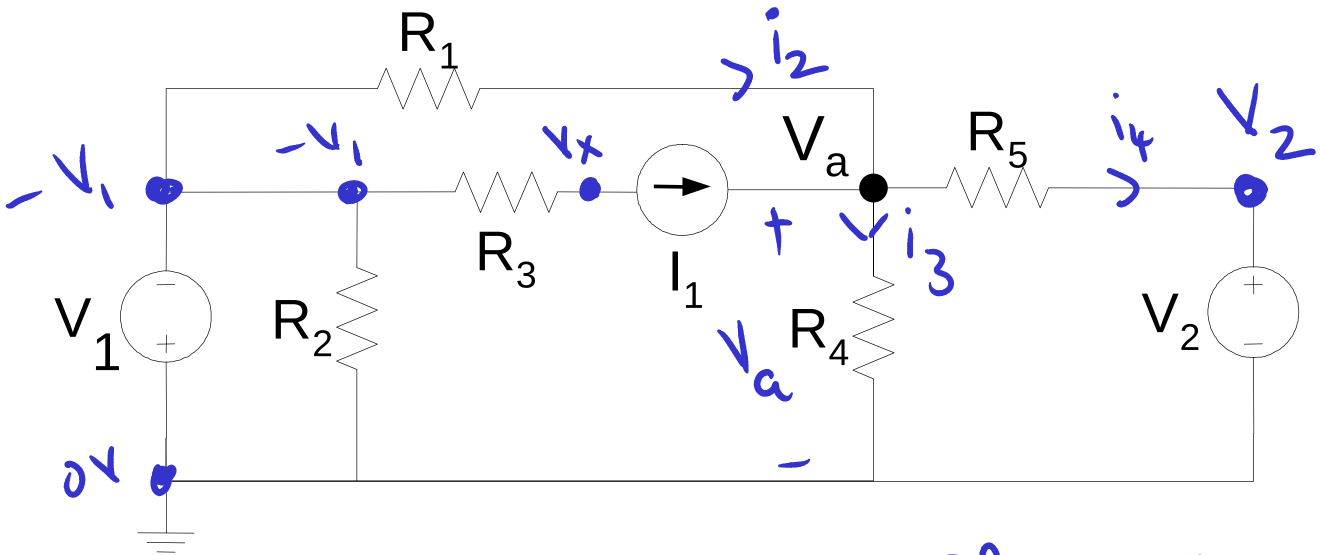
EE100Su08 Lecture #5 (July 2nd 2008)

- Outline
  - **Questions?**
  - **Lab notes:**
    - **Labs 2 and 3 have been shortened**
    - **Monday lab: go to your SECOND lab section next week.**
  - Node-Voltage Analysis: wrap up
  - Mesh analysis: read it, OPTIONAL
  - Superposition
  - Thevenin's Theorem

Nodal Analysis: Example #2



If you are systematic, not challenging ☺.

Challenges:

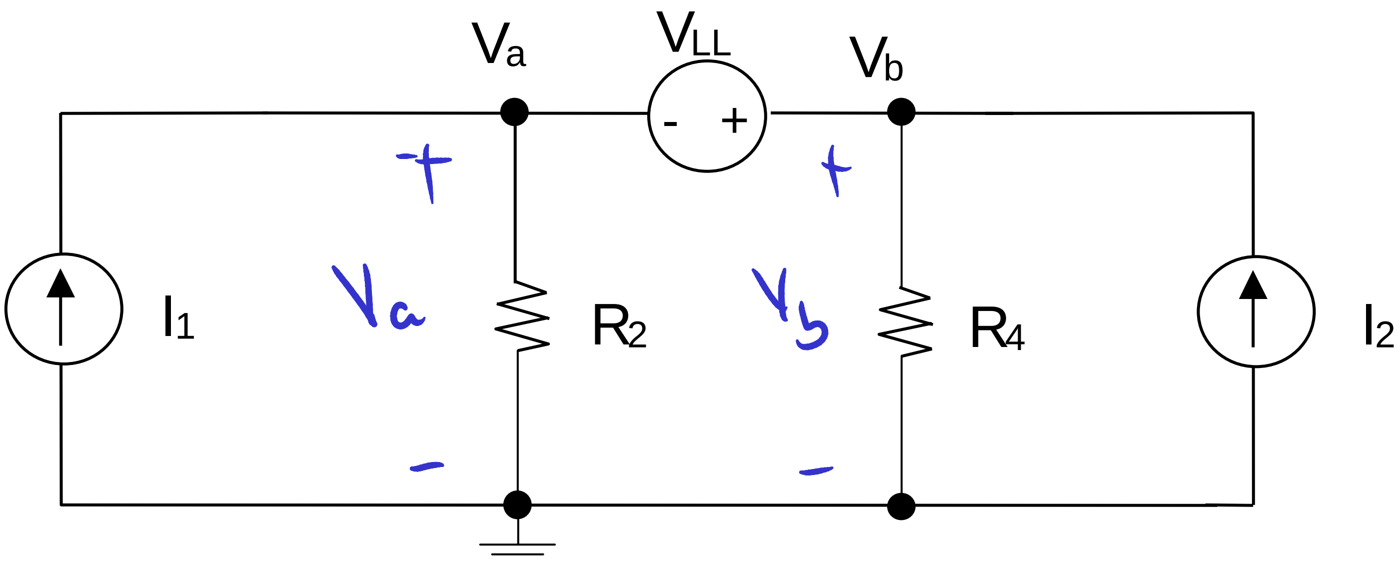
Determine number of nodes needed

Deal with different types of sources

@ V_a: $I_1 + i_2 = i_3 + i_4 \Rightarrow I_1 + \frac{-V_1 - V_a}{R_1} = \frac{V_a}{R_4} + \frac{V_a - V_2}{R_5}$

Nodal Analysis w/ "Floating Voltage Source"

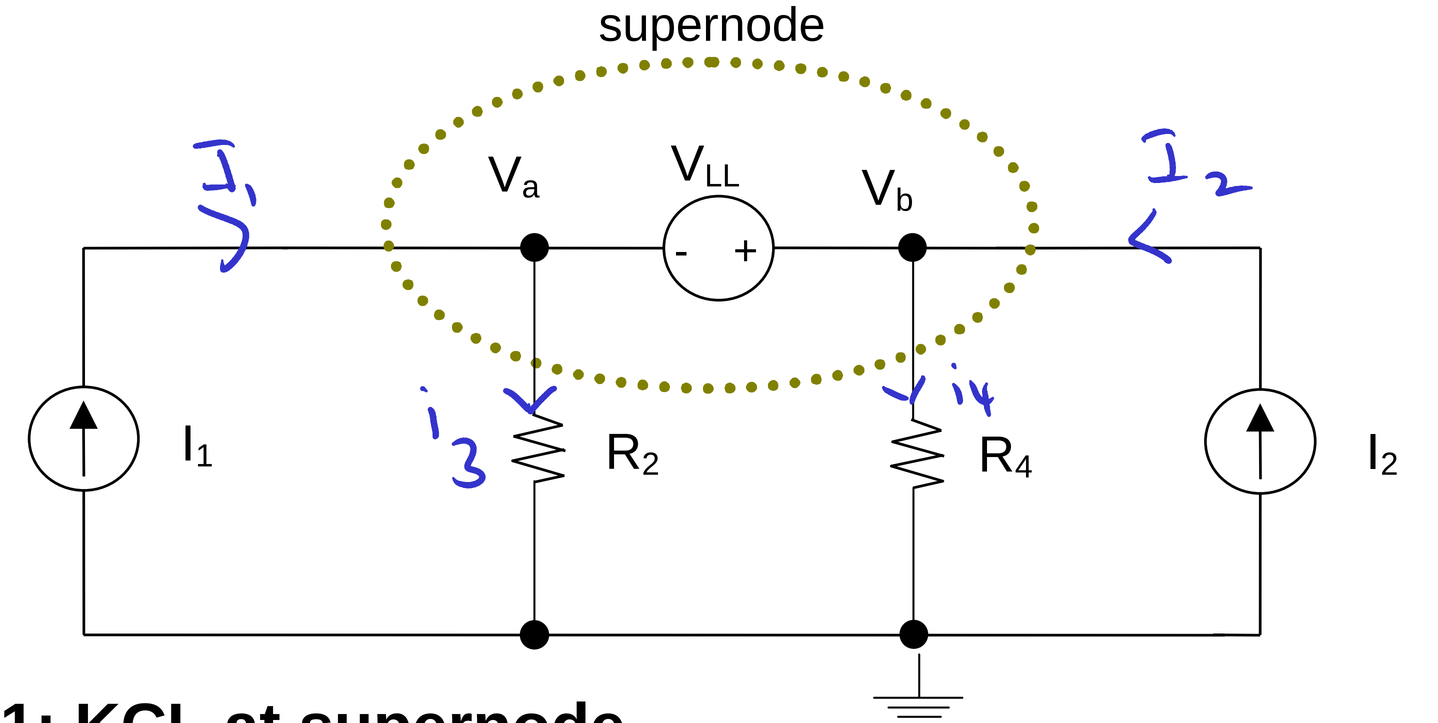
A "floating" voltage source is one for which neither side is connected to the reference node, e.g. V_{LL} in the circuit below:



Problem: We cannot write KCL at nodes a or b because there is no way to express the current through the voltage source in terms of V_a-V_b.

Solution: Define a "supernode" – that chunk of the circuit containing nodes a and b. Express KCL for this supernode. Incorporate voltage source constraint into KCL equation.

Nodal Analysis: Example #3



<u>Eq'n 1</u>: KCL at supernode

$$I_1 + I_2 = i_3 + i_4 \Rightarrow \boxed{I_1 + I_2 = \frac{V_a}{R_2} + \frac{V_b}{R_4}}$$

Substitute property of voltage source: $\boxed{V_b - V_a = V_{LL}}$

Superposition

A ***linear circuit*** is one constructed only of linear elements (linear resistors, **and linear capacitors and inductors,** linear dependent sources) and independent sources. Linear

means I-V charcteristic of elements/sources are straight lines when plotted

Principle of Superposition:

- In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone.

Superposition

Procedure:

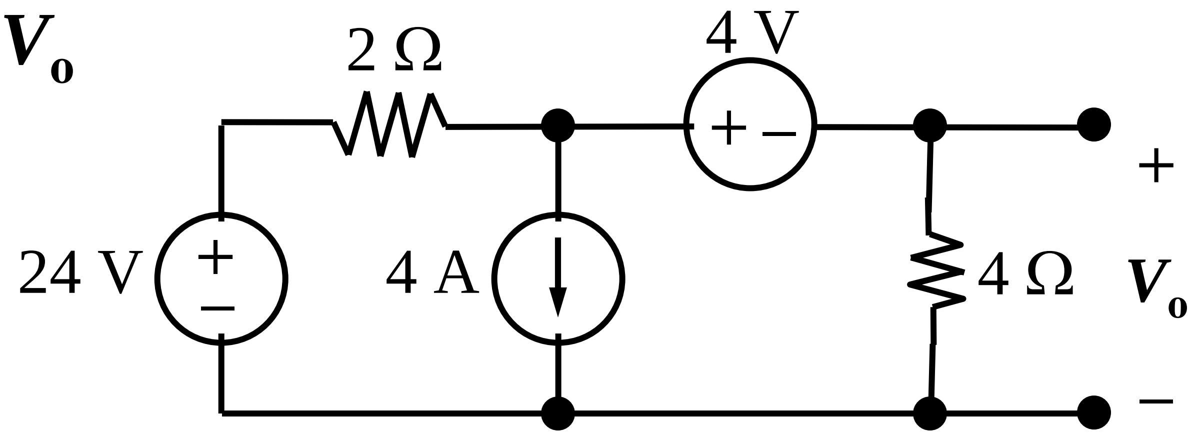
1. Determine contribution due to one independent source
   - Set all other sources to 0: Replace independent voltage source by short circuit, independent current source by open circuit
2. Repeat for each independent source
3. Sum individual contributions to obtain desired voltage or current

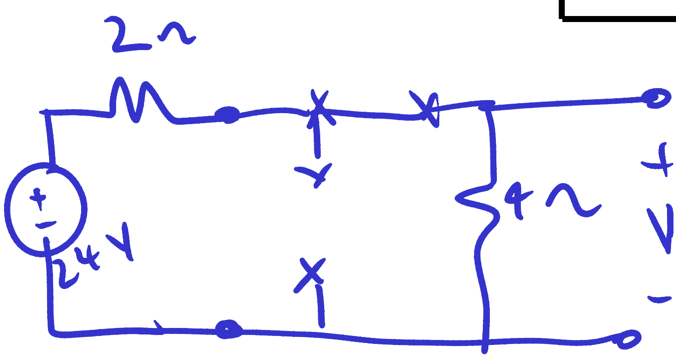
Open Circuit and Short Circuit

- Open circuit → i=0 ; Cut off the branch
- Short circuit → v=0 ; replace the element by wire
- Turn off an independent voltage source means
  - V=0
  - Replace by wire
  - Short circuit
- Turn off an independent current source means
  - i=0
  - Cut off the branch
  - open circuit

Superposition Example

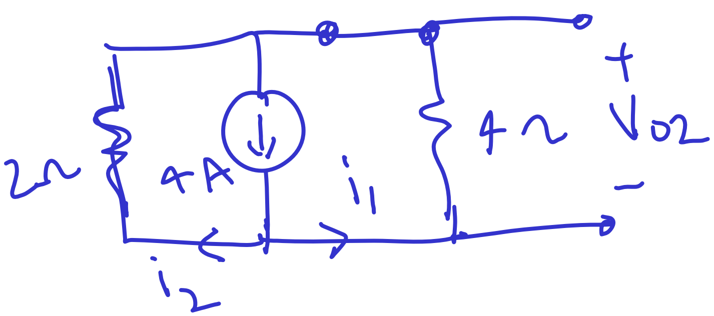
- Find V_o





$$V_{01} = \left(\frac{4}{4+2}\right)24 = \frac{4}{6}\cdot 24 = \underline{\underline{16V}}$$

$$\Rightarrow \boxed{V_{01} = 16V}$$

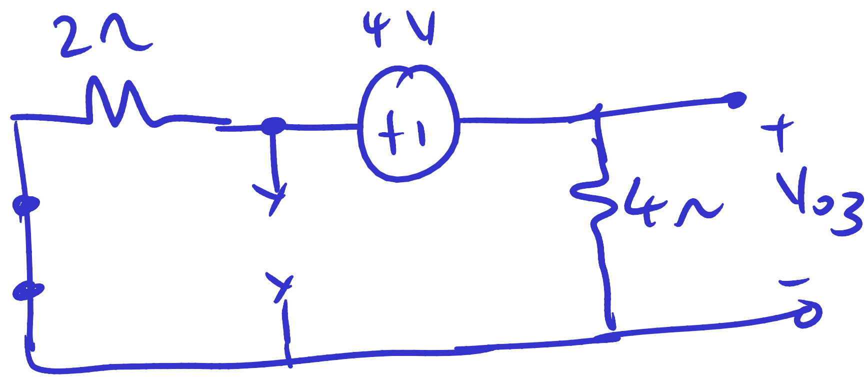


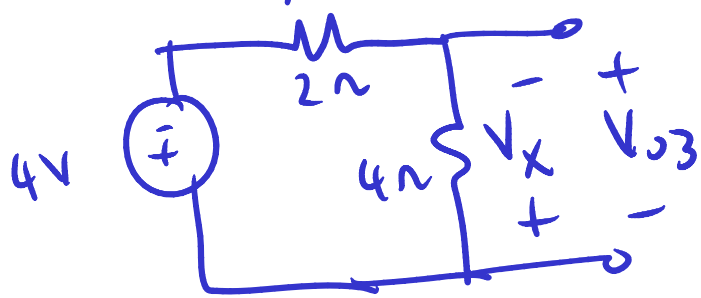
$$V_{02} = -i_1 \cdot 4 \qquad i_1 = \frac{2}{4+2}\cdot 4 = 8/6$$

$$= \underline{\underline{4/3}}\ A$$

$$\boxed{V_{02} = -16/3\ V}$$

Superposition example (contd.)





$$V_x = \left(\frac{4}{4+2}\right)4$$

[Voltage Divider]

$$V_{o3} = -V_x = -\frac{4}{6}\cdot 4 = \frac{-16}{6}\,V$$

$$= -8/3\,V$$

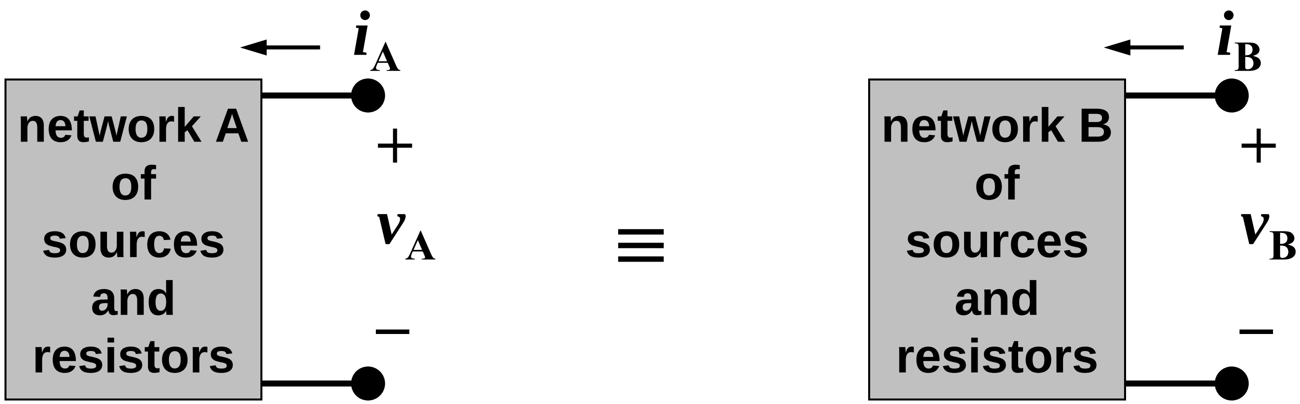
Superposition example (contd.)

$$V_o = V_{o1} + V_{o2} + V_{o3}$$

$$= 16 - \frac{16}{3} - 8/3 = 16 - \frac{24}{3} = \underline{\underline{8\ V}}$$

Equivalent Circuit Concept

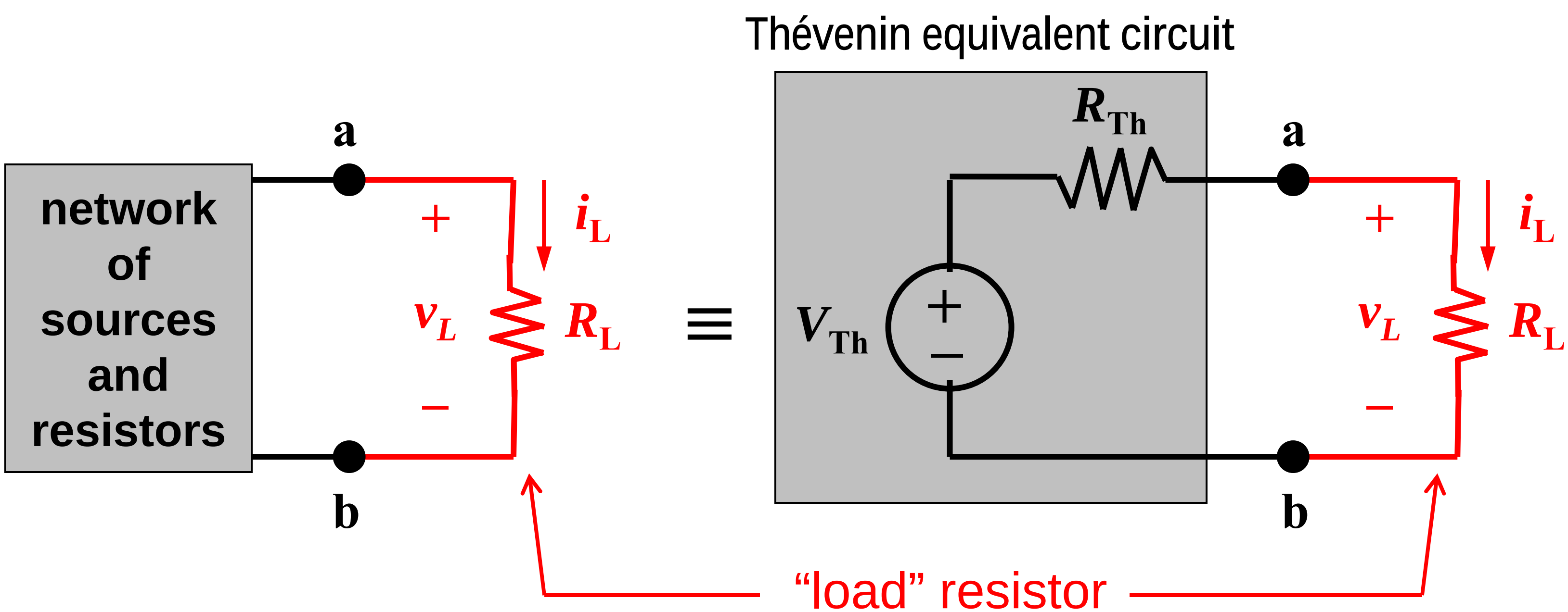
- A network of voltage sources, current sources, and resistors can be replaced by an ***equivalent circuit*** which has identical terminal properties (*I-V* characteristics) without affecting the operation of the rest of the circuit.



$$i_A(v_A) = i_B(v_B)$$

Thévenin Equivalent Circuit

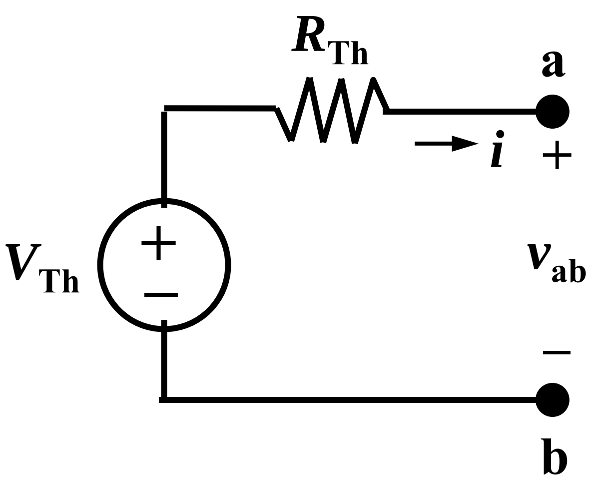
- Any* *linear* 2-terminal (1-port) network of indep. voltage sources, indep. current sources, and linear resistors can be replaced by an equivalent circuit consisting of an independent voltage source in series with a resistor without affecting the operation of the rest of the circuit.

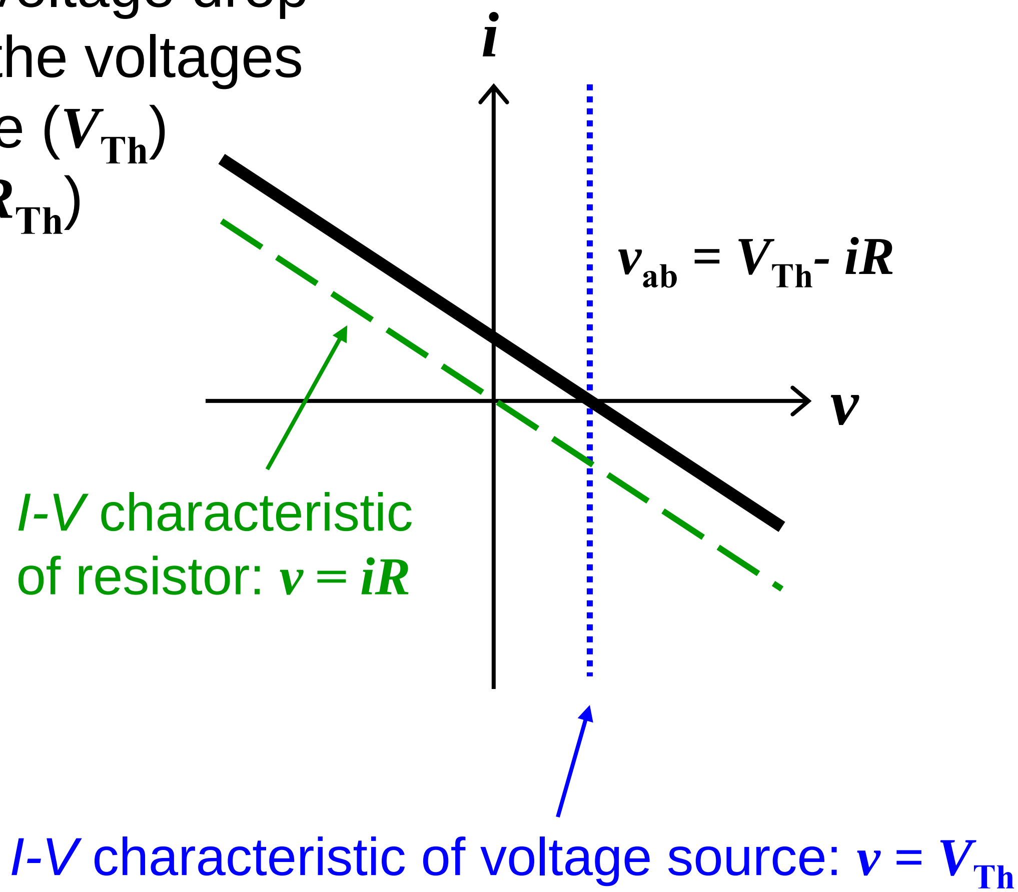


I-V Characteristic of Thévenin Equivalent

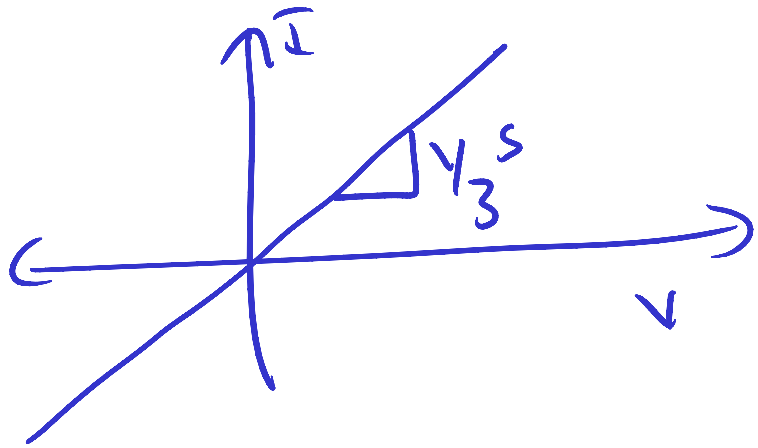
- **The *I-V* characteristic for the series combination of elements is obtained by adding their voltage drops:**

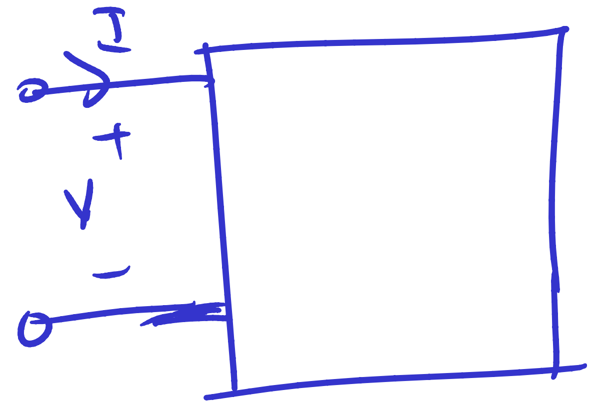
For a given current $\boldsymbol{i}$, the voltage drop $\boldsymbol{v}_{\mathbf{ab}}$ is equal to the sum of the voltages dropped across the source ($\boldsymbol{V}_{\mathbf{Th}}$) and across the resistor ($\boldsymbol{iR}_{\mathbf{Th}}$)





Intuitive ideas behind Thevenin's theorem

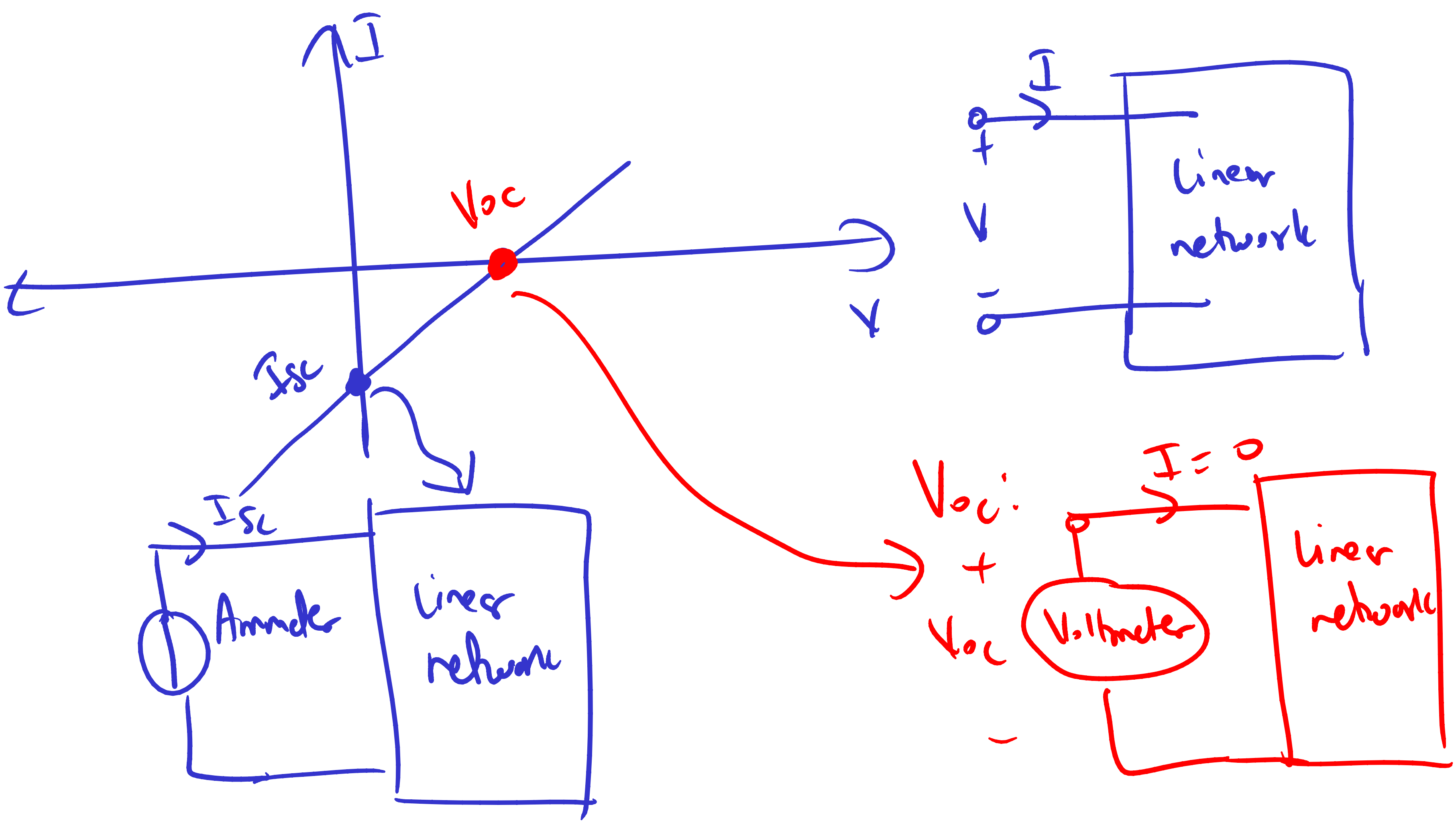




$R_{eq} = 3\,\Omega$. We do not know what is (are) the actual resistance (resistances) inside the box!

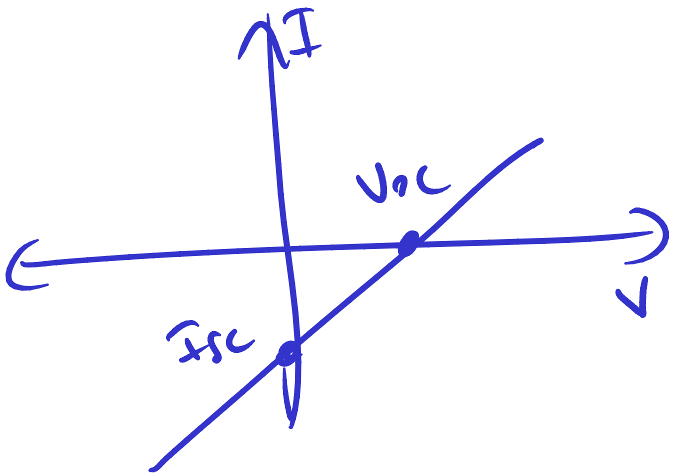
eg: $R_{eq} = 6 \| 6\,\Omega,\ 2+1\,\Omega,\ 0.5+2.5\,\Omega \ldots\ldots$

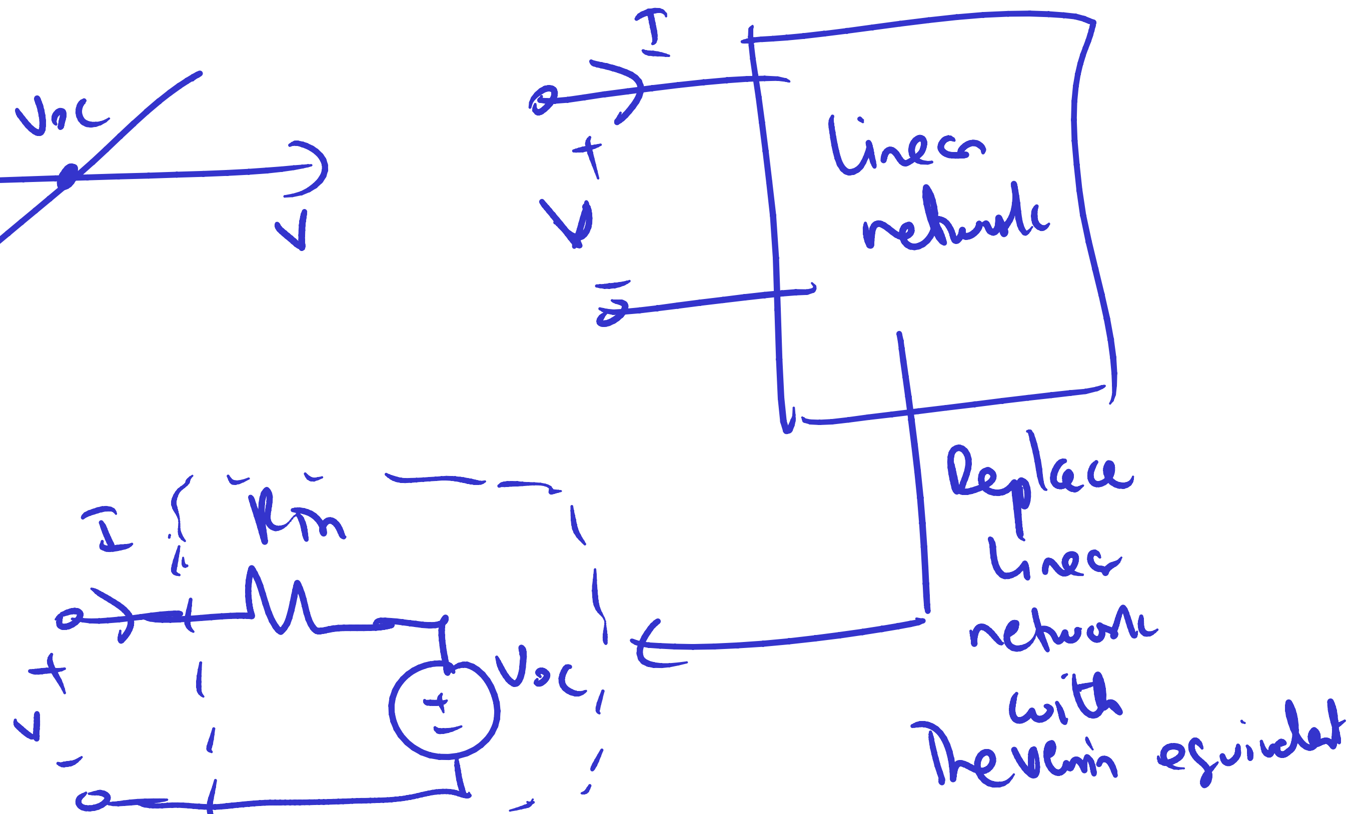
Intuitive idea behind Thevenin's theorem (contd.)



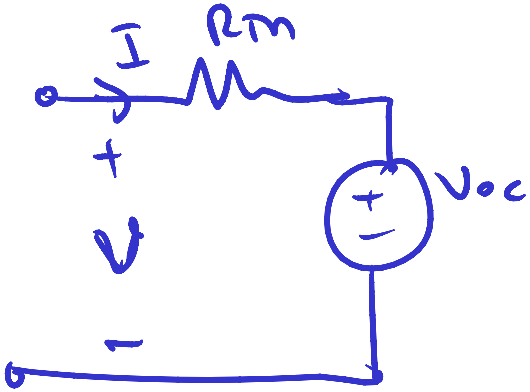
Intuitive ideas behind Thevenin's theorem (contd.)

Now, I have V_{oc} & I_{sc} $\Rightarrow$ $R_{Thevenin} = R_{Th} \triangleq \dfrac{-V_{oc}}{I_{sc}} \triangleq \dfrac{1}{slope}$

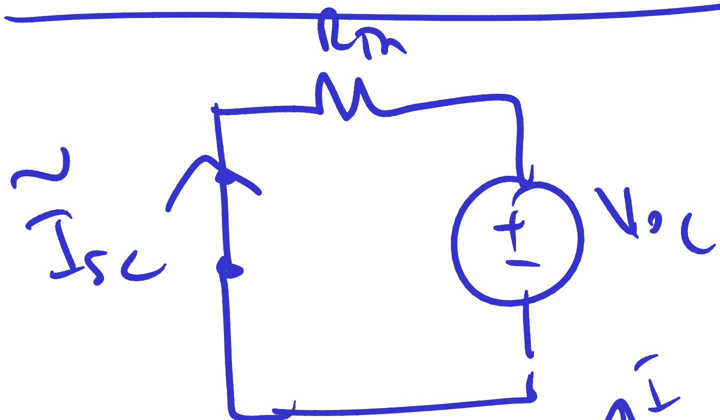




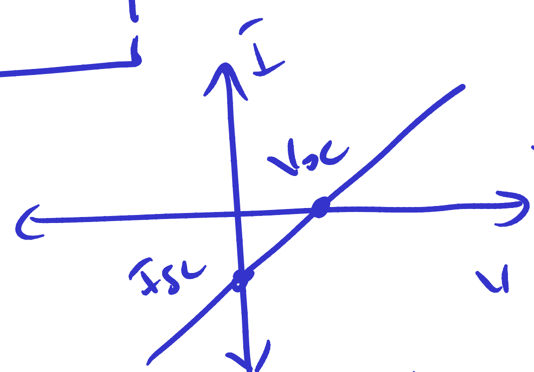
Intuitive ideas behind Thevenin's theorem (contd.)



$$\tilde{V}_{oc} = V_{oc}$$ ✓



$$\tilde{I}_{sc} = -\frac{V_{oc}}{R_{th}}$$



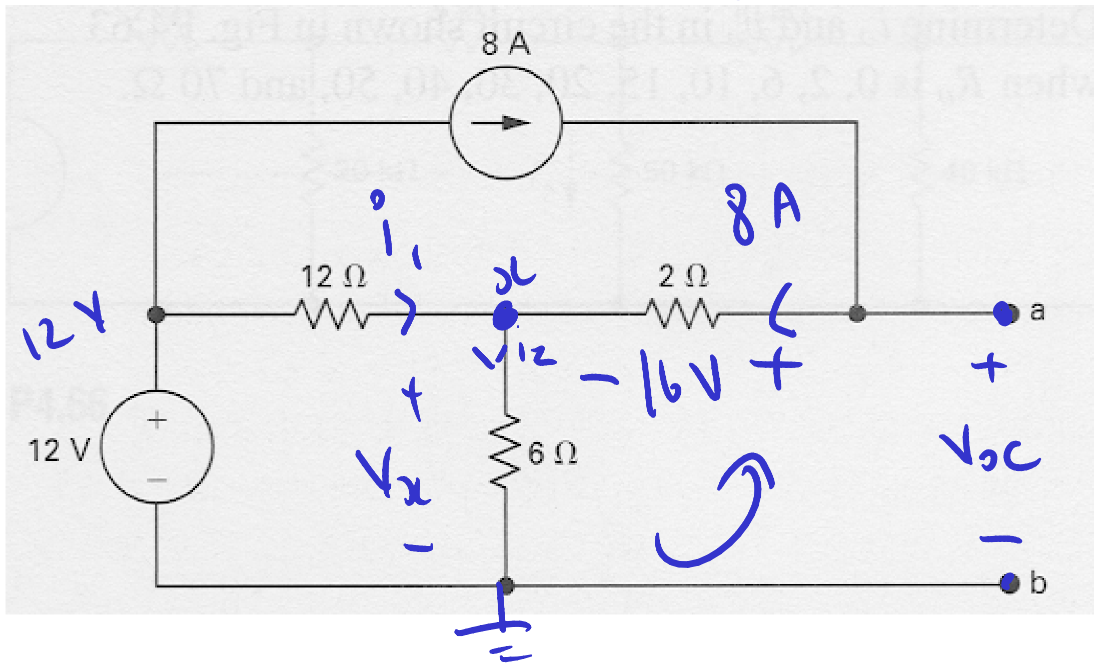
$$= I_{sc}$$ ✓

Thévenin Equivalent Example

Find the Thevenin equivalent with respect to the terminals a,b:

[p. 4.64]

HW #2 problem



Goal: Find V_{oc} & I_{sc}

Step 1: Finding V_{oc}:

$V_x = V_{oc} - 16$

Node eqn @ x: $i_1 + 8A = i_2 \Rightarrow \frac{12 - V_x}{12} + 8 = \frac{V_x}{6}$

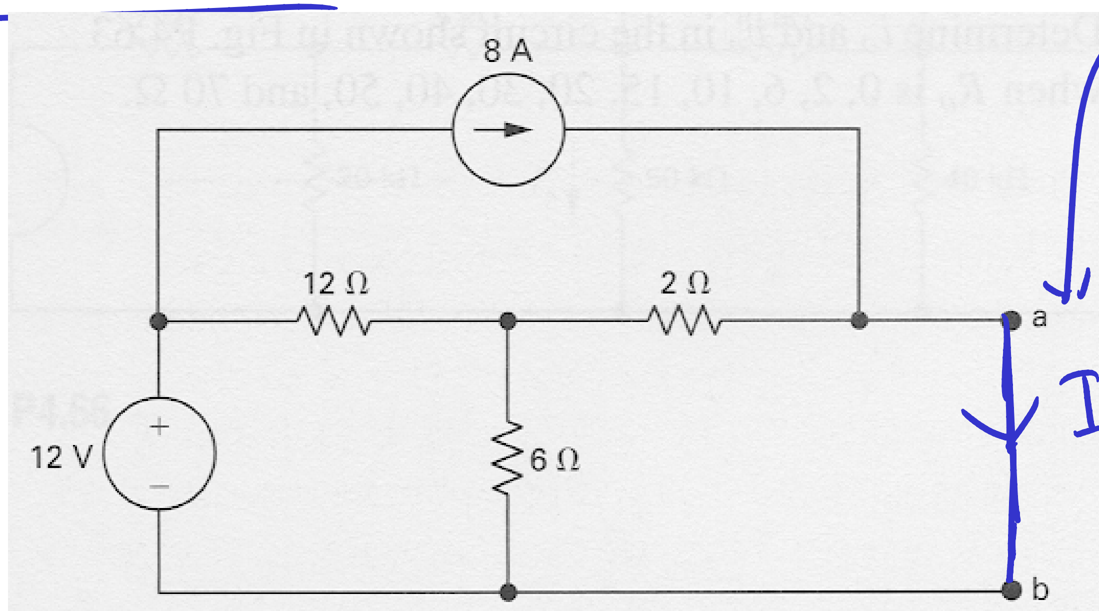
Thévenin Equivalent Example (contd.)

$\Rightarrow \quad 12 - V_x + 96 = 2V_x$

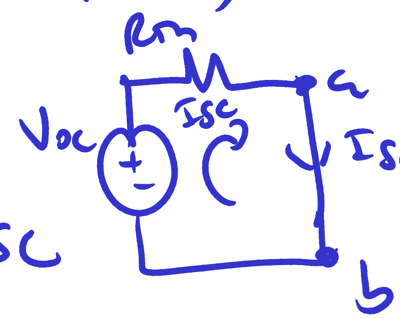
$\Rightarrow \quad V_x = \frac{108}{3} = \underline{\underline{36\,V}}$

$\therefore V_{oc} = V_x + 16 = \underline{\underline{52\,V}}$

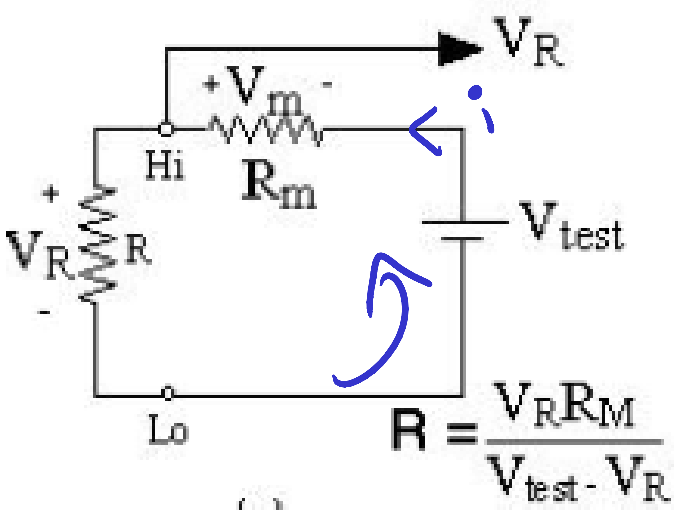
$\underline{\underline{I_{sc}}}$:



Direction of I_{sc} is from a to b, because:

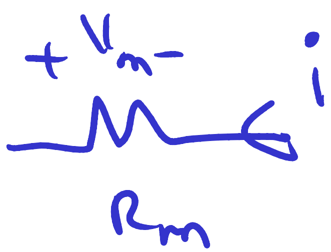


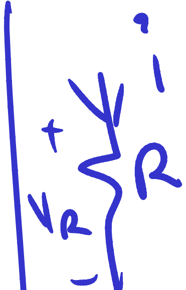
Aside: Prelab 1, question 3



(Q:) Prove: $R = \frac{V_R R_m}{V_{test} - V_R}$

KVL: $V_{test} + V_m - V_R = 0$





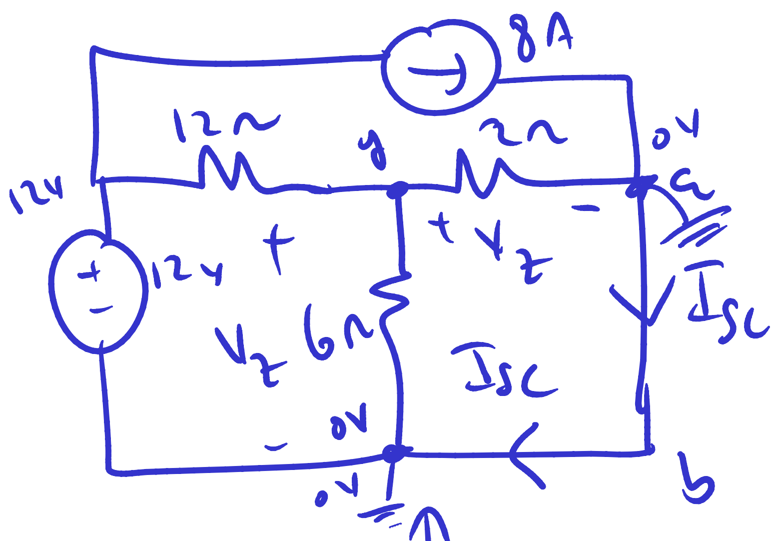
$V_m = -iR_m$ | $V_R = iR$

$\Rightarrow V_{test} + (-iR_m) - V_R = 0$

$\Rightarrow i = \frac{V_{test} - V_R}{R_m}$

$\therefore R = V_R / i$

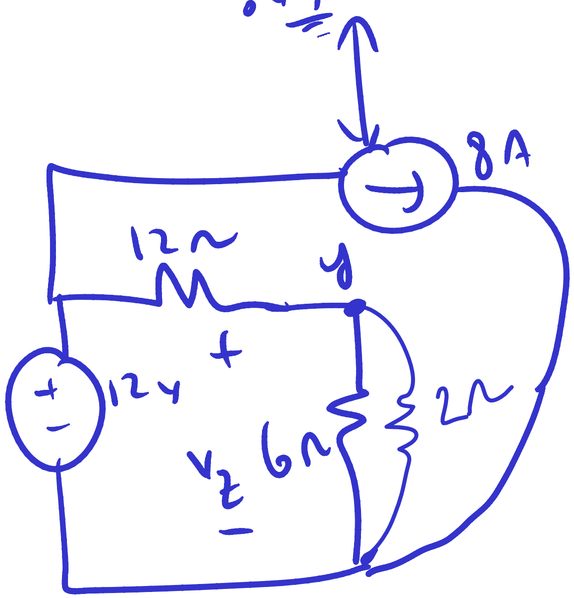
Thévenin Equivalent Example (contd.)

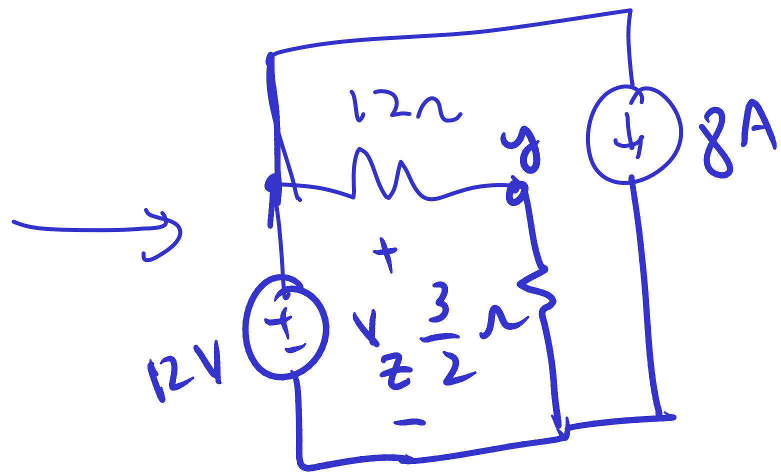


$$I_{sc} = 9\ A\ (??)$$

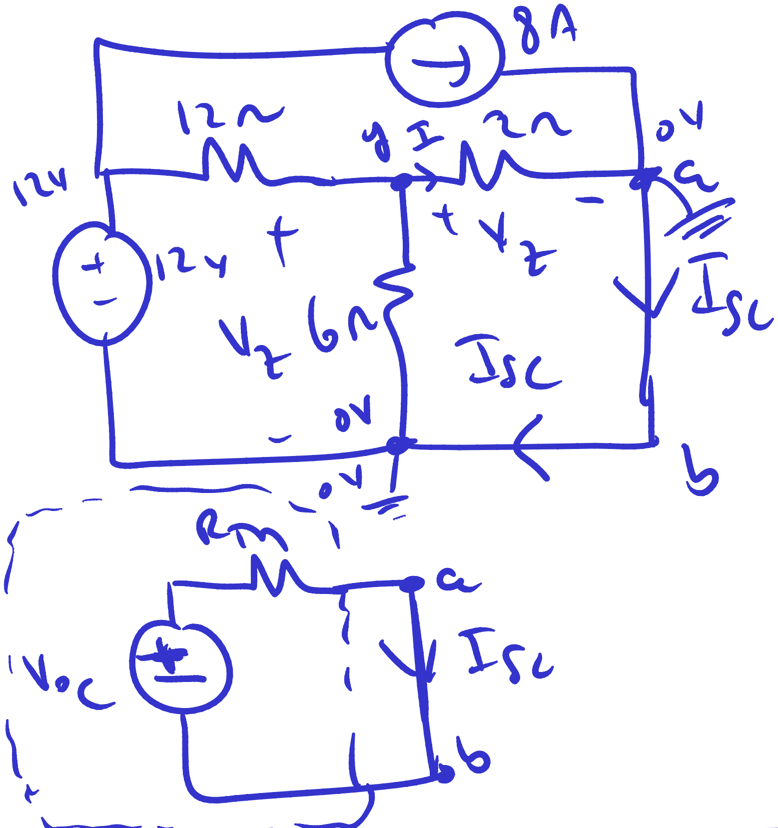
$$V_z = \left(\frac{\frac{3}{2}}{\frac{3}{2}+12}\right) 12\ V = \frac{36}{27}$$

$$= \frac{4}{3}\ V$$





Thévenin Equivalent Example (contd.)



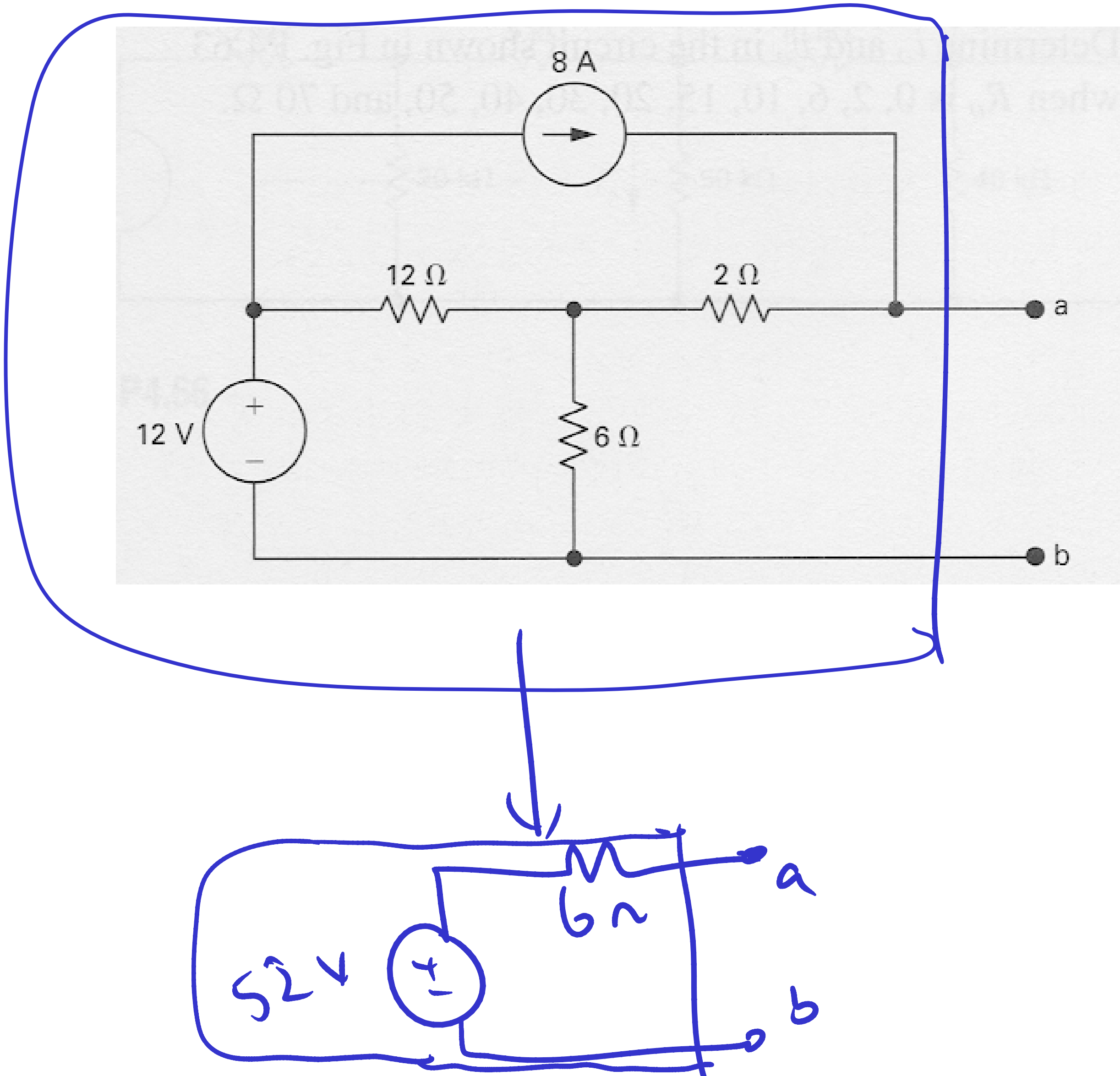
$$V_x = \frac{4}{3}\ \text{V}$$

$$I = \frac{V_x}{2} = \frac{2}{3}\ \text{A}$$

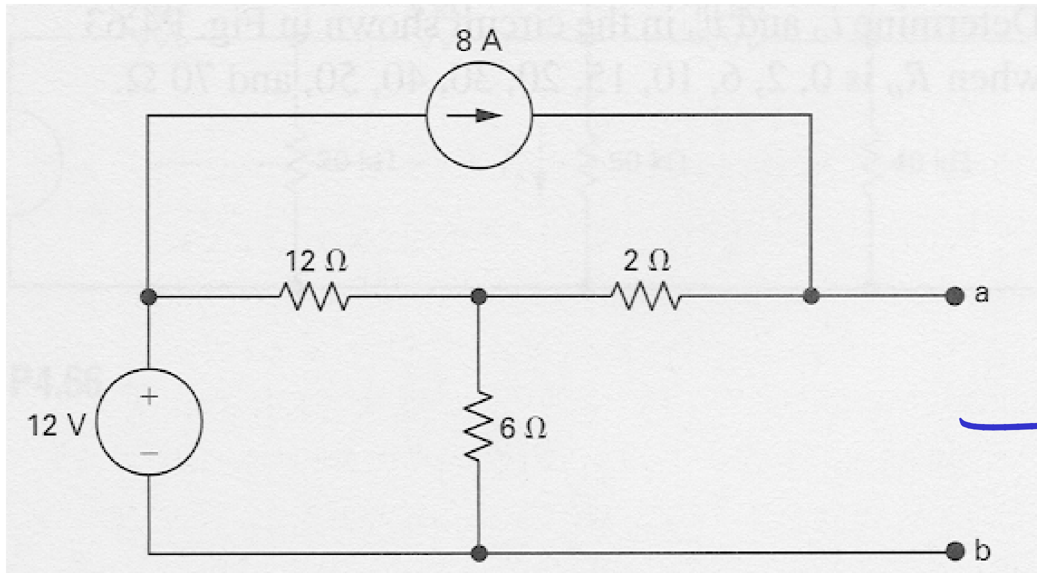
$$I_{sc} = 8 + 2/3 = 26/3\ \text{A}$$

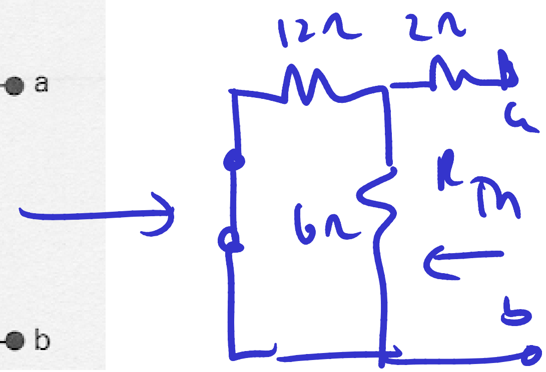
$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{52}{26/3} = \frac{\cancel{52}^{\,2} \cdot 3}{\cancel{26}} = 6\ \Omega$$

Thévenin Equivalent Example (contd.)



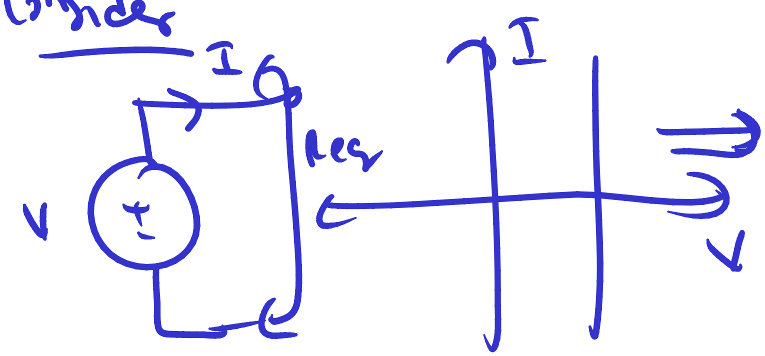
R_{Th} Calculation Example #1





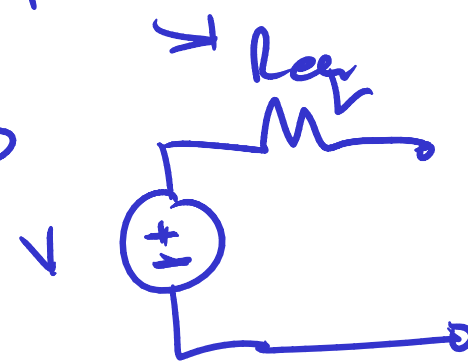
Set all independent sources to 0:

Consider



$\Rightarrow R_{eq} = 0$

ideally zero



$R_{Th} = \frac{12 \cdot 6}{18} + 2$

$= 4 + 2$

$= 6\,\Omega$