EE100Su08 Lecture #9 (July 16th 2008)

- Outline
  - HW #1s and Midterm #1 returned today
    - Midterm #1 notes
    - HW #1 and Midterm #1 regrade deadline: Wednesday, July 23rd 2008, 5:00 pm PST. Procedure:
      - HW #1: Bart's office hours
      - Midterm #1: Attach a note to the FRONT of your test with your complaint and drop it in HW box
  - Questions?
  - This week: Operational Amplifiers (Op-Amps)
    - Op-Amp Model
    - Negative Feedback for Stability
    - Components around Op-Amp define the Circuit Function
    - Nonlinear circuits
    - Op-Amp from 2-Port Blocks

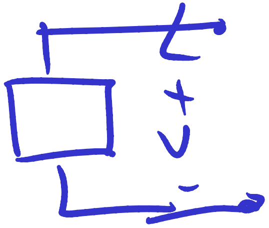
The Operational Amplifier

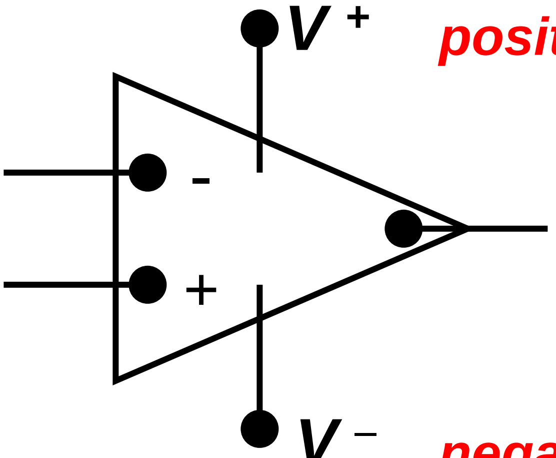
- The ***operational amplifier*** (“***op amp***”) is a basic building block used in analog circuits.
  - Its behavior is modeled using a dependent source.
  - When combined with resistors, capacitors, and inductors, it can perform various useful functions:
    - **amplification/scaling** of an input signal
    - **sign changing** (inversion) of an input signal
    - **addition** of multiple input signals
    - **subtraction** of one input signal from another
    - **integration** (over time) of an input signal
    - **differentiation** (with respect to time) of an input signal
    - **analog filtering**
    - **nonlinear functions** like exponential, log, sqrt, etc
  - Isolate input from output; allow cascading

Op Amp Terminals

- 3 signal terminals: 2 inputs and 1 output
- IC op amps have 2 additional terminals for DC power supplies
- Common-mode signal= $(v_1+v_2)/2$
- Differential signal = v_1-v_2

So far:





positive power supply

Inverting input v_2

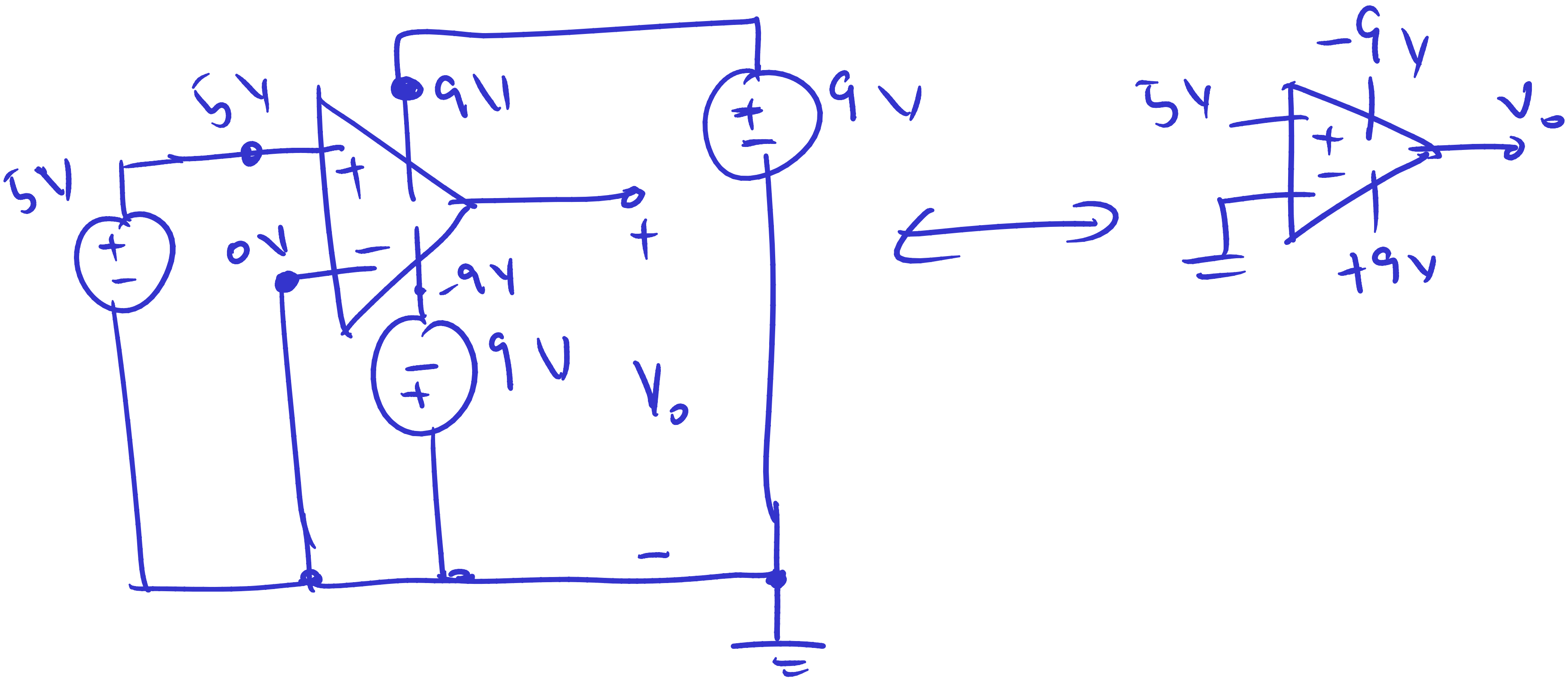
Non-inverting input v_1

v_0 output

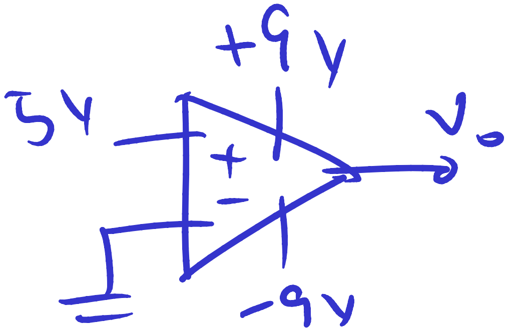
negative power supply

Op Amp "Notation" and Model

Ref: (1) Chapter 5 of your book] Read all of it!
(2) Intro. to Nonlinear Circuit Analysis]



Op Amp "Notation" and Model



$A = 10^6$

Assume op-amp is linear

$V_o = A(v_p - v_n)$

$= 10^6(5 - 0)$

$= 5$ MV

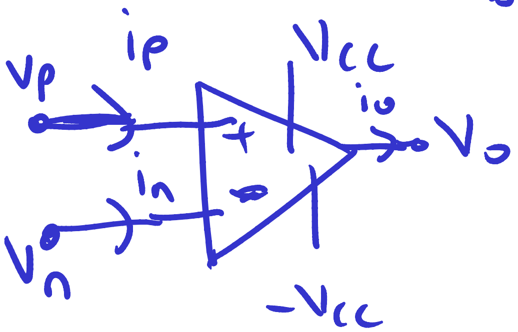
$V_o = 5$ MV > 9 V $= V_{cc}$

⇒ op-amp is railing at the positive

⇒ $\boxed{V_o = 9\text{ V}}$

[op-amp is not in linear region]

Model (p. 157 of book)



$(V_{cc} > 0)$

$V_o > V_{cc}$

⇕

$= V_{cc}$ if $v_p > v_n$

linear region

$\boxed{V_o = A(v_p - v_n) \text{ if } v_p \approx v_n}$

[$A \triangleq$ open-loop gain $\approx 10^6$ (HUGE)]

$= -V_{cc}$ if $v_p < v_n$

Op Amp "Notation" and Model

Consider: $v_o = A(v_p - v_n)$ — ①

For all practical circuits, $|v_o| << A$

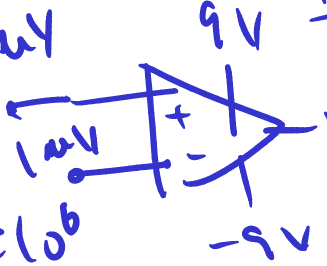
ex: $A = 10^6$, $v_o \in [9V, -9V]$

∴ ① ⇒ $v_p - v_n = \frac{v_o}{A}$

⇒ $v_p - v_n \approx 0$

⇒ $\boxed{v_p \approx v_n}$

ex:

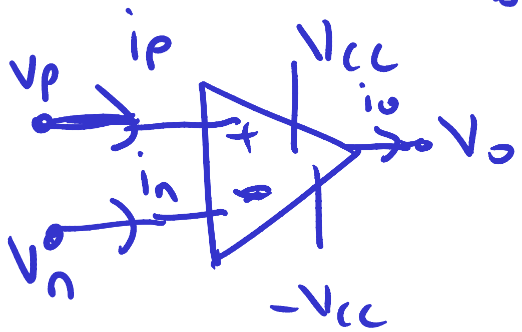


$A = 10^6$

$v_o = A(v_p - v_n)$

$= 10^6(5\mu V - 1\mu V) = 4V$

Model (p. 157 of book)



$(V_{cc} > 0)$

$v_o > V_{cc}$

⇕

$= V_{cc}$ if $v_p > v_n$

linear region

$\boxed{V_o = A(v_p - v_n) \text{ if } v_p \approx v_n}$

$A \triangleq$ open-loop gain $\approx 10^6$ (HUGE)

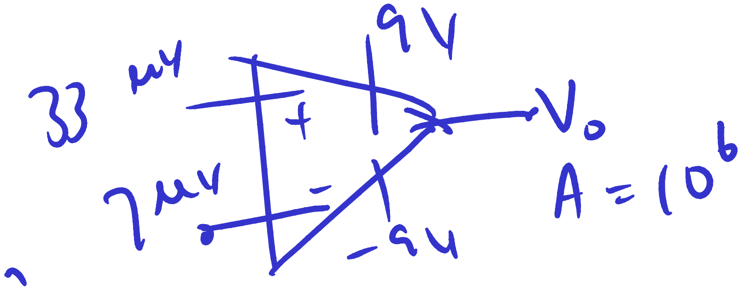
$= -V_{cc}$ if $v_p < v_n$

Op Amp “Notation” and Model

positive

+ve rail!

Assume op-amp is in linear region



$$V_o = A(v_p - v_n)$$

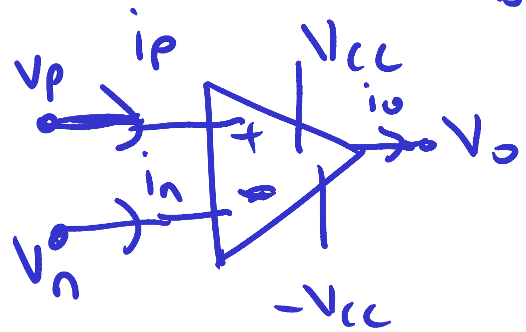
$$= 10^6(33\,\mu V - 7\,\mu V)$$

$$= 10^6(26\,\mu V)$$

$$= 26\,V \underline{\underline{>}} 9V = V_{cc}$$

$$\therefore \boxed{V_o = 9\,V}$$

Model (p. 157 of book)



$(V_{cc} > 0)$

$V_o > V_{cc}$

⇕

$= V_{cc}$ if $v_p > v_n$

linear region

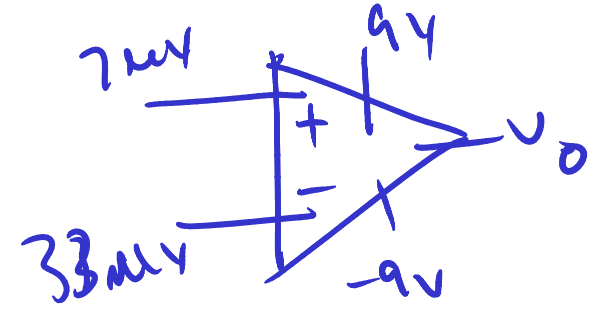
$$\boxed{V_o = A(v_p - v_n) \quad \text{if } v_p \approx v_n}$$

$A \triangleq$ open-loop gain $\approx 10^6$ (HUGE)

$= -V_{cc}$ if $v_p < v_n$

Op Amp "Notation" and Model

-ve rail: $A = 10^6$



Assume op-amp is linear.

$$v_o = A(v_p - v_n)$$

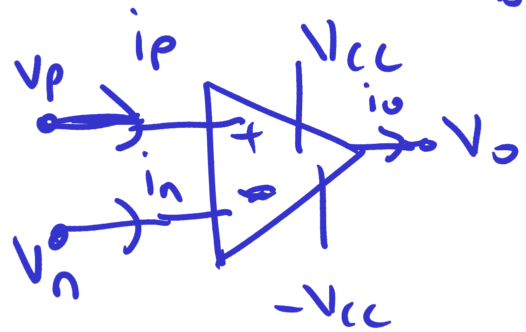
$$= 10^6(7mV - 33mV)$$

$$= -26V < -9V$$

$$\| \\ -V_{cc}$$

$$\therefore \boxed{v_o = -9V}$$

Model (p. 157 of book)



$(V_{cc} > 0)$

$v_o > V_{cc}$

$\Updownarrow$

$= V_{cc}$ if $v_p > v_n$

linear region

$\boxed{v_o = A(v_p - v_n) \text{ if } v_p \approx v_n}$

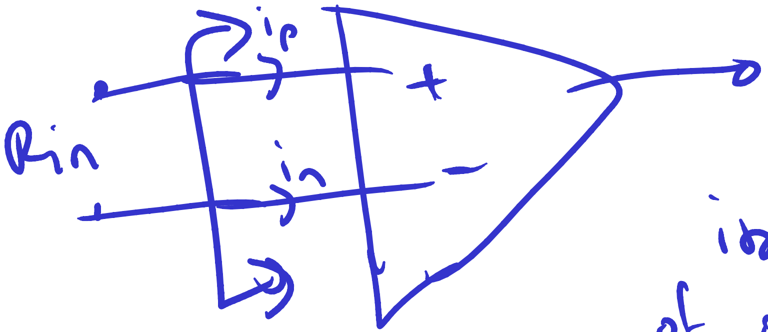
$A \triangleq$ open-loop gain $\approx 10^6$ (HUGE)

$= -V_{cc}$ if $v_p < v_n$

Op Amp "Notation" and Model

Op-amp model:

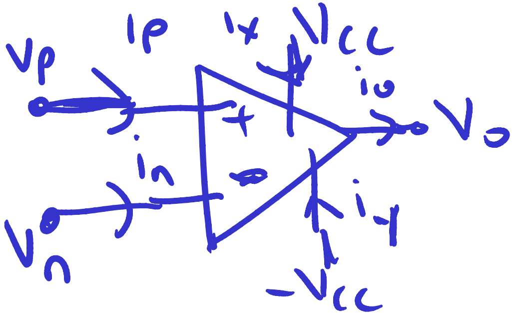
$i_p \approx i_n = 0$ always true irrespective of op-amp region of operation



But, i_x, i_y & i_o NEED NOT be zero!

(linear, +ve rail, -ve rail)

Model (p. 157 of book)



$(V_{CC} > 0)$

$V_o > V_{CC}$

$\Updownarrow$

$= V_{CC}$ if $v_p > v_n$

linear region

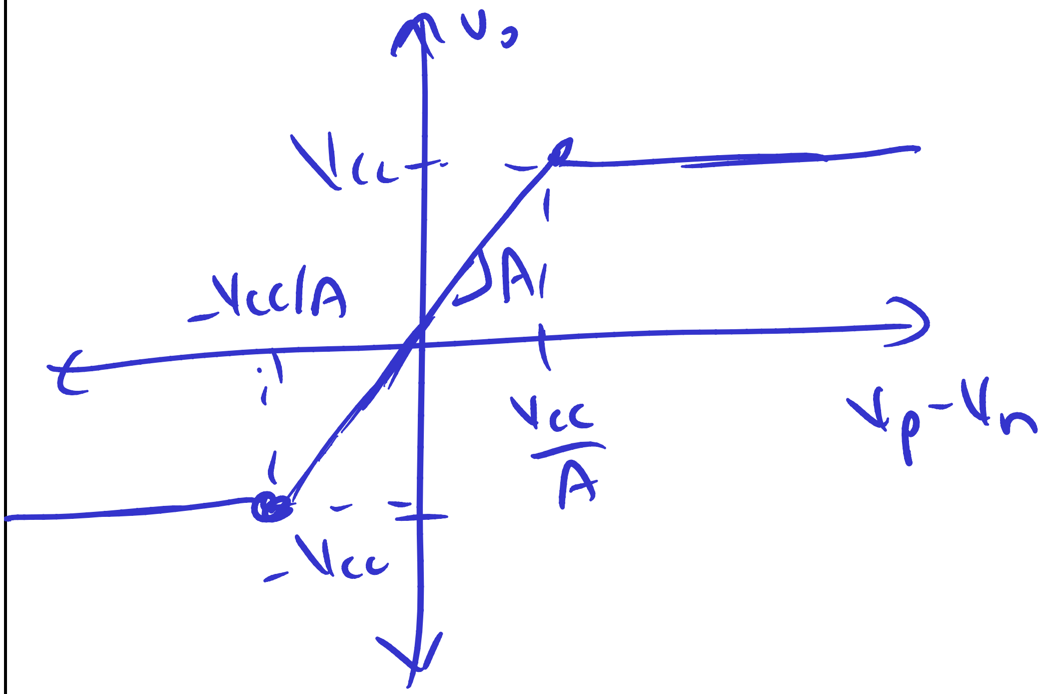
$$\boxed{V_o = A(v_p - v_n) \quad \text{if } v_p \approx v_n}$$

$A \triangleq$ open-loop gain $\approx 10^6$ (HUGE)

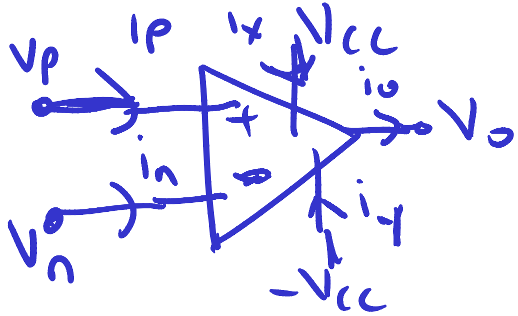
$= -V_{CC}$ if $v_p < v_n$

Op Amp "Notation" and Model

Voltage Transfer characteristic:



Model (p. 157 of book)



$(V_{cc} > 0)$

$V_o > V_{cc}$

⇕

$= V_{cc}$ if $v_p > v_n$

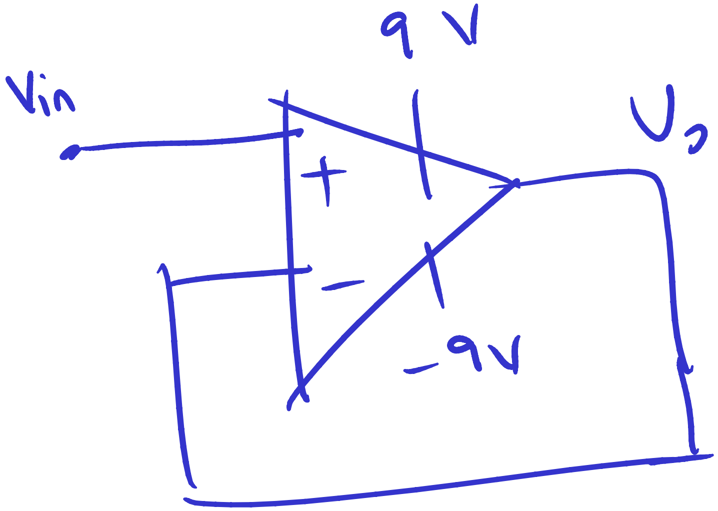
linear region

$$\boxed{V_o = A(v_p - v_n) \text{ if } v_p \approx v_n}$$

$A \triangleq$ open-loop gain $\approx 10^6$ (HUGE)

Book: $V_o = A(v_p - v_n)$ if $-V_{cc} < A(v_p - v_n) < V_{cc} \Rightarrow V_o = A(v_p - v_n)$ if $-\frac{V_{cc}}{A} < v_p - v_n < \frac{V_{cc}}{A}$

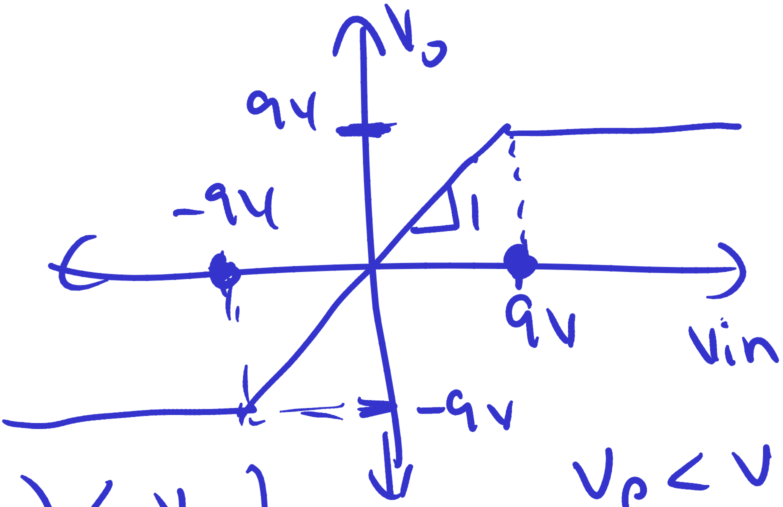
Useful op-amp circuits (1) Voltage follower.



[Negative feedback]

(Q:) Plot V_o vs V_{in}

[i.e. Voltage transfer characteristics]

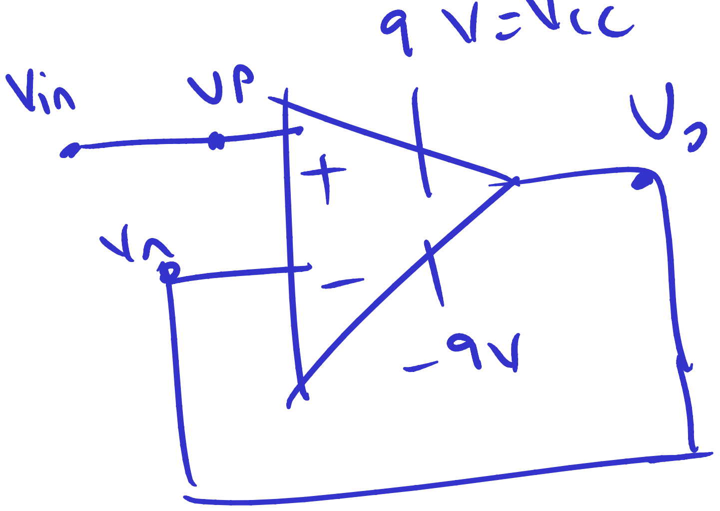


Book:

$$V_o = \begin{cases} -V_{cc} & A(v_p - v_n) < -V_{cc} \\ A(v_p - v_n) & v_p \approx v_n \\ V_{cc} & A(v_p - v_n) > V_{cc} \end{cases} \xrightarrow[A \to \infty]{} \begin{matrix} v_p < v_n \\ v_p = v_n \\ v_p > v_n \end{matrix}$$

↑ IDEALLY

Useful op-amp circuits (1) Voltage follower.



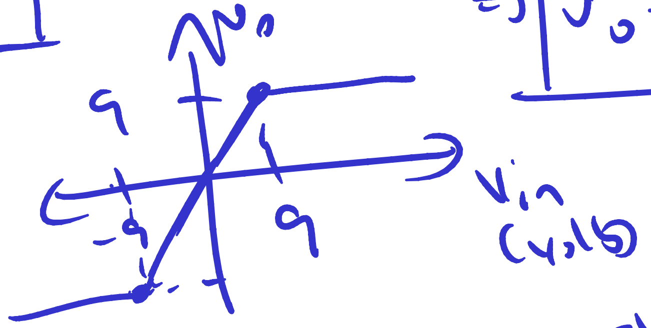
Assume op-amp is in +ve rail

$\Rightarrow V_o = 9\,V$ if $v_p > v_n$

$\Rightarrow V_o = 9\,V$ if $V_{in} > V_o$

$\Rightarrow \boxed{v_o = 9 \text{ if } v_{in} > 9}$

[Negative feedback]



Book:

$$V_o = \begin{cases} -V_{cc} & A(v_p - v_n) < -V_{cc} \\ A(v_p - v_n) & -V_{cc} < V_o < V_{cc} \\ V_{cc} & A(v_p - v_n) > V_{cc} \end{cases} \xrightarrow[A \to \infty]{}$$

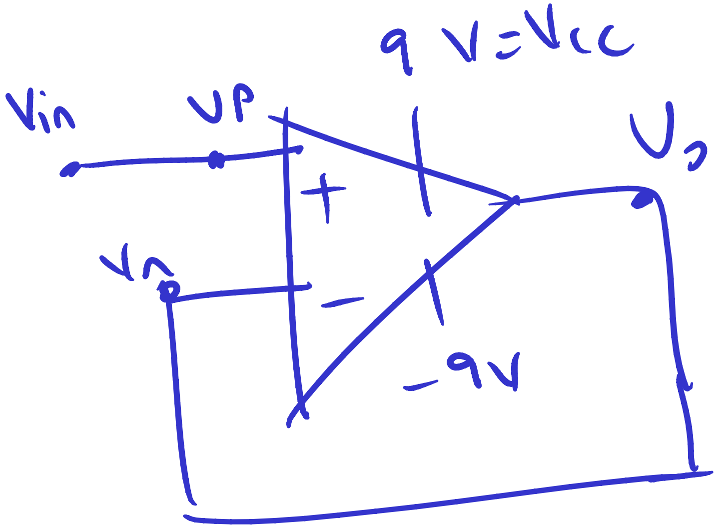
↑ IDEALLY

-ve rail ⇒ $v_p < v_n$

$v_p = v_n$

$v_p > v_n$ ← +ve rail

Useful op-amp circuits (1) Voltage follower.



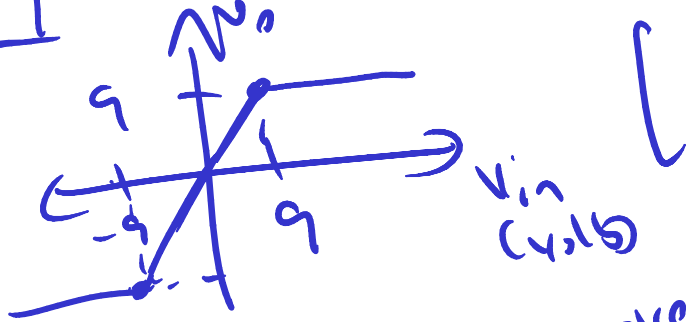
linear region: $V_o = A(v_p - v_n)$

$\Rightarrow v_p \approx v_n$

$\Rightarrow \boxed{V_{in} = V_o}$

$[-9 < V_o < 9]$

[Negative feedback]



Book:

$$V_o = \begin{cases} -V_{cc} & A(v_p - v_n) < -V_{cc} \\ A(v_p - v_n) & -V_{cc} < V_o < V_{cc} \\ V_{cc} & A(v_p - v_n) > V_{cc} \end{cases} \xrightarrow[A \to \infty]{}$$

$\uparrow$ IDEALLY

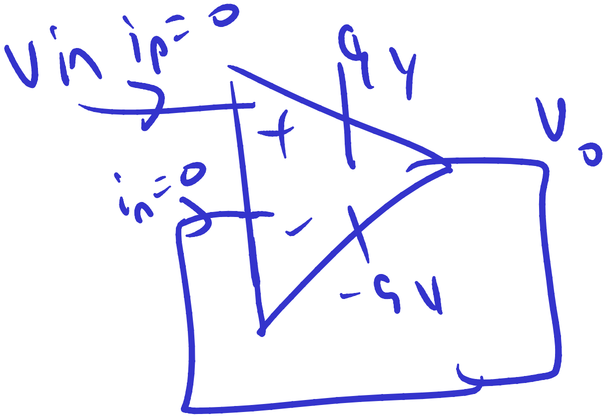
−ve rail $\Rightarrow v_p < v_n$

$v_p = v_n$

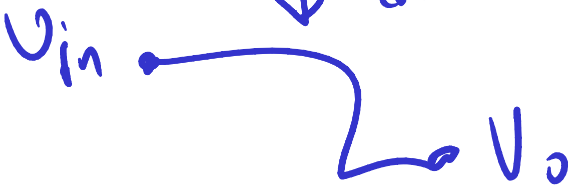
+ve rail $\rightarrow v_p > v_n$

Useful op-amp circuits (1) Voltage follower.

(Q:) How is this useful?

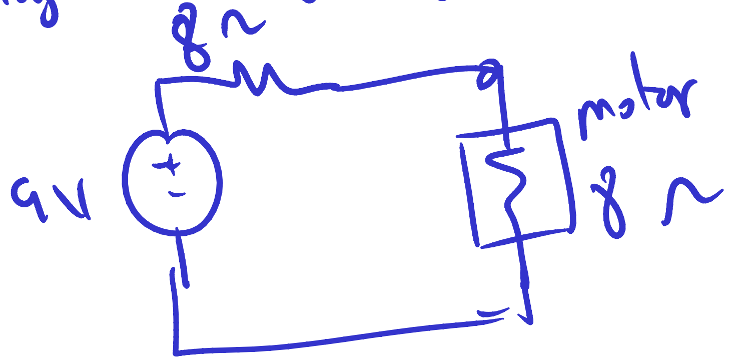


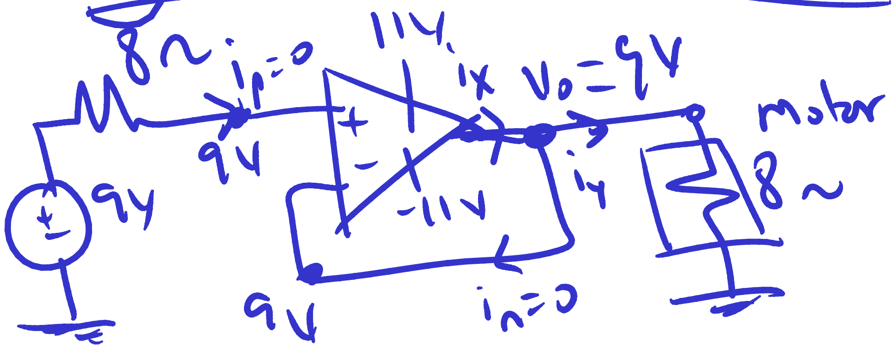
we could have done this!



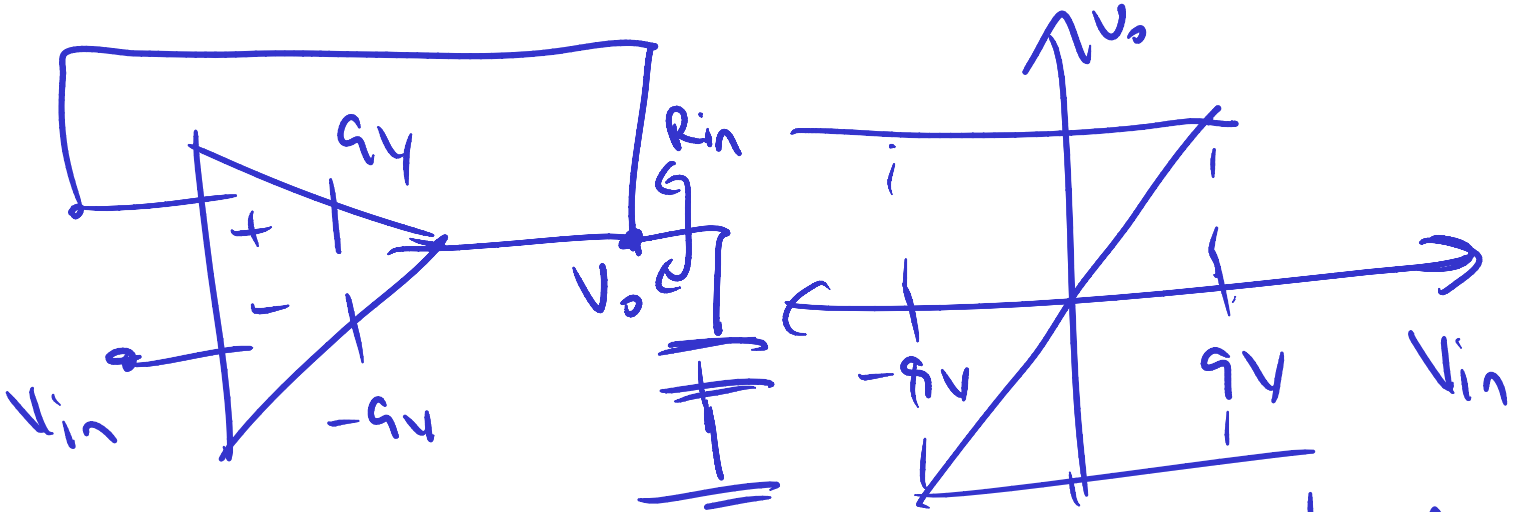
Useful because it avoids "loading" the source

ex:





Useful op-amp circuits (1) POSITIVE FEEDBACK



Assume op-amp is in +ve rail:

$V_o = 9V$ if $V_p > V_n$

$\Rightarrow V_o = 9V$ if $V_o > V_{in}$

$\Rightarrow V_o = 9V$ if $V_{in} < 9V$

Assume op-amp is in −ve rail

$V_o = -9V$ if $V_p < V_n$

$V_o = -9V$ if $V_o < V_{in}$

$\Rightarrow V_o = -9V$ if $V_{in} > -9$

Assume op-amp is saturated.

$V_o = V_{in}$ if

$-9V < V_o < 9$

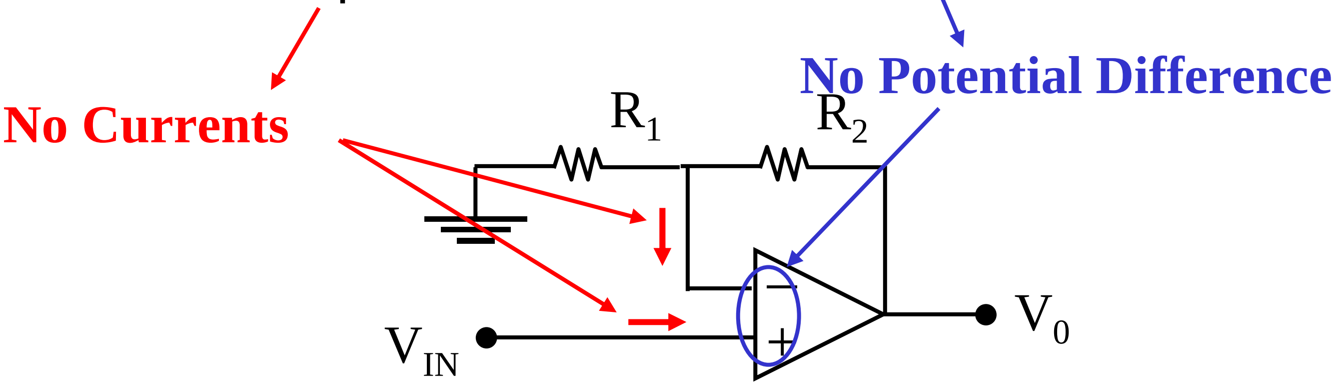
Summing-Point Constraint

- Check if under negative feedback
  - Small v_i result in large v_o
  - Output v_o is connected to the inverting input to reduce v_i
  - Resulting in $v_i=0$
- Summing-point constraint
  - $v_1= v_2$
  - $i_1 = i_2 =0$
- Virtual short circuit
  - Not only voltage drop is 0 (which is short circuit), input current is 0
  - This is different from short circuit, hence called "virtual" short circuit.

Ideal Op-Amp Analysis Technique

Assumption 1: The potential between the op-amp input terminals, $v_{(+)} - v_{(-)}$, equals zero.

Assumption 2: The currents flowing into the op-amp's two input terminals both equal zero.



No Currents

No Potential Difference

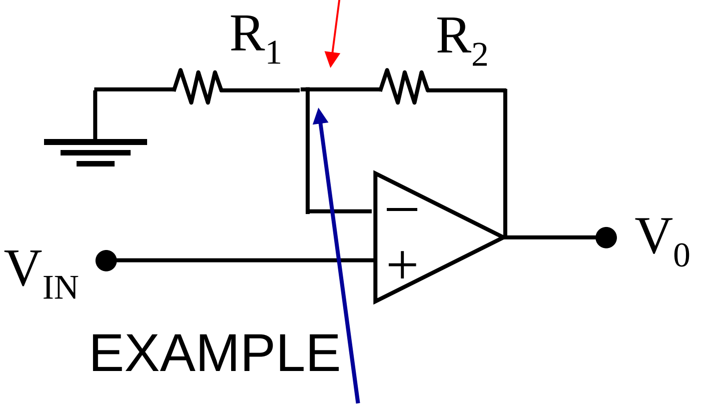
EXAMPLE

Ideal Op-Analysis: Non-Inverting Amplifier

Assumption 1: The potential between the op-amp input terminals, $v_{(+)} - v_{(-)}$, equals zero.

Assumption 2: The currents flowing into the op-amp's two input terminals both equal zero.

KCL with currents in only two branches



V_{IN} appears here

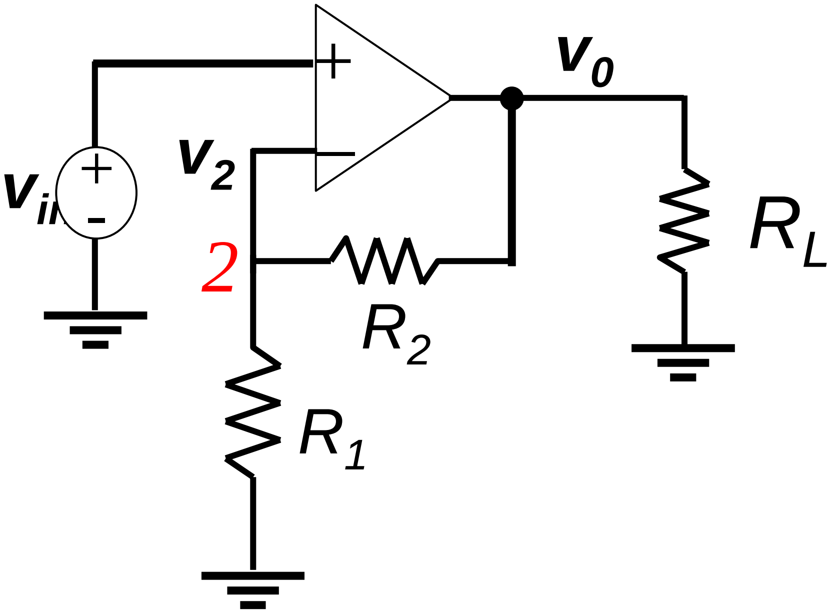
$$\frac{v_{in}}{R_1} + \frac{v_{in} - v_{out}}{R_2} = 0$$

$$v_{out} = \frac{R_1 + R_2}{R_1} v_{in}$$

Non-inverting Amplifier

Non-Inverting Amplifier

- Ideal voltage amplifier



$$Closed\ \ loop\ \ gain = A_v = \frac{v_o}{v_{in}}$$

$$v_1 = v_2 = v_{in}\ ,i_1 = i_2 = 0$$

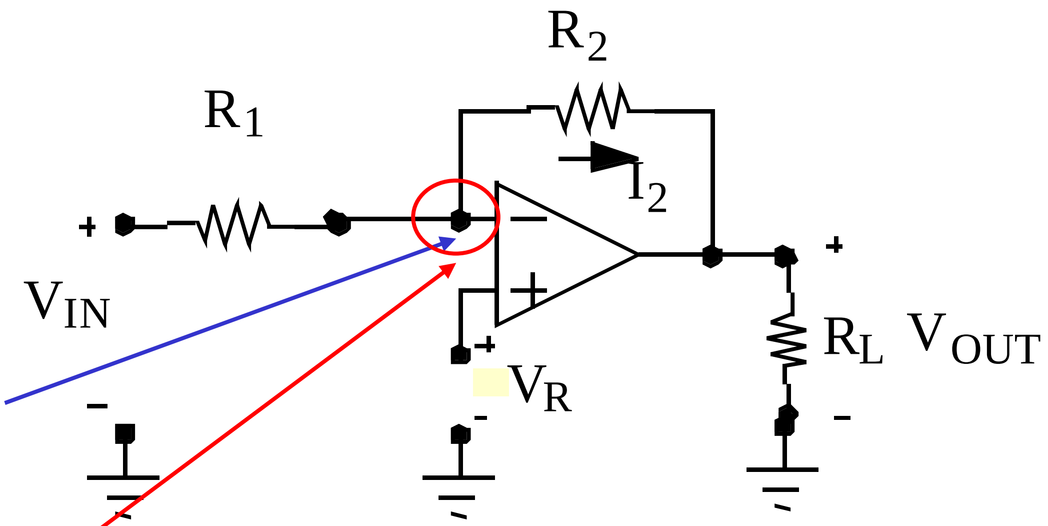
$$Use\ \ KCL\ \ At\ \ Node\ \ 2.$$

$$i = \frac{(v_0 - v_2)}{R_2} = \frac{(v_2 - 0)}{R_1}$$

$$A = \frac{v_o}{v_{in}} = \frac{(R_1 + R_2)}{R_1}$$

$$Input\ \ impedance = \frac{v_{in}}{i} \to \infty$$

Ideal Op-Amp Analysis: Inverting Amplifier



Voltage is V_R

Only two currents for KCL

$$\frac{V_R - V_{IN}}{R_1} + \frac{V_R - V_{OUT}}{R_2} = 0$$

$$V_{OUT} = V_R - \frac{R_2}{R_1}\left(V_{in} - V_R\right)$$

Inverting Amplifier with reference voltage

Inverting Amplifier

- Negative feedback → checked
- Use summing-point constraint

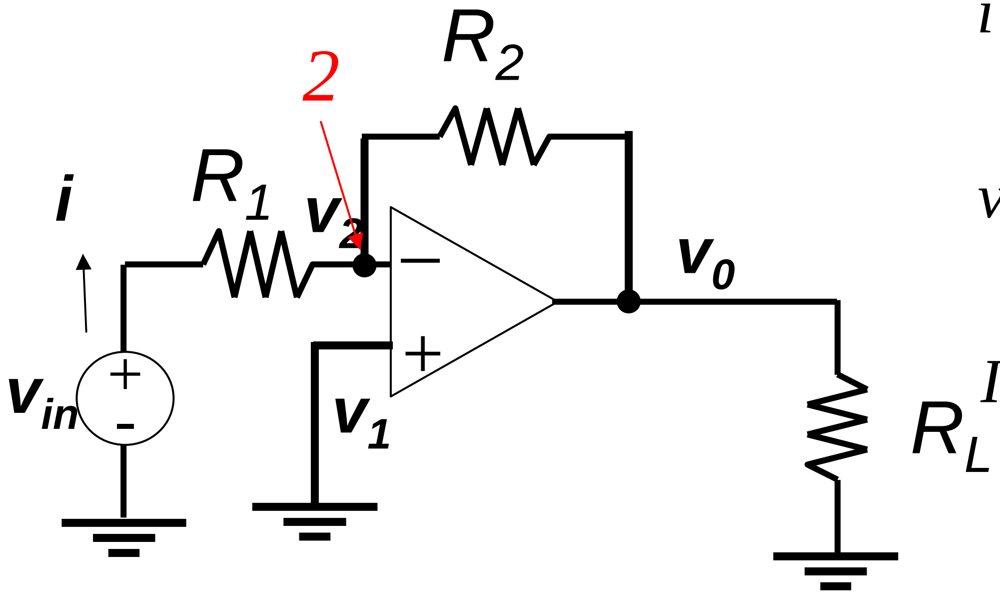
$$Closed\ \ loop\ \ gain = A_v = \frac{v_o}{v_{in}}$$

$$v_1 = v_2 = 0\ \ , i_1 = i_2 = 0$$

$$Use\ \ KCL\ \ At\ \ Node\ \ 2.$$

$$i = \frac{(v_{in} - v_2)}{R_1} = \frac{(v_{out} - v_2)}{R_2}$$

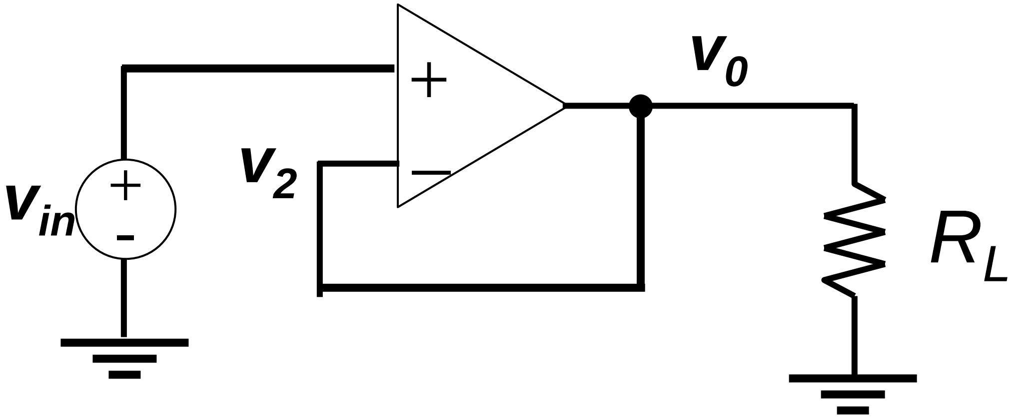
$$v_o = -\frac{R_2 v_o}{R_1}$$



$$Input\ \ impedance = \frac{v_{in}}{i} = R_1$$

Ideal voltage source – independent of load resistor

Voltage Follower



$$R_2 = 0$$

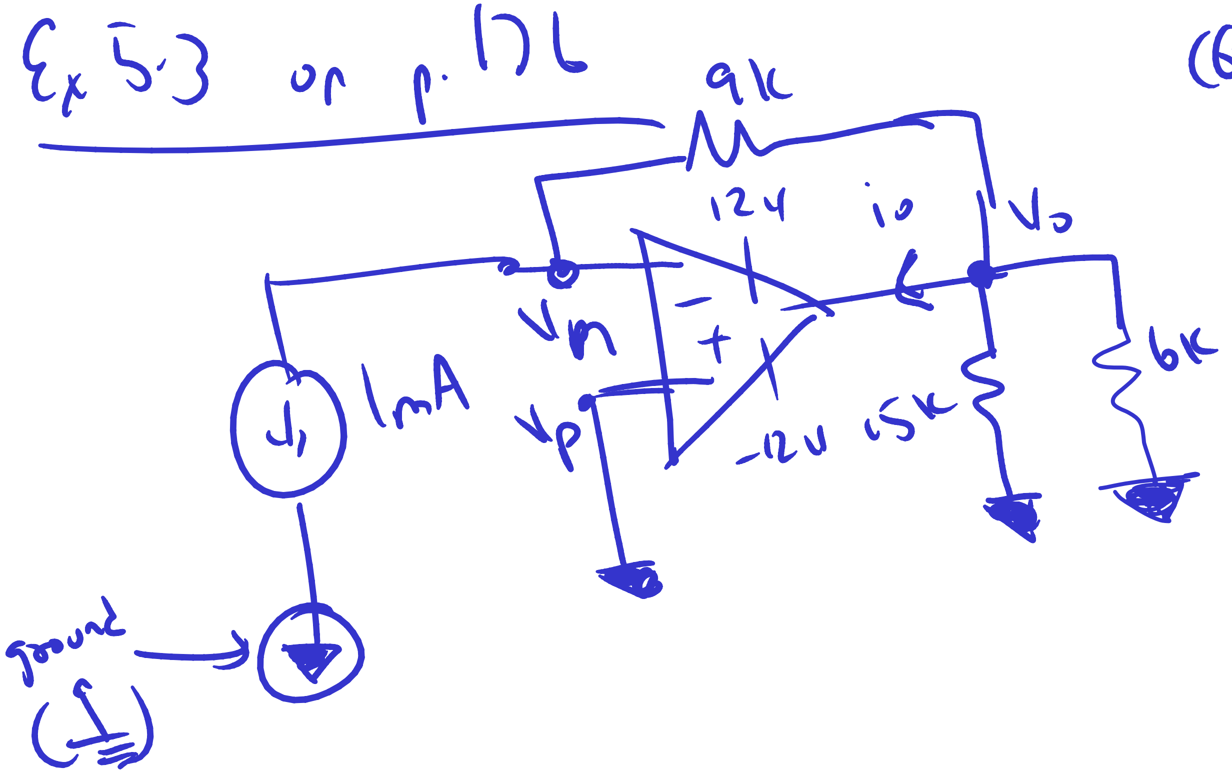
$$R_1 \to \infty$$

$$i = \frac{(v_0 - v_2)}{R_2} = \frac{(v_2 - 0)}{R_1}$$

$$A = \frac{v_o}{v_{in}} = \frac{(R_1 + R_2)}{R_1} = 1 + \frac{R_2}{R_1} = 1$$

Op-amps in negative feedback:

Ex 5.3 on p. 176



(Q:) Find i_o

Rule of thumb for −ve feedback:

(1) Use summing point constraint:

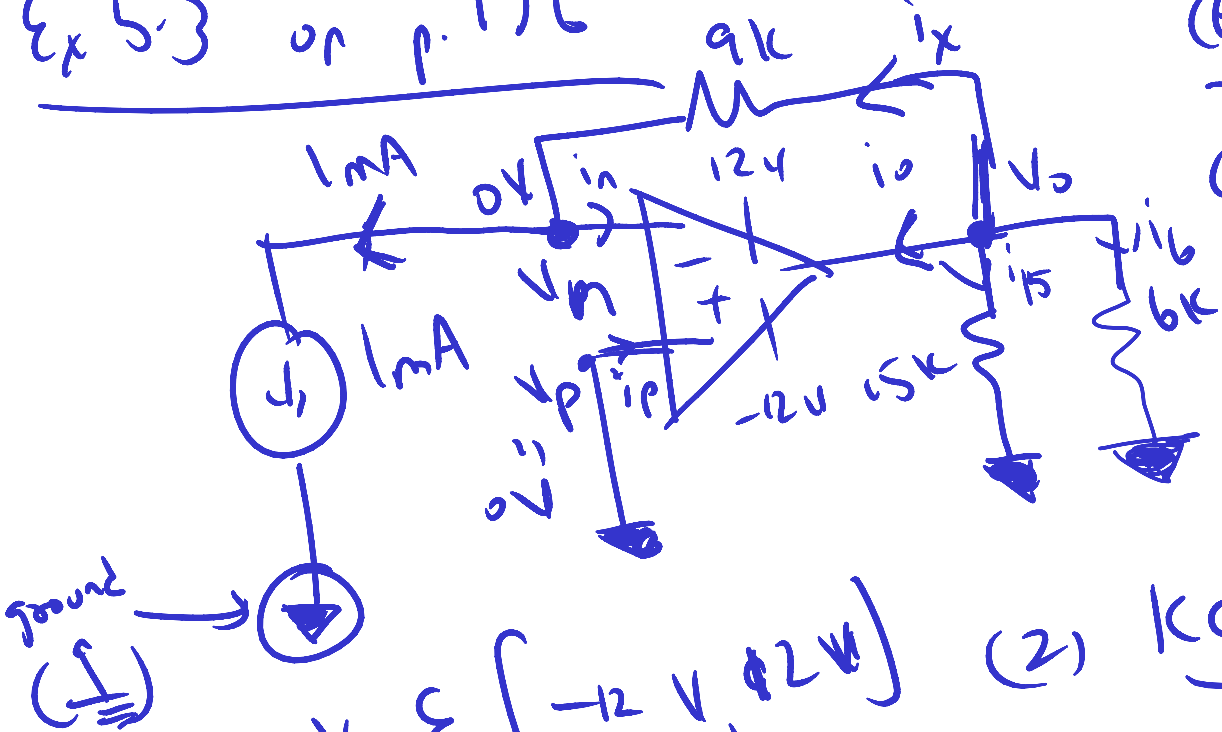
(i) Assume op-amp is linear: $V_p = V_n$ &

Check if

$-V_{cc} < V_o < V_{cc}$

Op-amps in negative feedback:

Ex 5.3 on p. 176



(Q:) Find i_o

(1) Assume: $V_p = V_n = 0V$

$i_n = i_p = 0$

(2) KCL @ V_n:

$1mA + \cancel{i_n}^{0} = i_x$

$\Rightarrow 1mA = \frac{V_o - \cancel{V_n}^{0}}{9K}$

$\Rightarrow \boxed{V_o = 9V}$

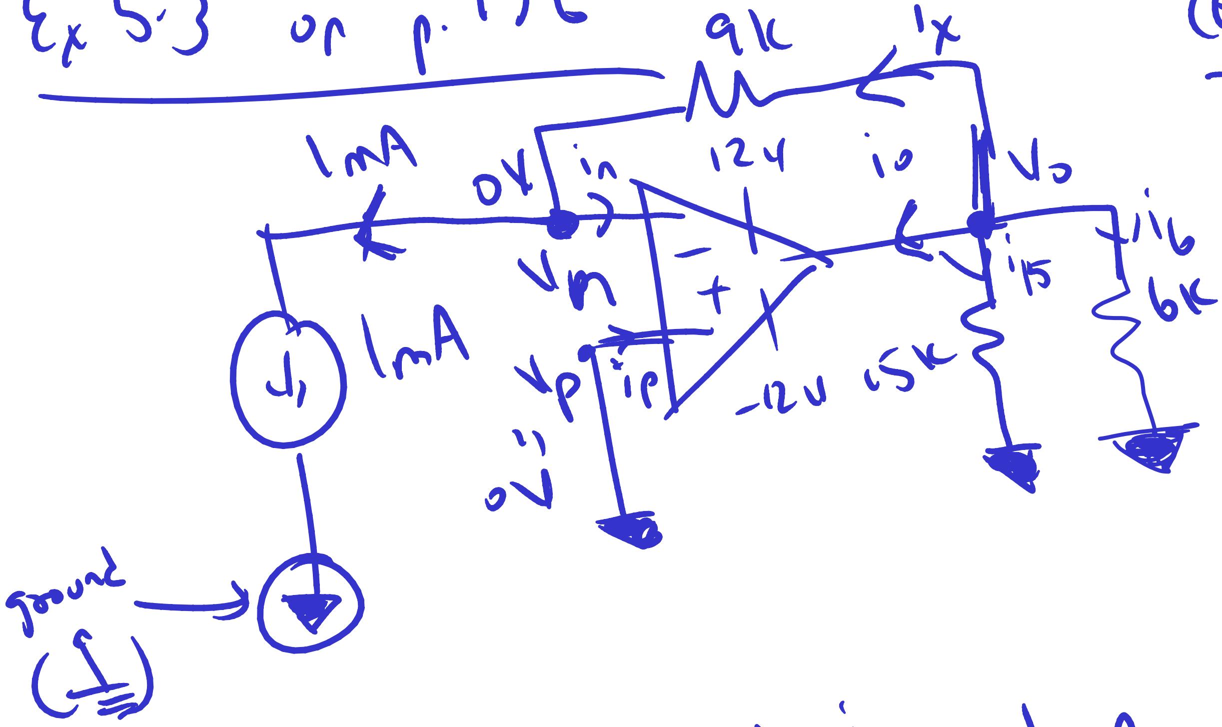
Notice: $V_o \in [-12 V, 12 V]$ ✓

$\Rightarrow V_p = V_n = 0 V$

Op-amps in negative feedback:

Ex 5.3 on p. 176

(Q:) Find i_o



KCL @ V_o:

$i_o + i_x + i_{15} + i_6 = 0$

$\Rightarrow i_o + 1mA + \frac{V_o}{15k} + \frac{V_o}{6k} = 0$

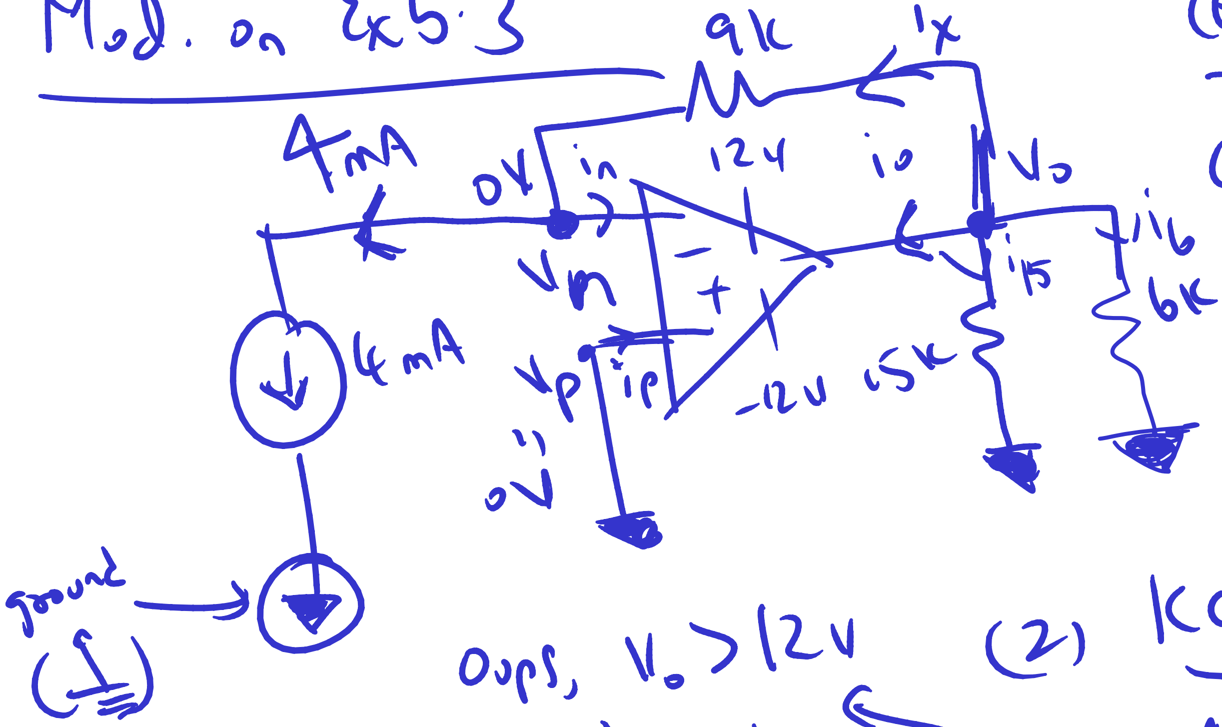
$\Rightarrow i_o = -1mA - \frac{9}{15} mA - \frac{9}{6} mA$

$= -1 ma - 0.6 mA - 1.5 mA$

$\boxed{i_o = -3.1 \text{ mA}}$

Op-amps in negative feedback:

Mod. on Ex 5.3



(Q:) Find i_o

(1) Assume: $V_p = V_n = 0V$

$i_n = i_p = 0$

(2) KCL @ V_n:

$$4mA + \overset{0}{\cancel{i_n}} = i_x$$

$$4mA = \frac{V_o - \overset{0}{\cancel{V_n}}}{9K}$$

$\Rightarrow \boxed{V_o = 36 V}$

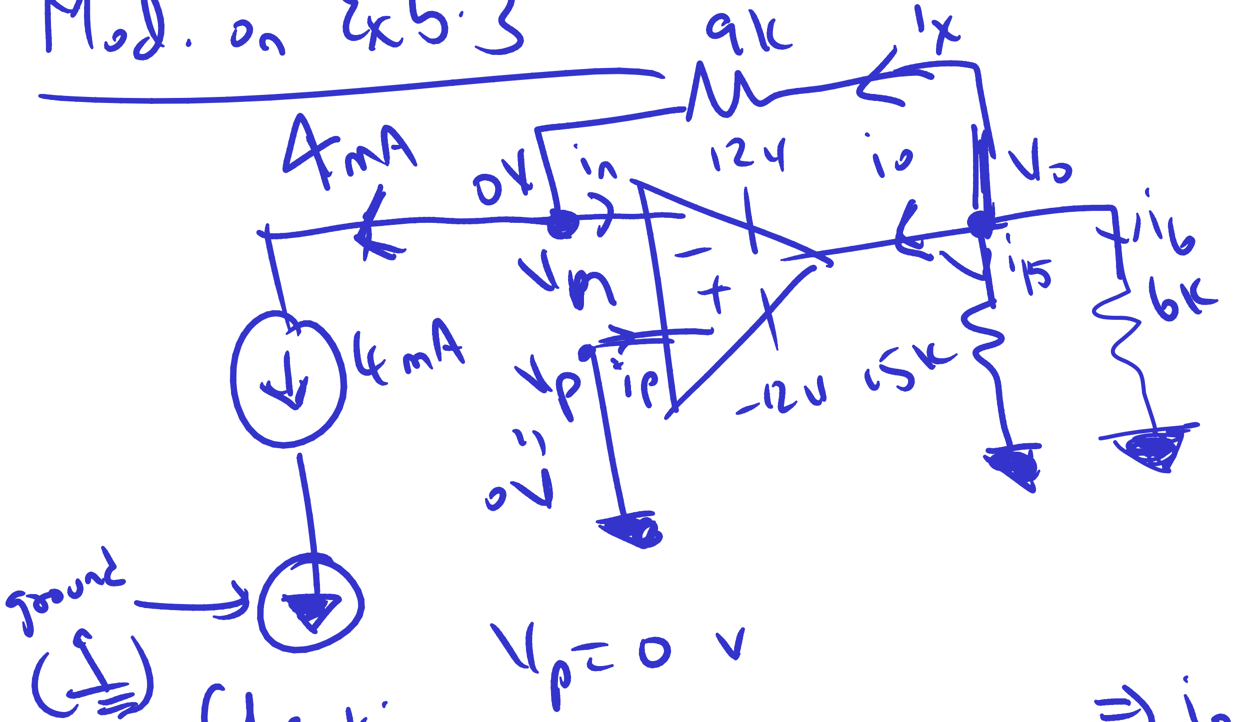
Oops, $V_o > 12V$

⇒ our assumption that $V_p = V_n = 0$ is wrong!

$\Rightarrow \boxed{V_o = 12 V}$

Opamps in negative feedback:

Mod. on Ex5.3



$V_o = 12\ V$

∴ KCL @ V_o:

$i_o + i_x + i_{15} + i_6 = 0$

$\Rightarrow i_o = -4mA - \frac{12}{15}mA - \frac{12}{6}mA$

$\Rightarrow i_o = 4mA - 0.8mA - 2mA$

$\boxed{i_o = -6.8\ mA}$

Check:

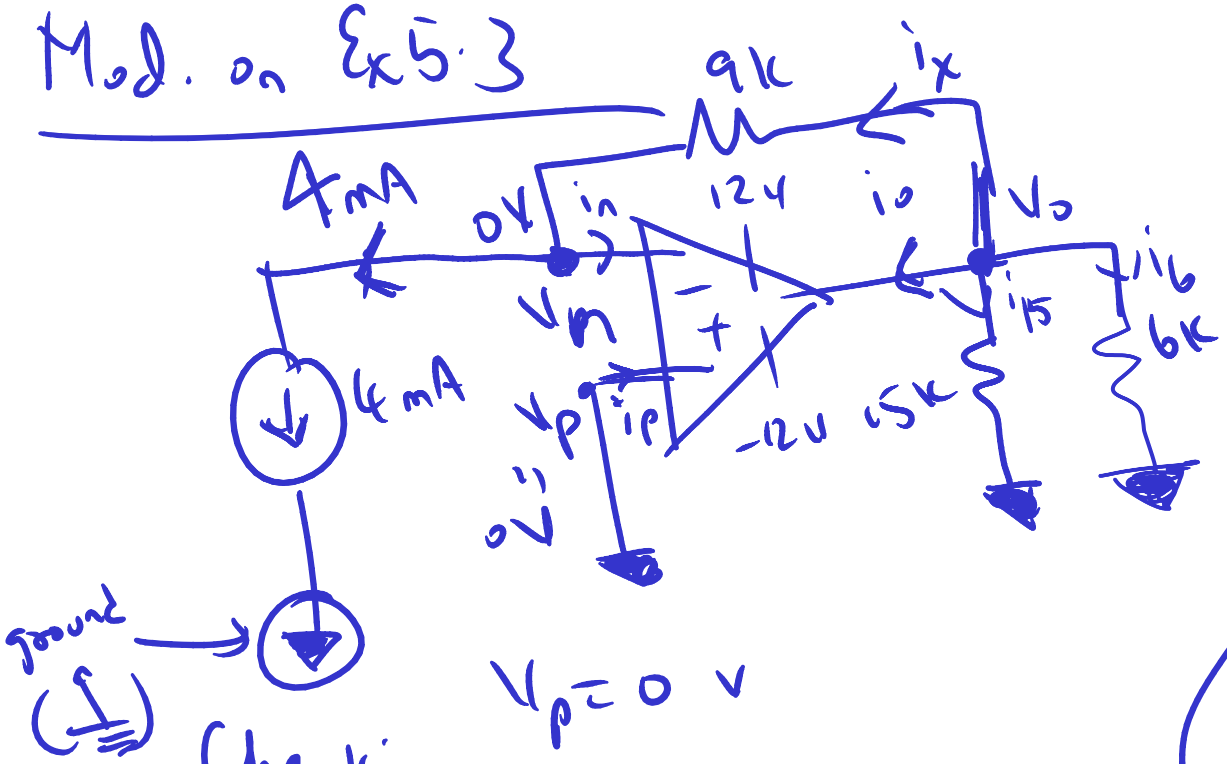
$V_p = 0\ V$

$i_x = \frac{V_o - V_n}{9k}$

$\Rightarrow (4\ mA)(9\ k) = V_o - V_n \Rightarrow V_n = 12 - 36 = \underline{\underline{-24V}}$

Op-amps in negative feedback:

Mod. on Ex 5.3



Check:

$V_p = 0\ v$

$$i_x = \frac{V_o - V_n}{9k}$$

$\Rightarrow (4\ mA)(9\ k) = V_o - V_n \Rightarrow V_n = 12 - 36 = \underline{\underline{-24V}}$

$\Rightarrow \therefore V_o = \overset{\text{open-loop gain}}{A}(v_p - v_n)$

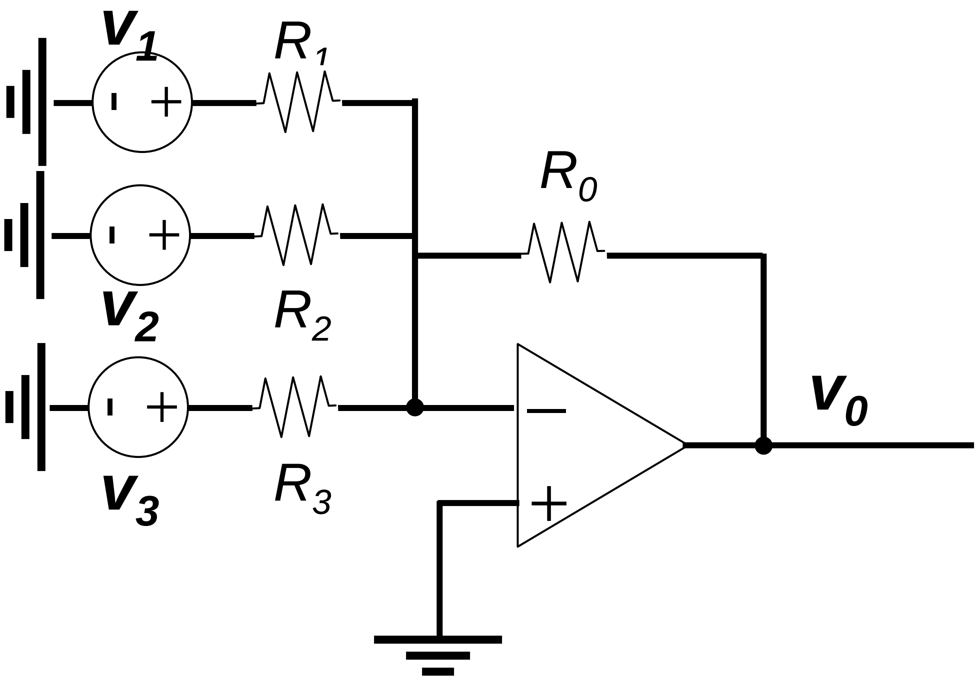
$= 10^6(0 - (-24))$

$= 10^6 \cdot 24$

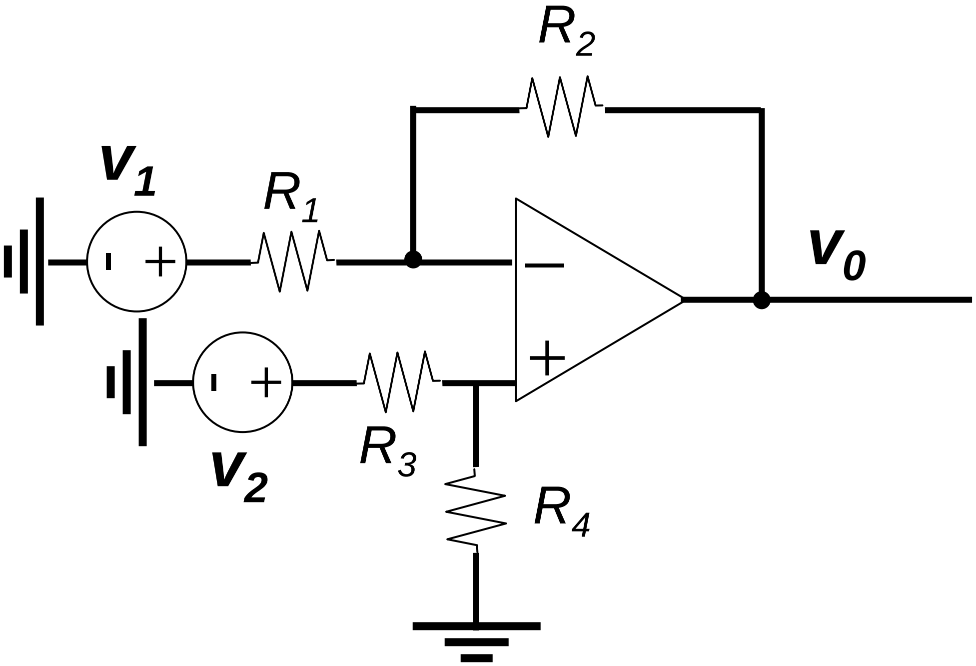
$\downarrow$

really big +ve number

Summing Amplifier

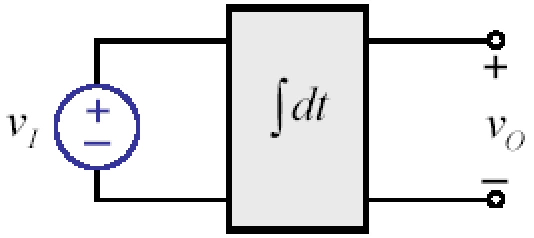


Difference Amplifier

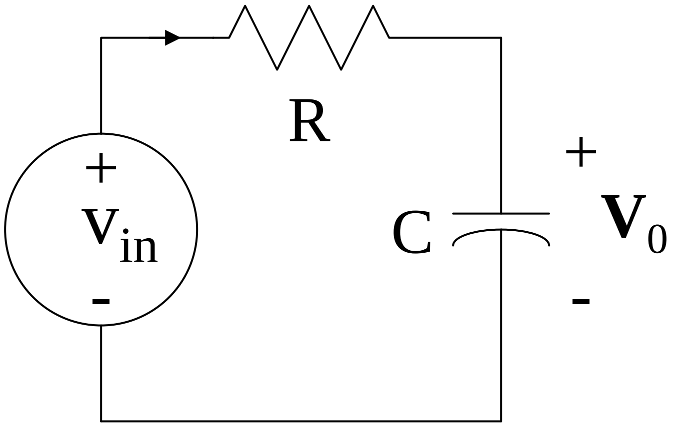


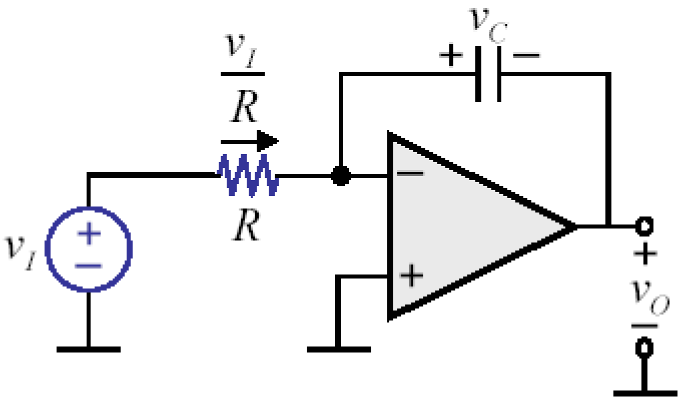
Integrator

- Want $v_o = K\int v_{in}dt$



- What is the difference between:



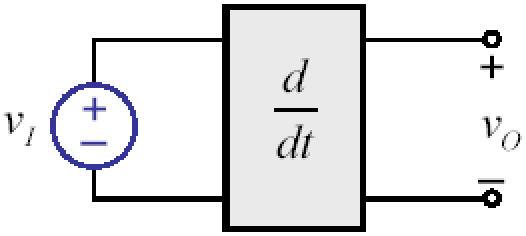


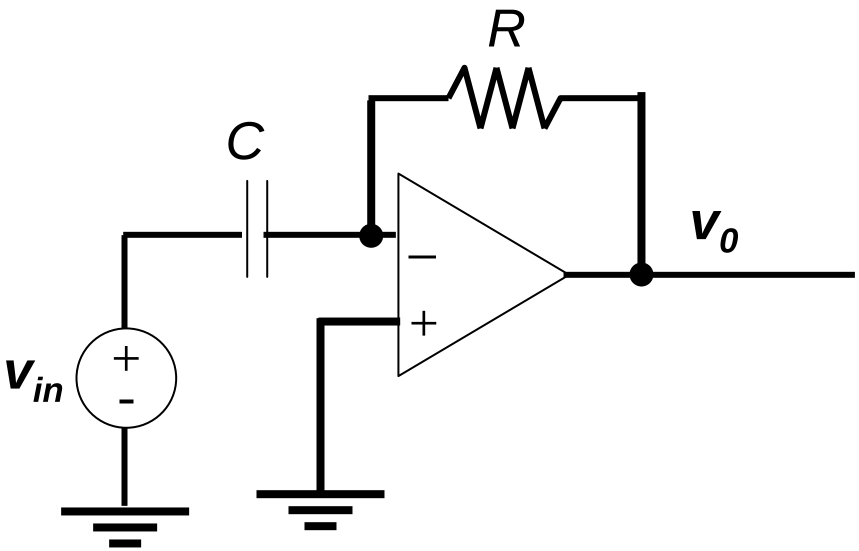
$$v_O \approx \frac{1}{RC}\int_{-\infty}^{t} v_I \, dt$$

$$v_O = -\frac{1}{C}\int_{-\infty}^{t} \frac{v_I}{R} dt$$

Differentiator

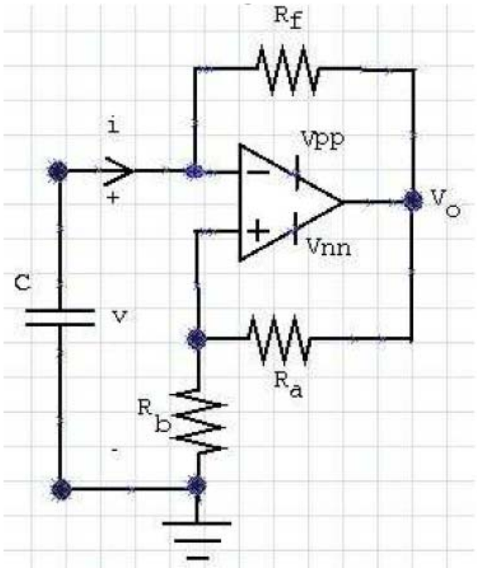
- Want





Nonlinear Opamp Circuits

- Start reading through online notes: “Introduction to nonlinear circuit analysis”.
- Outline:
  - Differences between positive and negative feedback.
  - Oscillator circuit.

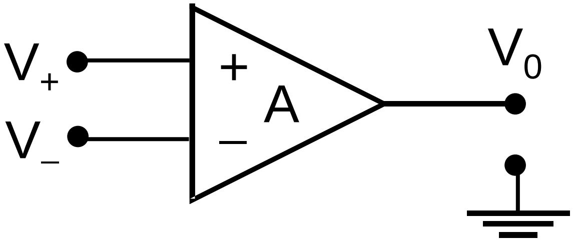


High Quality Dependent Source In an Amplifier

AMPLIFIER SYMBOL

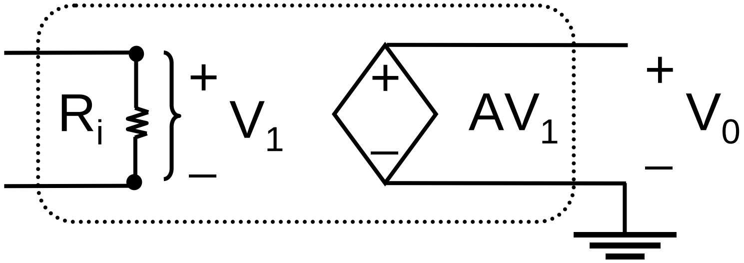
Differential Amplifier

$$V_0 = A(V_+ - V_-)$$



AMPLIFIER MODEL

Circuit Model *in linear region*



V_0 depends only on input $(V_+ - V_-)$

See the utility of this: this Model when used correctly mimics the behavior of an amplifier but omits the complication of the many many transistors and other components.

Model for Internal Operation

- A is differential gain or open loop gain
- Ideal op amp

$$A \to \infty$$

$$R_i \to \infty$$

$$R_o = 0$$

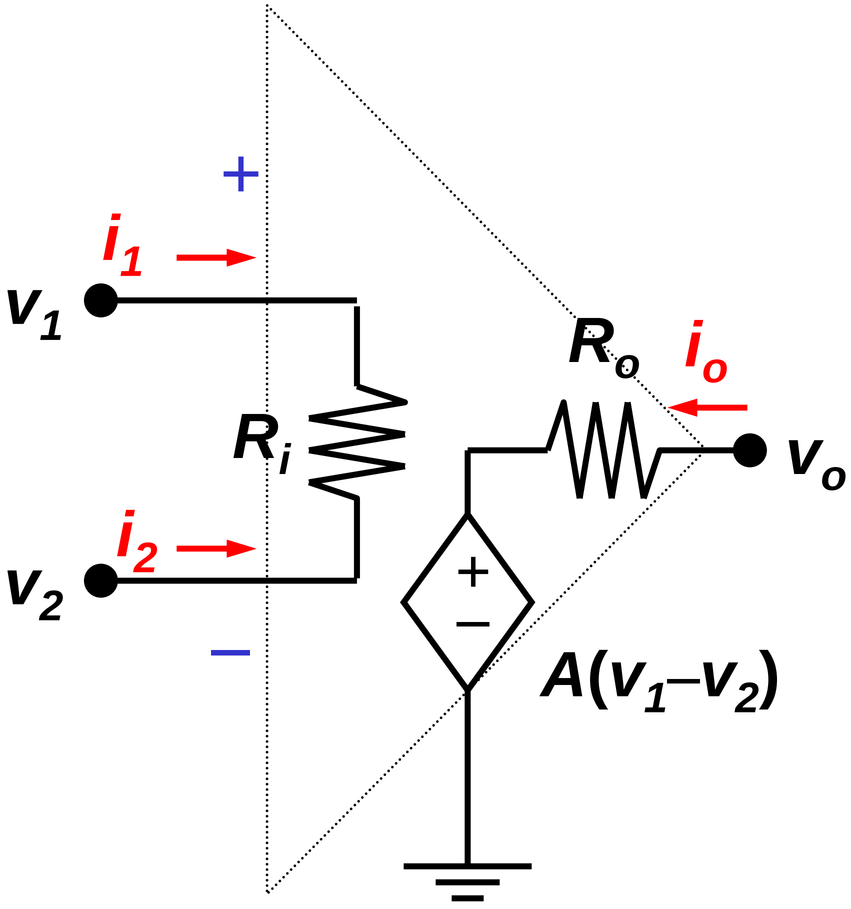
  - Common mode gain = 0

$$v_{cm} = \frac{(v_1 + v_2)}{2} \quad , v_d = v_1 - v_2$$

$$v_o = A_{cm} v_{cm} + A_d v_d$$

$$Since \quad v_o = A(v_1 - v_2) \quad , A_{cm} = 0$$

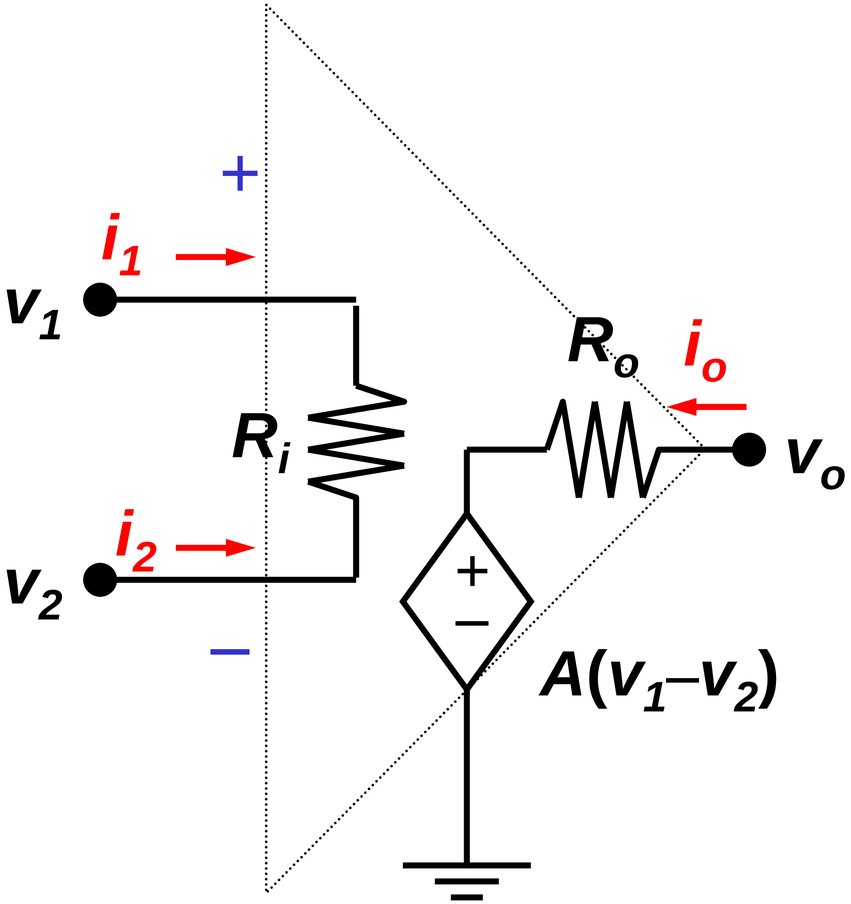
- Circuit Model



Model and Feedback

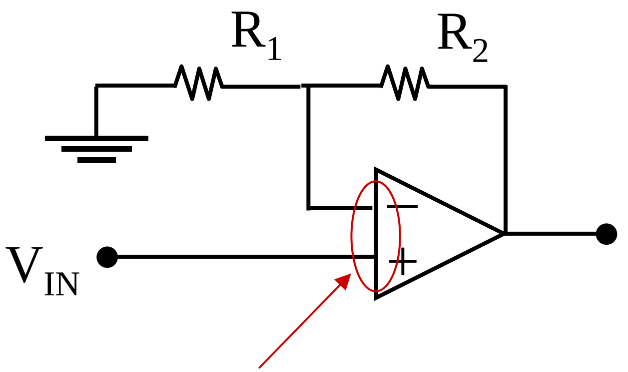
- Negative feedback
  - connecting the output port to the negative input (port 2)
- Positive feedback
  - connecting the output port to the positive input (port 1)
- Input impedance: R looking into the input terminals
- Output impedance: Impedance in series with the output terminals

- Circuit Model

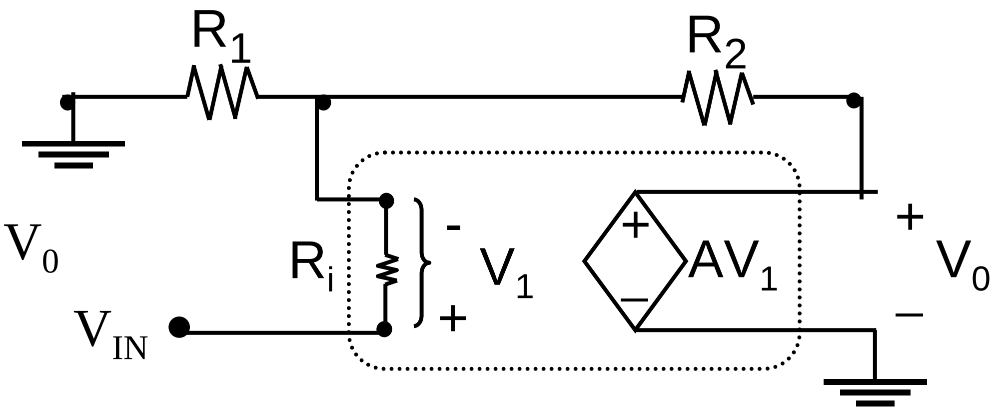


Op-Amp and Use of Feedback

A very high-gain differential amplifier can function in an extremely linear fashion as an operational amplifier by using negative feedback.



Summing Point



Circuit Model

Negative feedback ⇒ **Stabilizes** the output

Hambley Example pp. 644 for Power Steering

We can show that that for $A \rightarrow \infty$ and $R_i \rightarrow \infty$,

$$V_0 \cong V_{IN} \cdot \frac{R_1 + R_2}{R_1}$$

Stable, finite, and independent of the properties of the OP AMP !

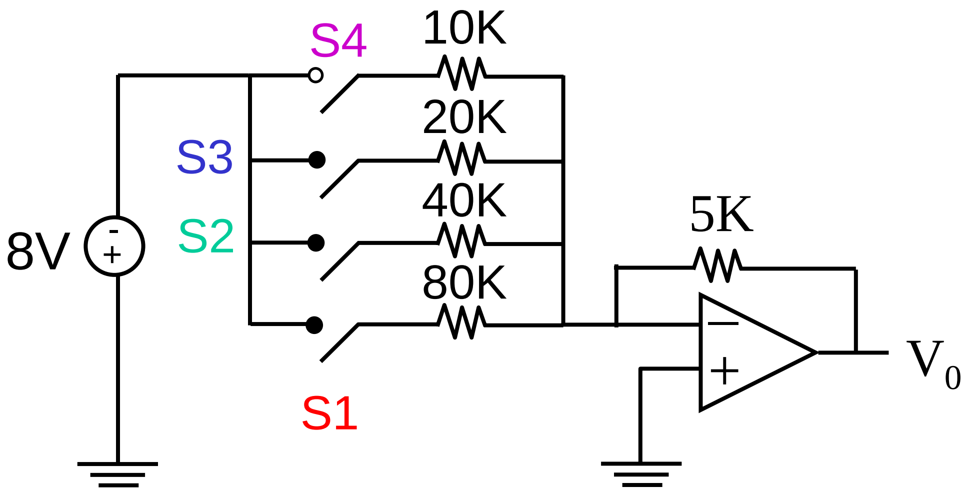
Application: Digital-to-Analog Conversion

A DAC can be used to convert the digital representation of an audio signal into an analog voltage that is then used to drive speakers -- so that you can hear it!

Binary number	Analog output (volts)
0 0 0 0	0
0 0 0 1	.5
0 0 1 0	1
0 0 1 1	1.5
0 1 0 0	2
0 1 0 1	2.5
0 1 1 0	3
0 1 1 1	3.5
1 0 0 0	4
1 0 0 1	4.5
1 0 1 0	5
1 0 1 1	5.5
1 1 0 0	6
1 1 0 1	6.5
1 1 1 0	7
1 1 1 1	7.5

↑ MSB ↑ LSB

“Weighted-adder D/A converter”



4-Bit D/A

(Transistors are used as electronic switches)

S1 closed if LSB =1
S2 " if next bit = 1
S3 " if " " = 1
S4 " if MSB = 1

Characteristic of 4-Bit DAC