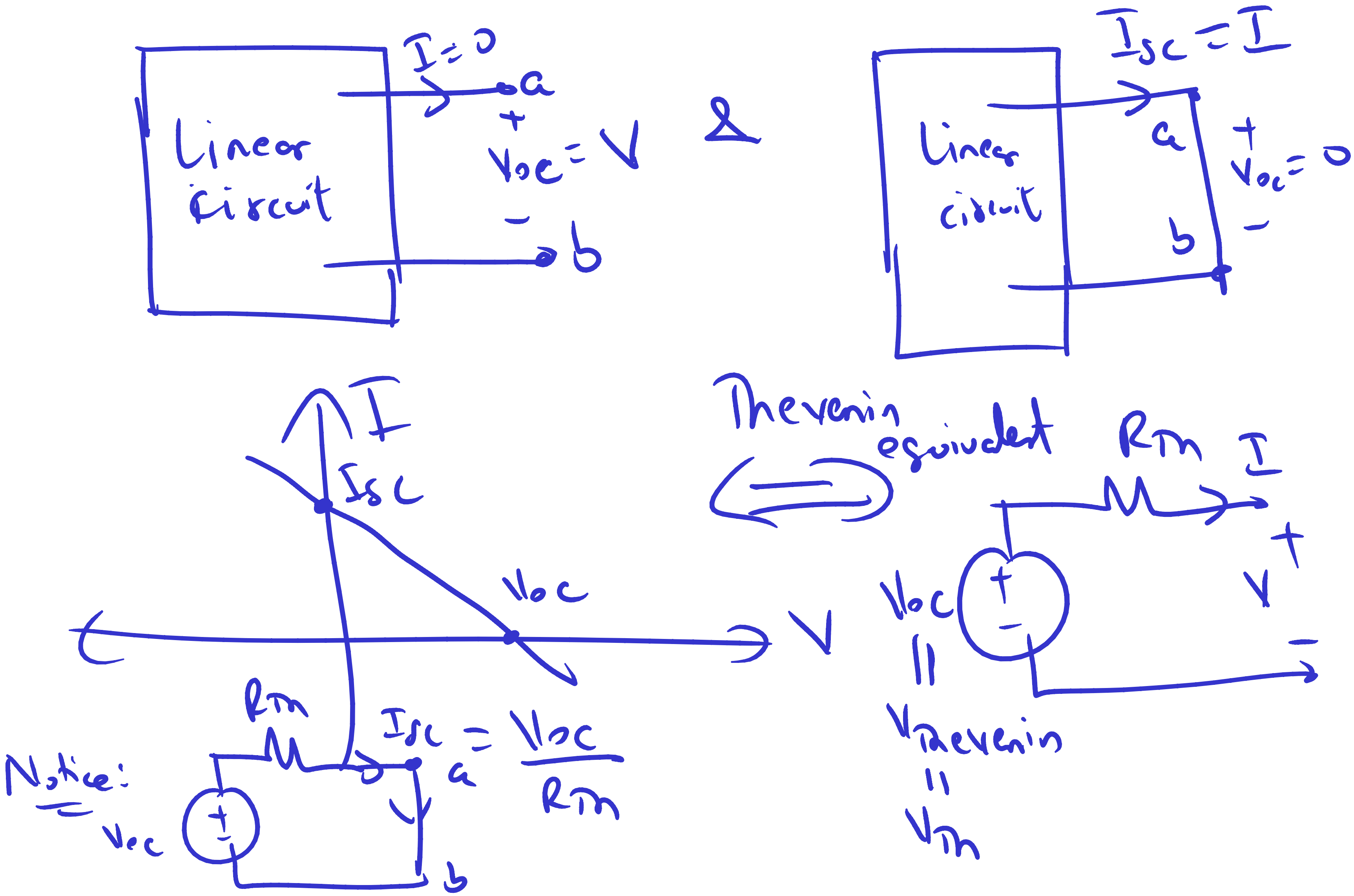
EE100Su08 Lecture #6 (July 7th 2008)

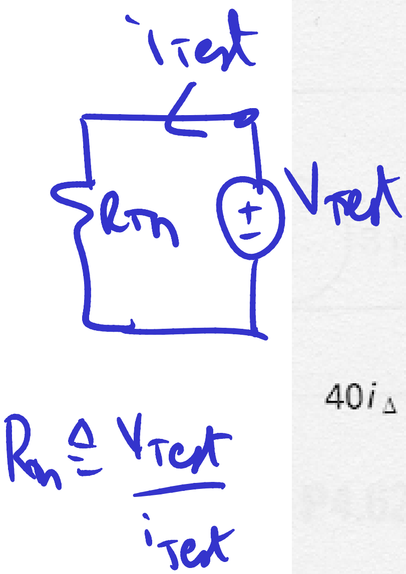
- Outline
  - **Today:**
    - **Midterm on Monday, 07/14/08 from 2 – 4 pm**
      - **Second room location changed to 120 Latimer**
    - **Questions?**
    - **Chapter 4 wrap up**
      - Thevenin and Norton
      - Source Transformations
      - Miscellaneous:
        - » Maximum Power Transfer theorem
    - **Chapter 6 wrap up**
      - **Capacitors (definition, series and parallel combination)**
      - **Inductors (definition, series and parallel combination)**
    - **Chapter 7: Intuitive Introduction**

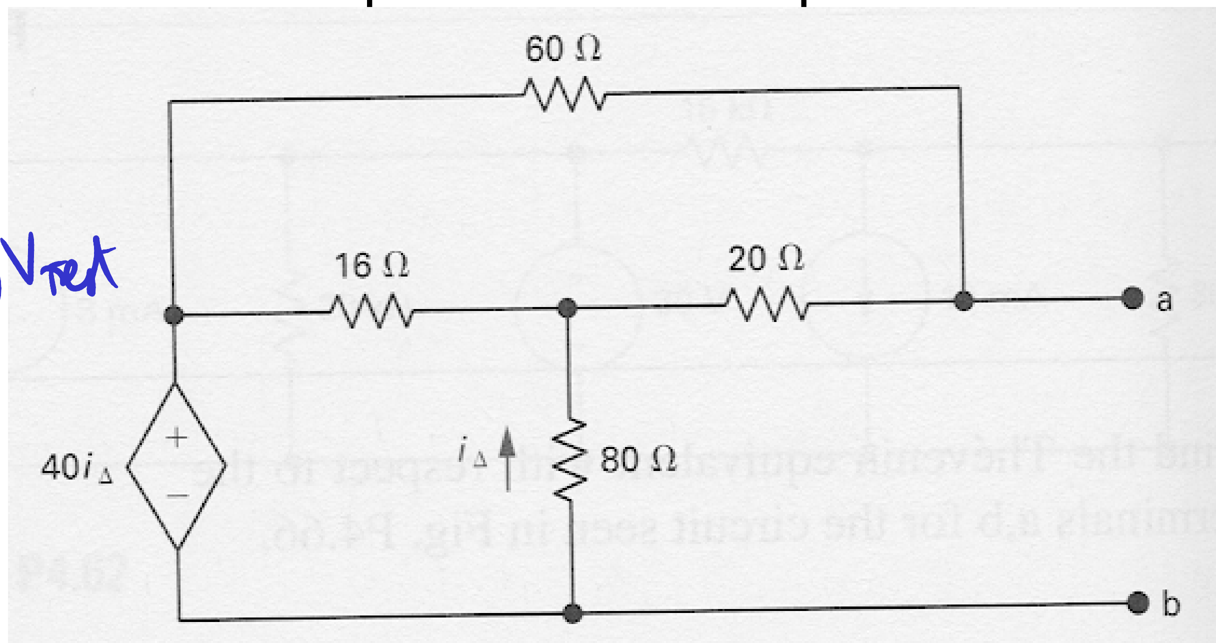
Recap: Thevenin Equivalents

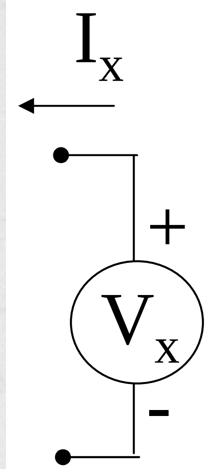


R_{Th} Calculation Example #2

Find the Thevenin equivalent with respect to the terminals a,b:



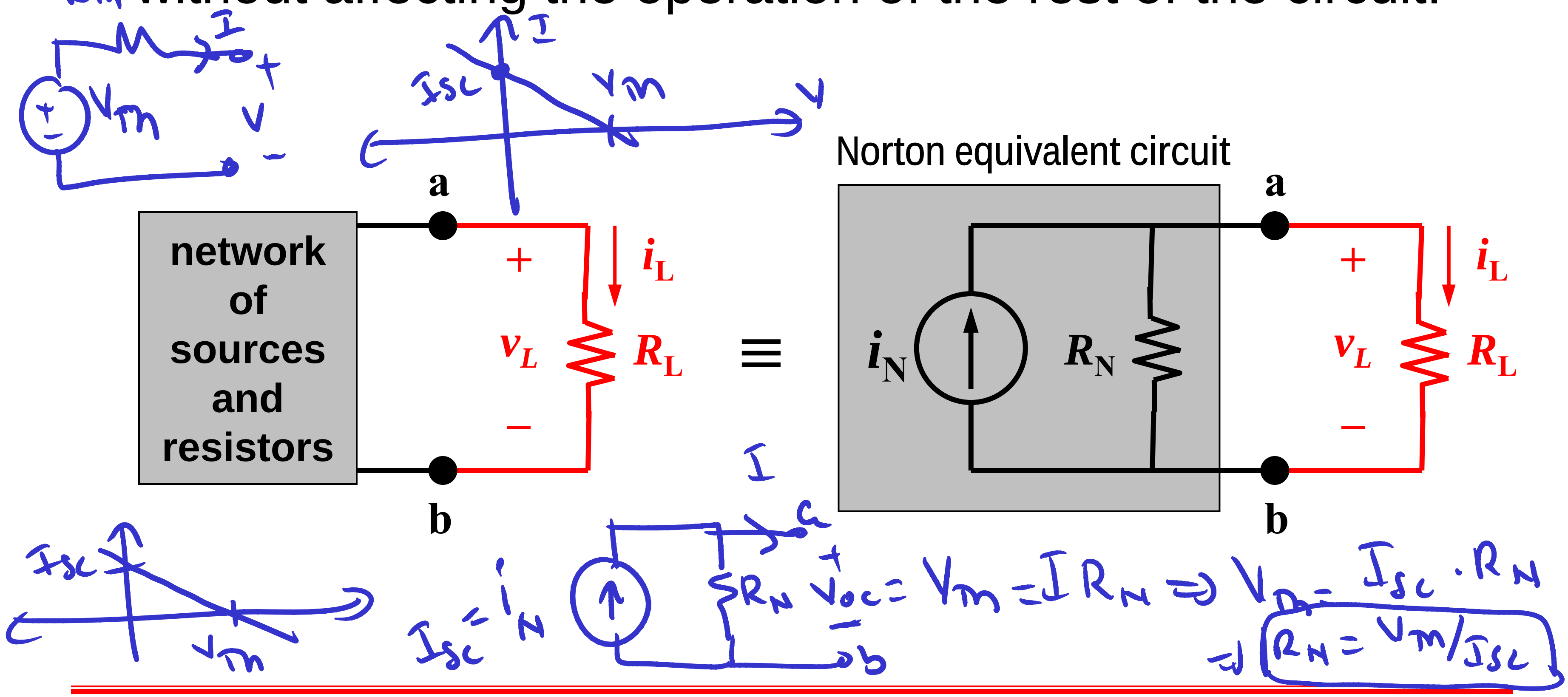




Since there is no independent source and we cannot arbitrarily turn off the dependence source, we can add a voltage source V_x across terminals a-b and measure the current through this terminal I_x . $R_{th}= V_x/ I_x$

Norton Equivalent Circuit

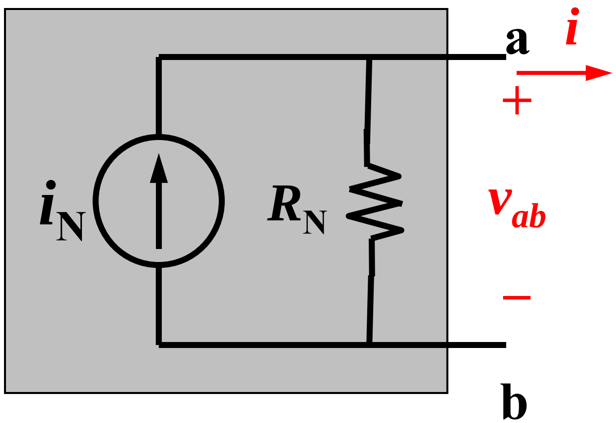
- Any* *linear* 2-terminal (1-port) network of indep. voltage sources, indep. current sources, and linear resistors can be replaced by an equivalent circuit consisting of an independent current source in parallel with a resistor without affecting the operation of the rest of the circuit.

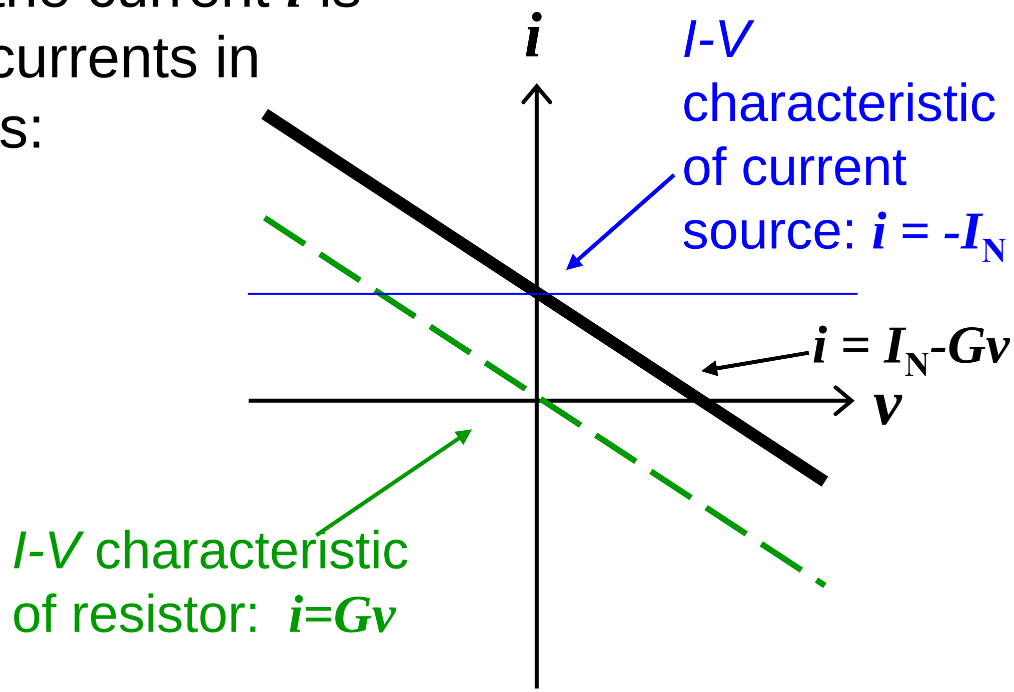


I-V Characteristic of Norton Equivalent

- **The *I-V* characteristic for the parallel combination of elements is obtained by adding their currents:**

For a given voltage v_{ab}, the current i is equal to the sum of the currents in each of the two branches:





Finding I_N and $R_N = R_{Th}$

Analogous to calculation of Thevenin Eq. Ckt:

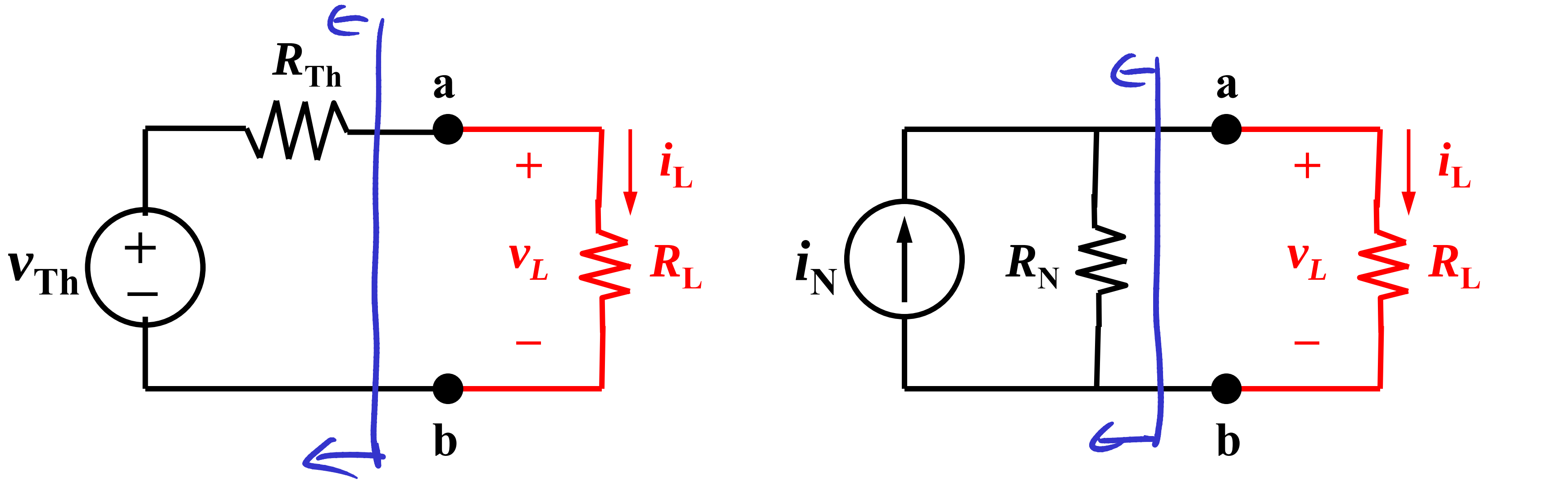
1) Find o.c voltage and s.c. current

$$I_N \equiv i_{sc} = V_{Th}/R_{Th}$$

2) Or, find s.c. current and Norton (Thev) resistance

Source Transforms: Finding I_N and R_N

- We can derive the Norton equivalent circuit from a Thévenin equivalent circuit simply by making a “source transformation”:



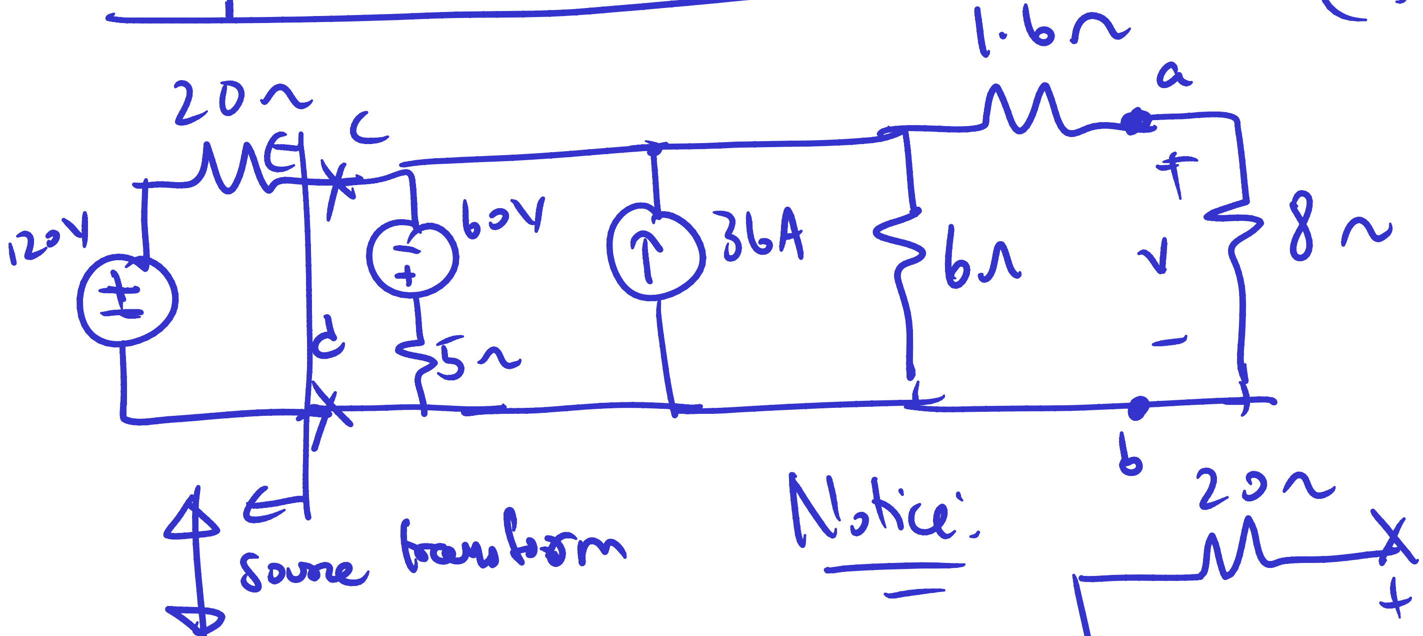
$$R_N = R_{Th} = \frac{v_{oc}}{i_{sc}}; \quad i_N = \frac{v_{Th}}{R_{Th}} = i_{sc}$$

Source Transformations

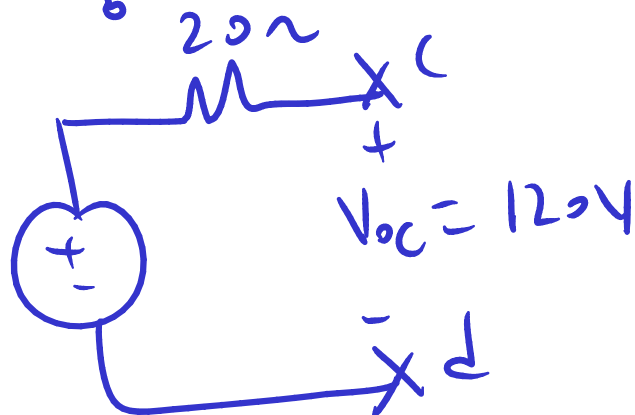
(example)

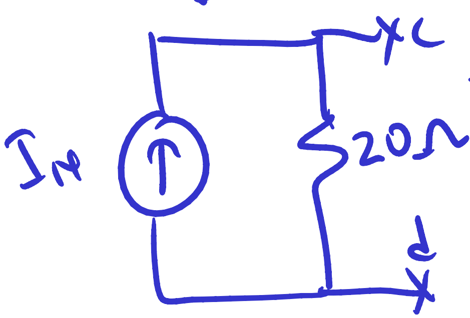
Drill problem 4.15 on p. 119

(a) Find v.



Notice:

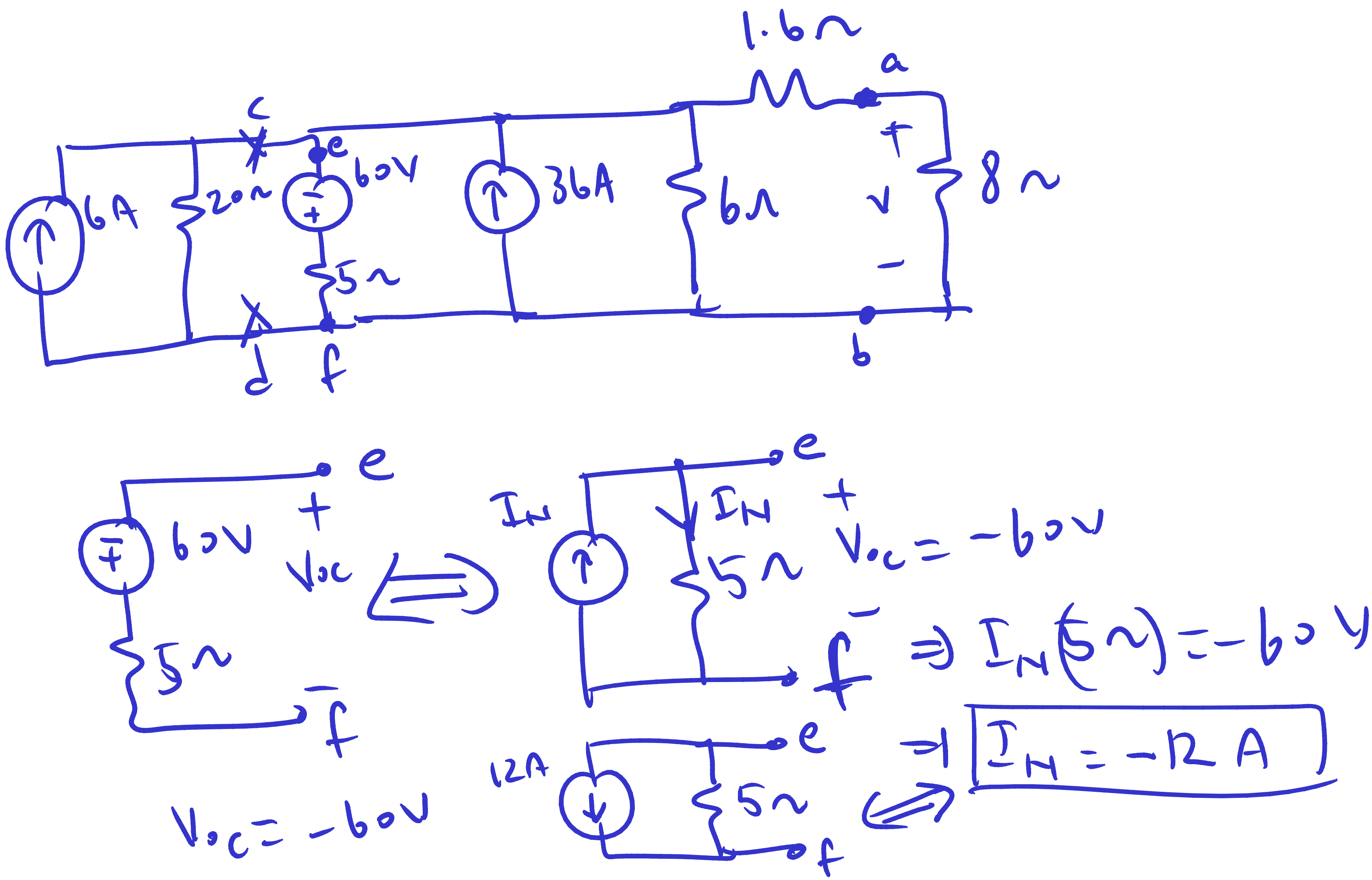




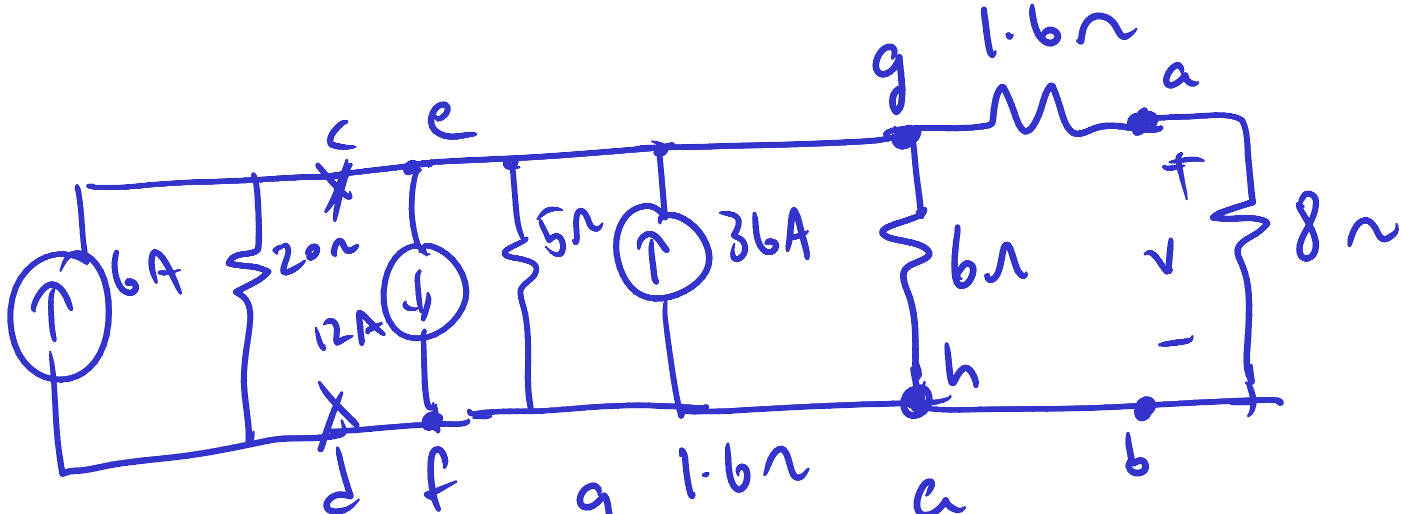
$V_{oc} = I_N \cdot 20$ 120V

$\Rightarrow I_N = \frac{120}{20} = 6A$

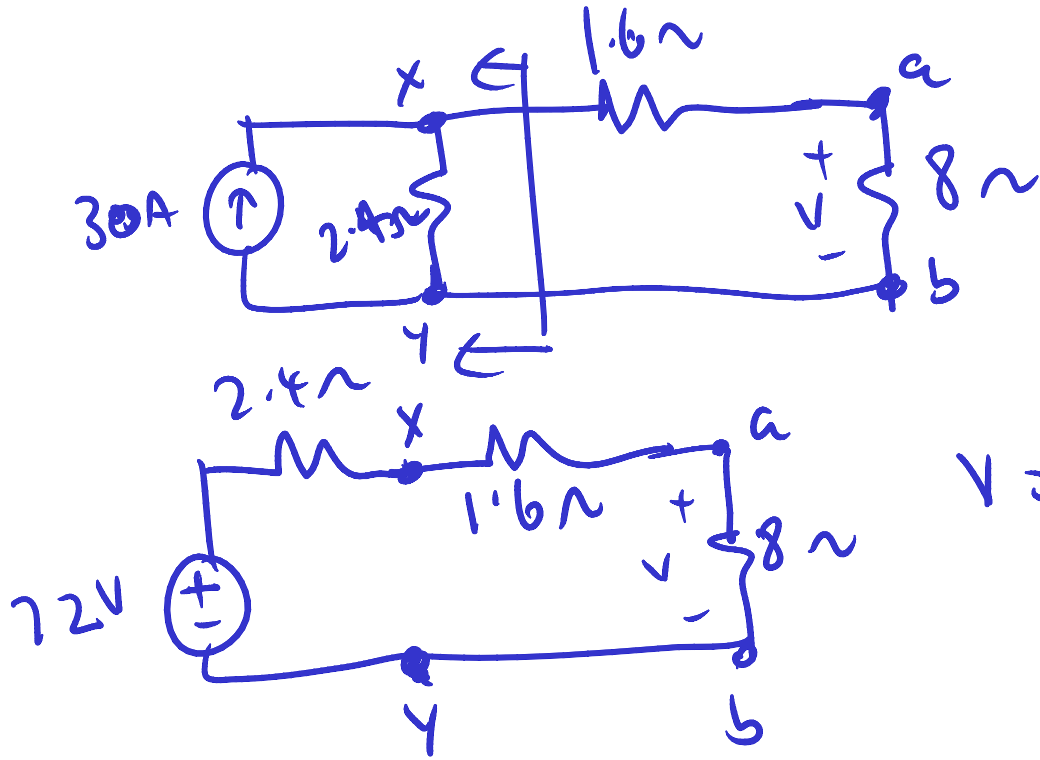
Source Transformations [example contd.]



Source Transformations [example contd.]



Source Transformations [example contd.]

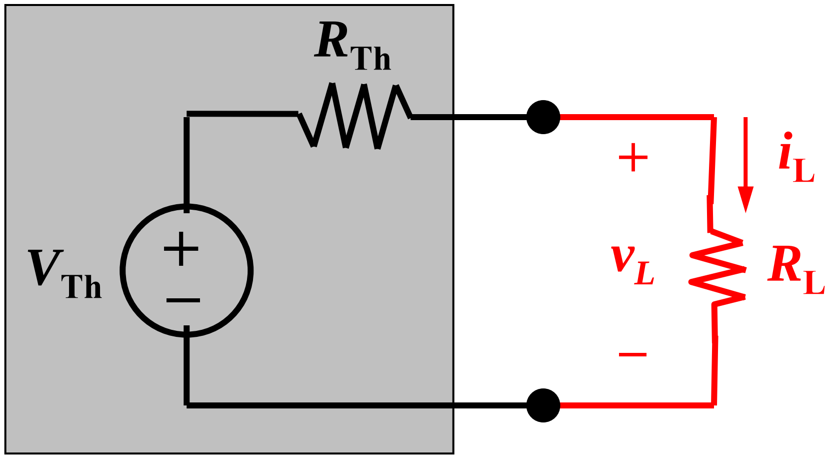


$$v = \left(\frac{8}{8+1.6+2.4}\right) 72$$

$$= \frac{8}{12} \cdot 72 = 48\ \text{V}$$

Maximum Power Transfer Theorem

Thévenin equivalent circuit



Power absorbed by load resistor:

$$p = i_L^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

To find the value of R_L for which p is maximum, set $\frac{dp}{dR_L}$ to 0:

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right] = 0$$

$$\Rightarrow (R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L) = 0$$

$$\Rightarrow \boxed{R_{Th} = R_L}$$

A resistive load receives maximum power from a circuit if the load resistance equals the Thévenin resistance of the circuit.

Summary of Techniques for Circuit Analysis -1

- Resistor network (Chapter 3)
  - Parallel resistors
  - Series resistors
  - Voltage Divider
  - Current Divider
  - Voltmeters and Ammeters
  - Y-delta conversion: OPTIONAL

Summary of Techniques for Circuit Analysis -2

- Node Analysis (Chapter 4)
  - Node voltage is the unknown
  - Solve for KCL
  - Floating voltage source using super node
- Superposition
  - Leave one independent source on at a time
  - Sum over all responses
  - Voltage off → SC
  - Current off → OC
- Mesh Analysis: OPTIONAL
  - Loop current is the unknown
  - Solve for KVL
  - Current source using super mesh
- Thevenin and Norton Equivalent Circuits
  - Solve for OC voltage
  - Solve for SC current
- Source Transforms:
  - Voltage sources in series with a resistance can be converted to current source in parallel with a resistance.

Comments on Dependent Sources

- Node-Voltage Method
  - Dependent current source:
    - treat as independent current source in organizing node eqns
    - substitute constraining dependency in terms of defined node voltages.
  - Dependent voltage source:
    - treat as independent voltage source in organizing node eqns
    - Substitute constraining dependency in terms of defined node voltages.

Comments on Dependent Sources (contd.)

A dependent source establishes a voltage or current whose value depends on the value of a voltage or current at a specified location in the circuit.

(device model, used to model behavior of transistors & amplifiers)

To specify a dependent source, we must identify:

1. the controlling voltage or current (must be calculated, in general)
2. the relationship between the controlling voltage or current and the supplied voltage or current
3. the reference direction for the supplied voltage or current

The relationship between the dependent source and its reference cannot be broken!

– Dependent sources cannot be turned off for various purposes (*e.g.* to find the Thévenin resistance, or in analysis using Superposition).

Chapters 6

- Outline
  - The capacitor
  - The inductor

The Capacitor

Two conductors (a,b) separated by an insulator:

difference in potential = V_{ab}

=> equal & opposite charge ***Q*** on conductors

$$Q = CV_{ab}$$

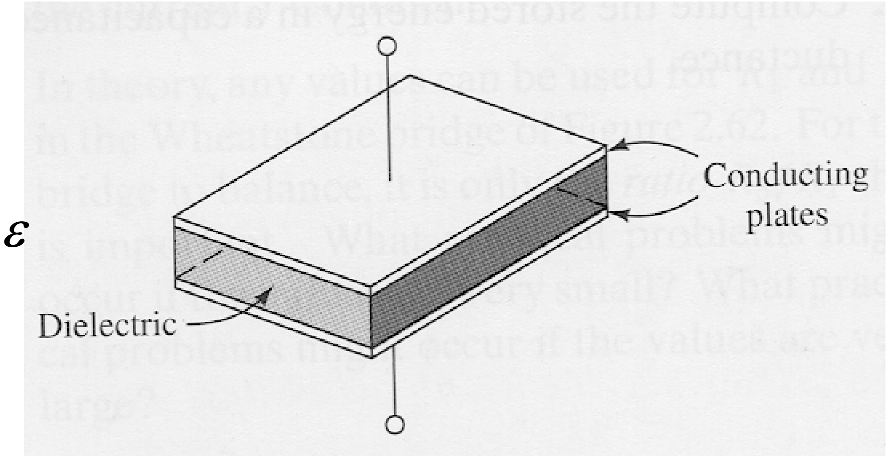
(stored charge in terms of voltage)

where ***C*** is the **capacitance** of the structure,

- positive (+) charge is on the conductor at higher potential

Parallel-plate capacitor:

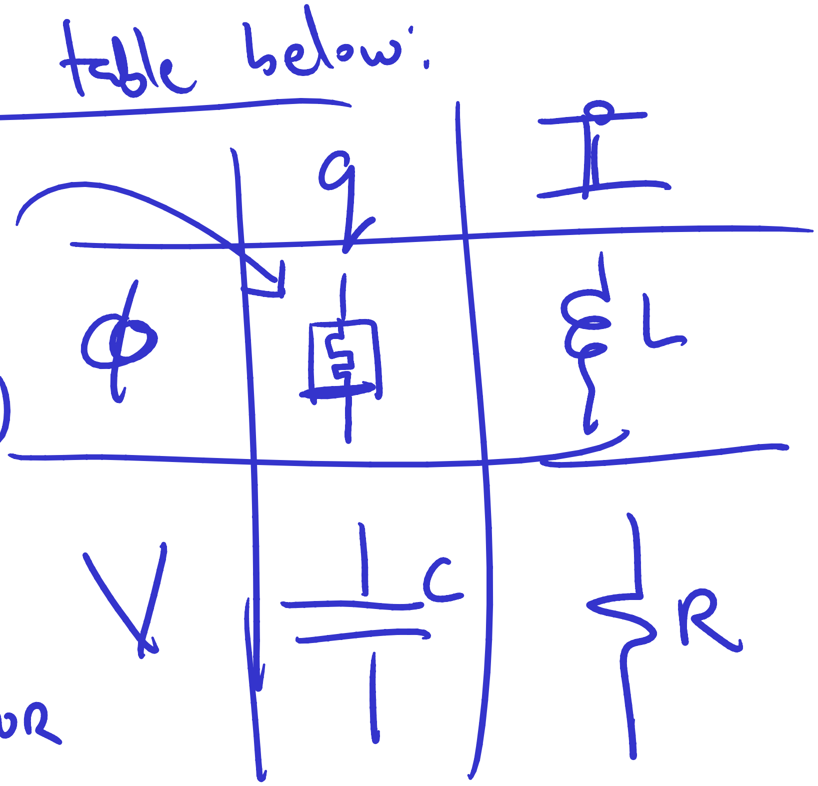
- area of the plates = ***A (m²)***
- separation between plates = ***d (m)***
- ***dielectric permittivity*** of insulator = ε ***(F/m)***



=> capacitance $C = \frac{A\varepsilon}{d}$ ***F***

A note on circuit variables

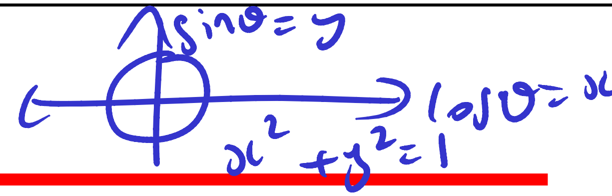
Consider table below:



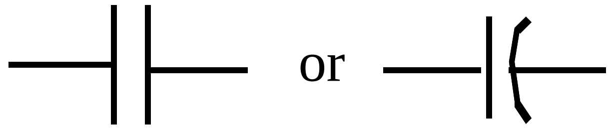
Leon O. Chua (my advisor) proposed the MEMRISTOR in 1971

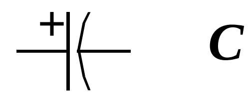
Point: interesting to think about the 4th circuit element, practically useless unless you are working on the nanoscale [Williams et. al., Nature 2008]

Capacitor



Symbol:





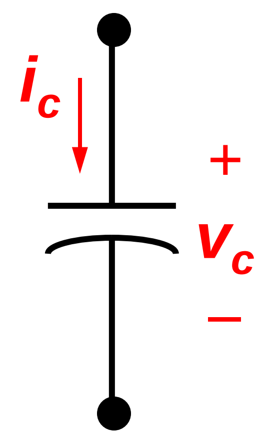
Electrolytic (polarized) capacitor

Units: Farads (Coulombs/Volt)

(typical range of values: 1 pF to 1 μF; for "supercapa-citors" up to a few F!)

Current-Voltage relationship:

$$i_c = \frac{dQ}{dt} = C\frac{dv_c}{dt} + v_c\frac{dC}{dt}$$



If C (geometry) is unchanging, $i_C = C\, dv_C/dt$

Note: Q (v_c) must be a continuous function of time

Voltage in Terms of Current

$$Q(t) = \int_0^t i_c(t)dt + Q(0)$$

$$v_c(t) = \frac{1}{C}\int_0^t i_c(t)dt + \frac{Q(0)}{C} = \frac{1}{C}\int_0^t i_c(t)dt + v_c(0)$$

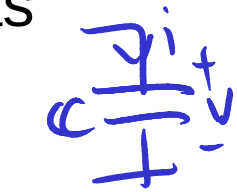
Uses: Capacitors are used to store energy for camera flashbulbs, in filters that separate various frequency signals, and they appear as undesired "parasitic" elements in circuits where they usually degrade circuit performance

Stored Energy

CAPACITORS STORE ELECTRIC ENERGY

You might think the energy stored on a capacitor is $\boldsymbol{QV = CV^2}$, which has the dimension of Joules. But during charging, the average voltage across the capacitor was only half the final value of $\boldsymbol{V}$ for a linear capacitor.

Thus, energy is $$\frac{1}{2}QV = \boxed{\frac{1}{2}CV^2}.$$



Note:

$p = vi$

$= v\,C\frac{dv}{dt}$

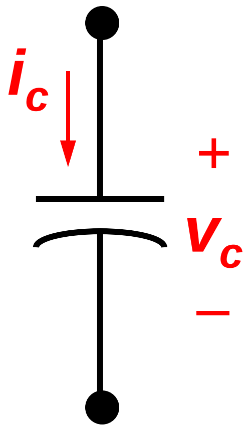
$p = Cv\frac{dv}{dt}$

$\Rightarrow E = \int p\,dt$

$= C\int v\,dv$

<u>Example</u>: A 1 pF capacitance charged to 5 Volts has ½(5V)² (1pF) = 12.5 pJ
(A 5F supercapacitor charged to 5 volts stores 63 J; if it discharged at a constant rate in 1 ms energy is discharged at a 63 kW rate!)

A more rigorous derivation

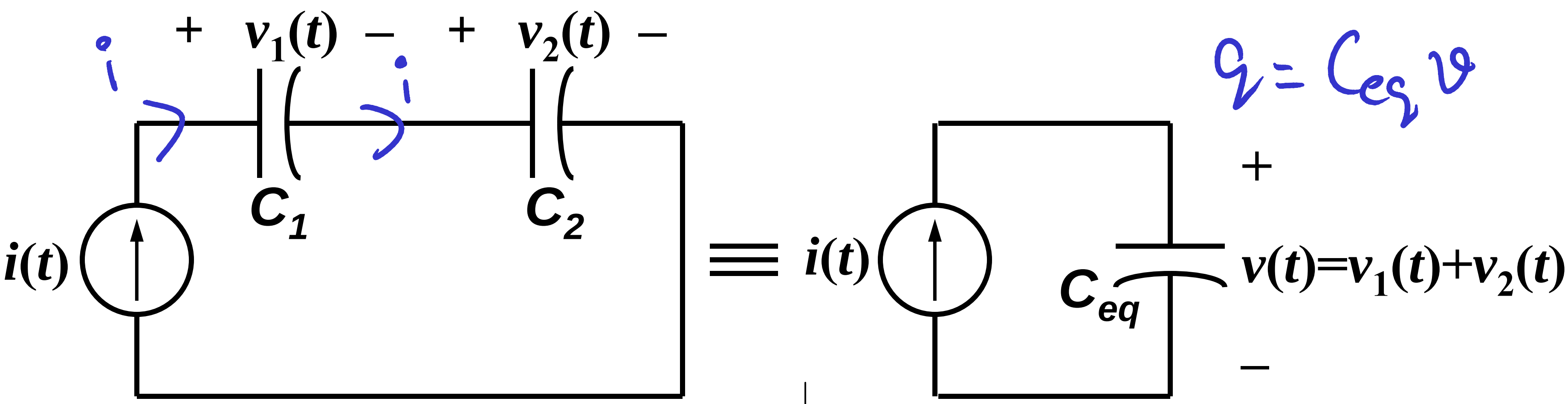


This derivation holds independent of the circuit!

$$w = \int_{t=t_{Initial}}^{t=t_{Final}} v_c \cdot i_c \, dt = \int_{v=V_{Initial}}^{v=V_{Final}} v_c \frac{dQ}{dt} dt = \int_{v=V_{Initial}}^{v=V_{Final}} v_c \, dQ$$

$$w = \int_{v=V_{Initial}}^{v=V_{Final}} C v_c \, dv_c = \frac{1}{2} C V_{Final}{}^2 - \frac{1}{2} C V_{Initial}{}^2$$

Capacitors in Series



$q = C_{eq} v$

$\Rightarrow q = C_1 v_1$

$q = C_2 v_2$

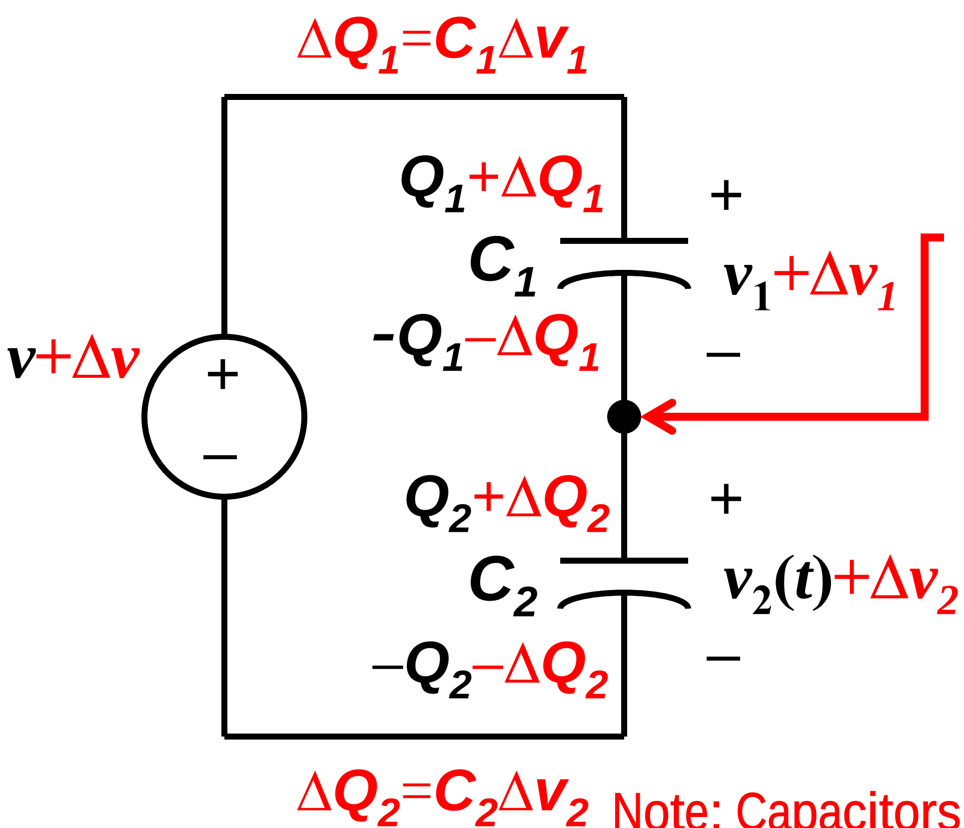
$q = C_{eq}(v_1 + v_2)$

$\Rightarrow q = C_{eq}\left[\frac{q}{C_1} + \frac{q}{C_2}\right]$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitive Voltage Divider

Q: Suppose the voltage applied across a series combination of capacitors is changed by Δv. How will this affect the voltage across each individual capacitor?



$$\Delta v = \Delta v_1 + \Delta v_2$$

Note that no net charge can can be introduced to this node. Therefore, $-\Delta Q_1 + \Delta Q_2 = 0$

$$\Rightarrow C_1 \Delta v_1 = C_2 \Delta v_2$$

$$\Delta v_2 = \frac{C_1}{C_1 + C_2} \Delta v$$

Note: Capacitors in series have the same incremental charge.

Inductor

Symbol: 

L

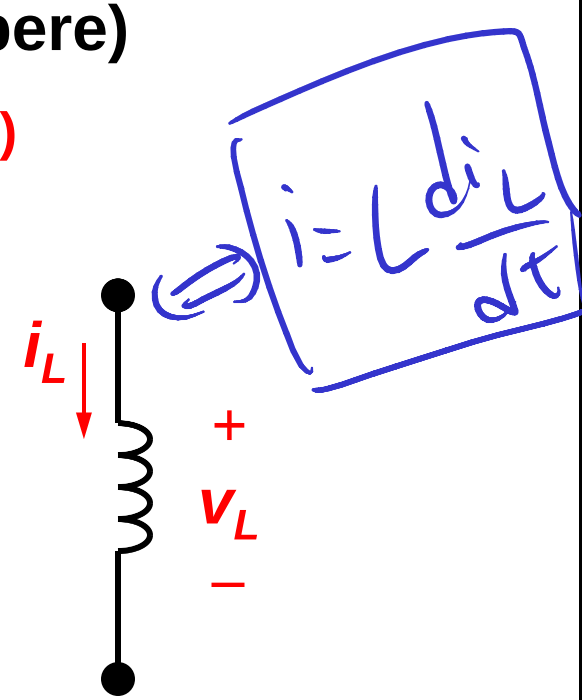
Units: **Henrys (Volts • second / Ampere)**

(typical range of values: μH to 10 H)

Current in terms of voltage:

$$di_L = \frac{1}{L} v_L(t)dt$$

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(\tau)d\tau + i(t_0)$$



Note: i_L **must be a continuous function of time**

Stored Energy

INDUCTORS STORE MAGNETIC ENERGY

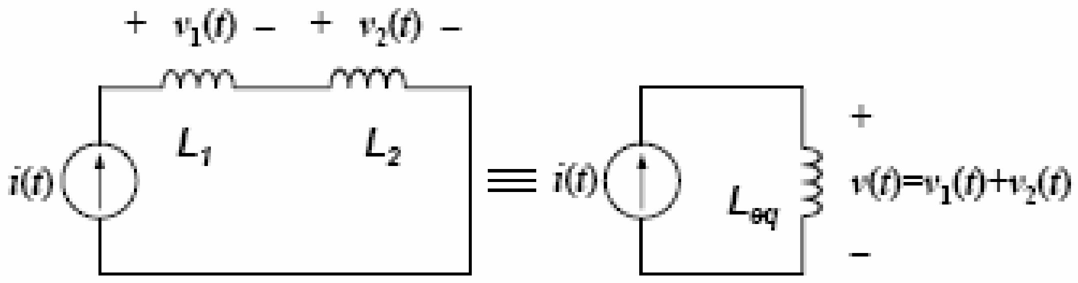
Consider an inductor having an initial current $i(t_0) = i_0$

$$p(t) = v(t)i(t) = \left(L\frac{di}{dt}\right)i$$

$$w(t) = \int_{t_0}^{t} p(\tau)d\tau =$$

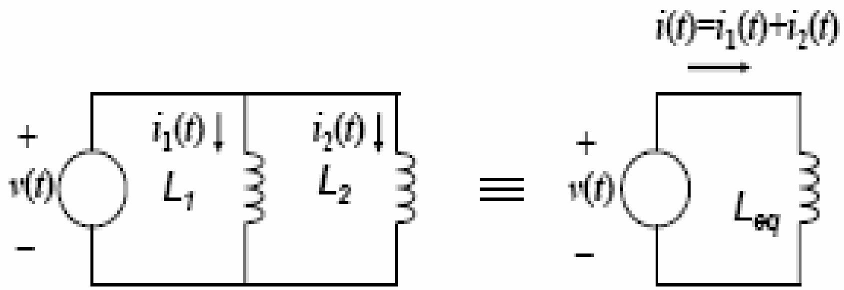
$$w(t) = \frac{1}{2}Li^2 - \frac{1}{2}Li_0^{\ 2}$$

Inductors in Series and Parallel



Common Current

$$L_{eq} = L_1 + L_2$$



Common Voltage

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Summary

Capacitor

$$i = C\frac{dv}{dt}; w = \frac{1}{2}Cv^2$$

wednesday

v cannot change instantaneously

i can change instantaneously

Do not short-circuit a charged capacitor (-> infinite current!)

n cap.'s in series: $\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$

n cap.'s in parallel: $C_{eq} = \sum_{i=1}^{n} C_i$

In steady state (not time-varying), a capacitor behaves like an open circuit.

Inductor

$$v = L\frac{di}{dt}; w = \frac{1}{2}Li^2$$

i cannot change instantaneously

v can change instantaneously

Do not open-circuit an inductor with current (-> infinite voltage!)

n ind.'s in series: $L_{eq} = \sum_{i=1}^{n} L_i$

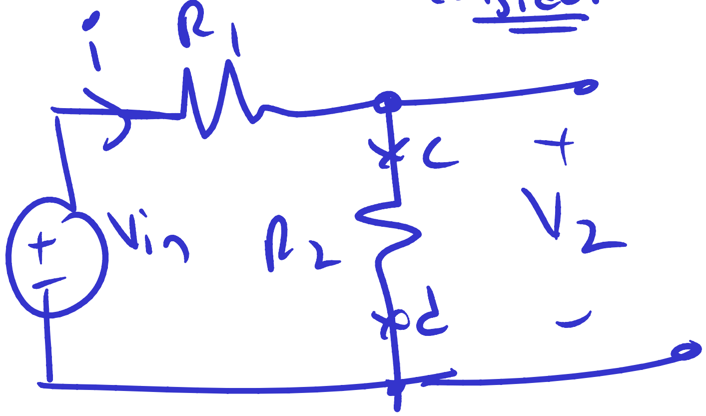
n ind.'s in parallel: $\frac{1}{L_{eq}} = \sum_{i=1}^{n} \frac{1}{L_i}$

In steady state, an inductor behaves like a short circuit.

Chapter 7: Intuitive Introduction

differential equation

Consider:



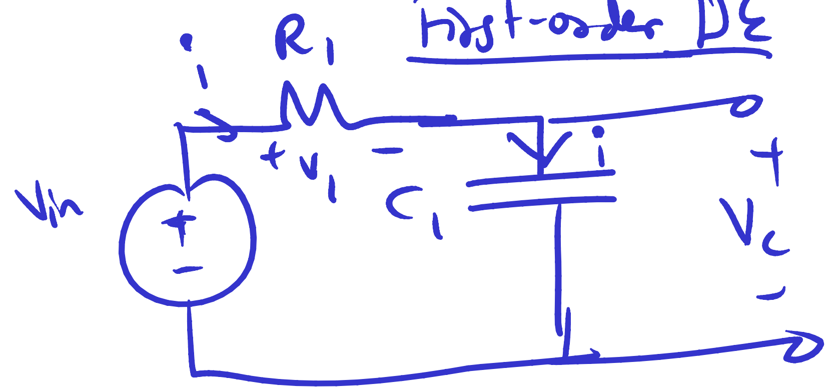
$$V_2 = V_{in} \cdot \frac{R_2}{R_1 + R_2}$$

[voltage divider]

As $R_2 \to \infty$ (open circuit cd),

$$V_2 \to V_{in}$$

First-order DE



(Q:) Given $v_{in}(t)$, find $v_c(t)$.

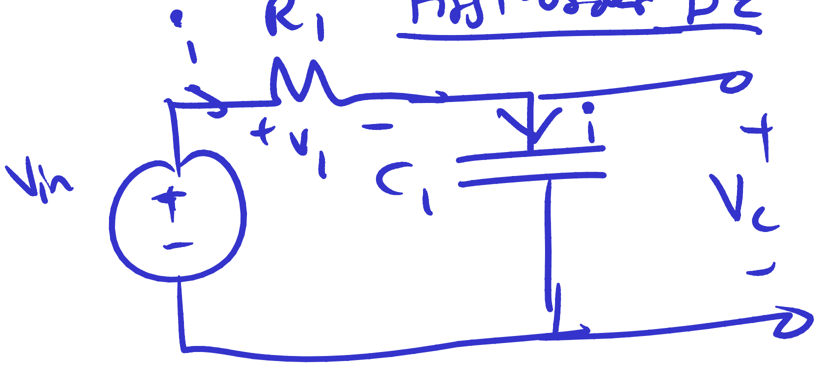
$$V_{in} = V_1 + V_c \quad (KVL)$$

$$= iR_1 + V_c$$

$$\Rightarrow V_{in}(t) = \left(C_1 \frac{dv_c}{dt}\right) R_1 + V_c(t)$$

Chapter 7: Intuitive Introduction

First-order DE



Note: $\frac{d^2v}{dt^2} \neq \left(\frac{dv}{dt}\right)^2$

(Q:) Given $v_{in}(t)$, find $v_c(t)$.

$V_{in} = V_1 + V_c$ (KVL)

$= iR_1 + V_c$

$\Rightarrow \boxed{V_{in}(t) = \left(C_1 \frac{dv_c}{dt}\right) R_1 + V_c(t)}$

(Q:) $x + 3 = 8$, Find the value of x?

(A:) We don't know since the domain of x is unspecified

Suppose: $V_{in} = 7$

$\Rightarrow 7 = R_1 C_1 \frac{dv_c(t)}{dt} + V_c(t)$

$V_c(t) = 7$?

$\Rightarrow$ seems to work!

Chapter 7: Intuitive Introduction

Notice $v_c(t) = 7$ V is not the correct solution because we ignored boundary conditions. [assumed $v_c(0) = 7$ V]

→ we will continue on Wednesday.