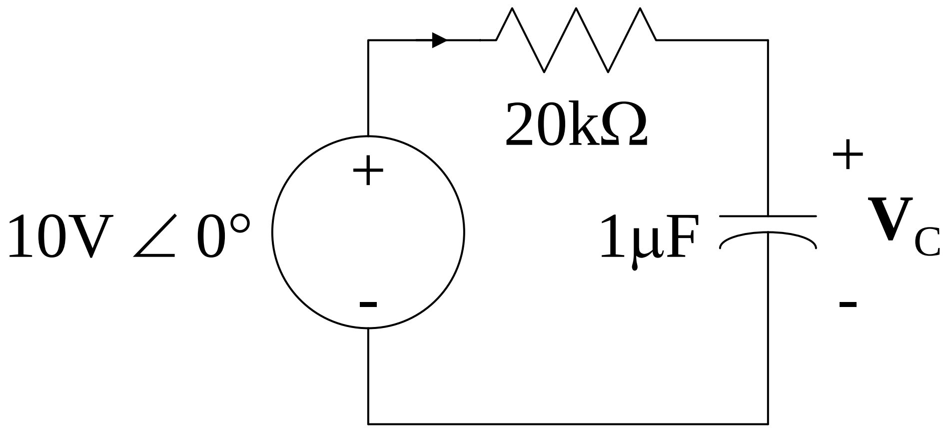
EE100Su08 Lecture #12 (July 23rd 2008)

- Outline
  - MultiSim licenses on Friday
  - HW #1, Midterm #1 regrade deadline: TODAY (5:00 pm PST).
  - QUESTIONS?
  - Circuit analysis using complex impedances: Examples
  - Frequency Response and Bode plots
- Reading
  - Chapter 9 from your book (skip 9.10, 9.11 (duh)), Appendix E* (skip second-order resonance bode plots)
  - Chapter 1 from your reader (skip second-order resonance bode plots)

Example: Single Loop Circuit



f=60 Hz, $\mathbf{V}_C$=?

$\omega = 377$ rad/sec

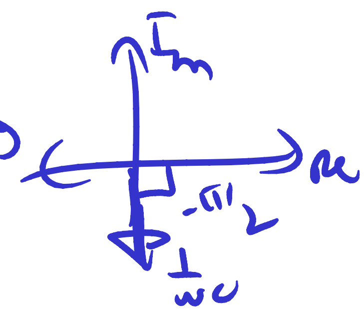
How do we find $\mathbf{V}_C$?

First compute impedances for resistor and capacitor:

$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°

$\mathbf{Z}_C$ = 1/j (2πf x 1μF) = 2.65kΩ ∠ -90°

$\frac{1}{j\omega C} = \frac{j}{j^2 \omega C} = \frac{-j}{\omega C} = 0 + \frac{-j}{\omega C}$



Circuit Analysis Using Complex Impedances

- Suitable for AC steady state.
- KVL

$$v_1(t) + v_2(t) + v_3(t) = 0$$

$$V_1 \cos\left(\omega t + \theta_1\right) + V_2 \cos\left(\omega t + \theta_2\right) + V_3 \cos\left(\omega t + \theta_3\right) = 0$$

$$\mathrm{Re}\left[V_1 e^{j(\omega t + \theta_1)} + V_2 e^{j(\omega t + \theta_2)} + V_3 e^{j(\omega t + \theta_3)}\right] = 0$$

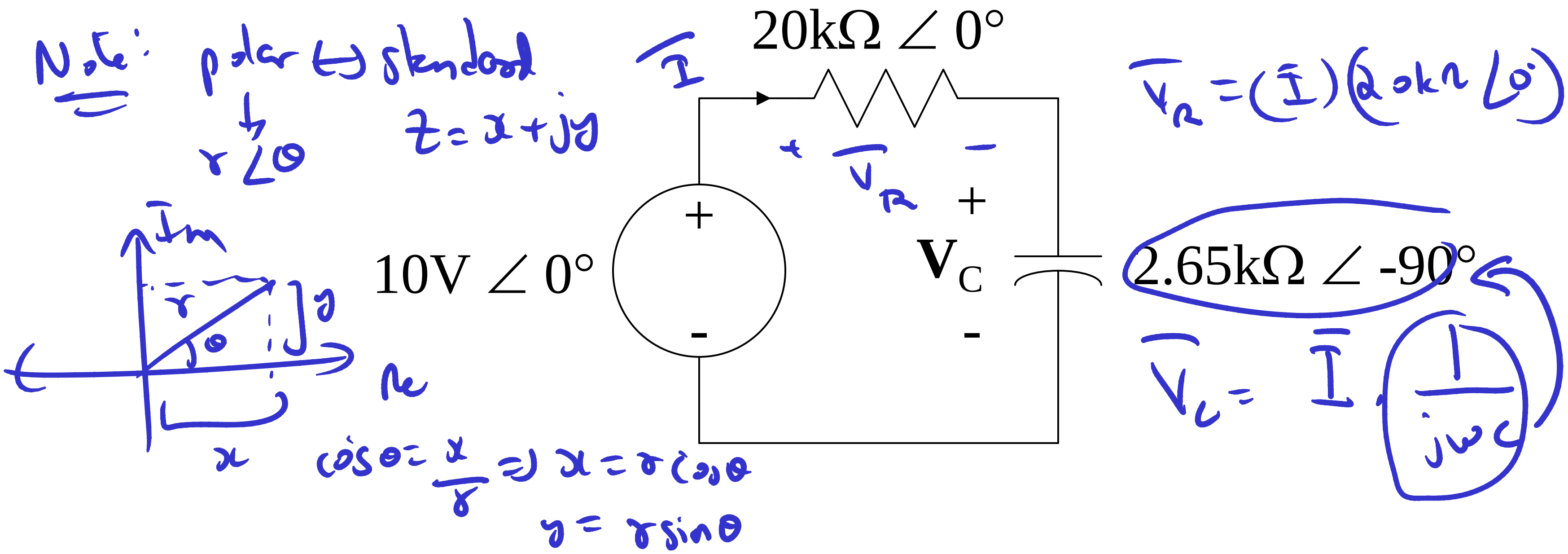
Phasor Form KVL

$$V_1 e^{j(\theta_1)} + V_2 e^{j(\theta_2)} + V_3 e^{j(\theta_3)} = 0$$

$$\mathbf{V_1} + \mathbf{V_2} + \mathbf{V_3} = 0$$

- Phasor Form KCL $\mathbf{I_1} + \mathbf{I_2} + \mathbf{I_3} = 0$
- Use complex impedances for inductors and capacitors and follow same analysis as in chap 2.

Impedance Example

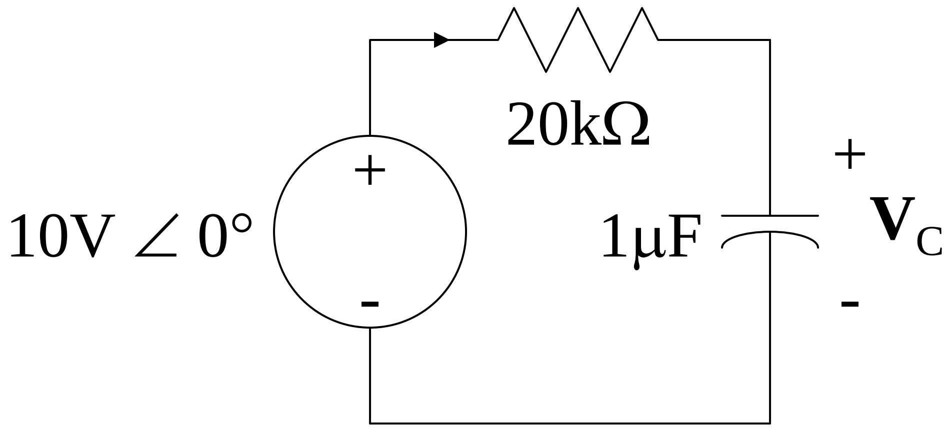


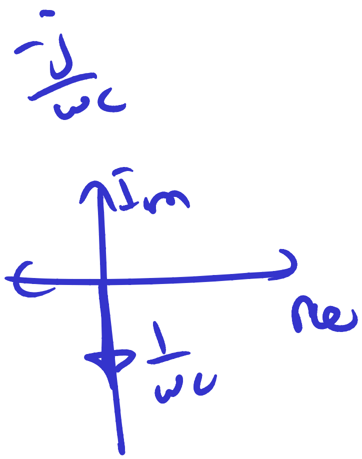
Now use the voltage divider to find $\mathbf{V}_C$:

$$\mathbf{V}_C = 10\text{V} \angle 0° \left(\frac{2.65\text{k}\Omega \angle -90°}{2.65\text{k}\Omega \angle -90° + 20\text{k}\Omega \angle 0°} \right)$$

$$\mathbf{V}_C = 1.31\text{V} \angle -82.4°$$

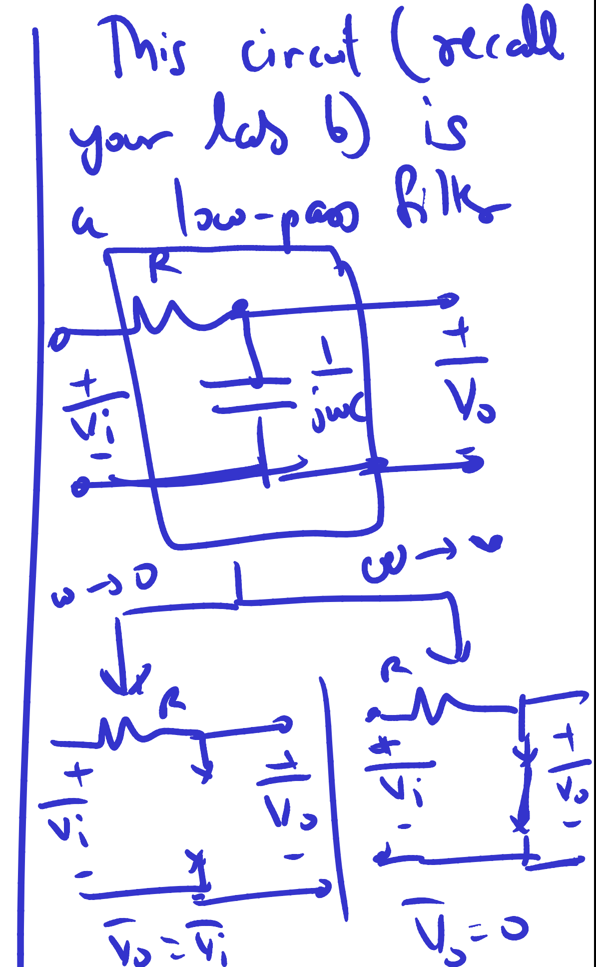
What happens when ω changes?

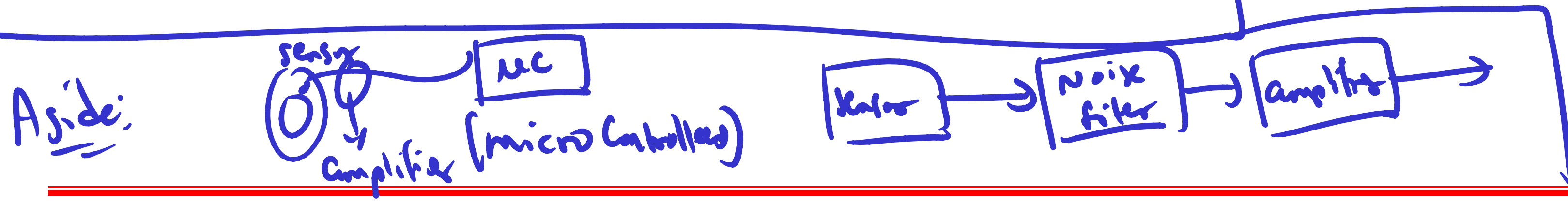




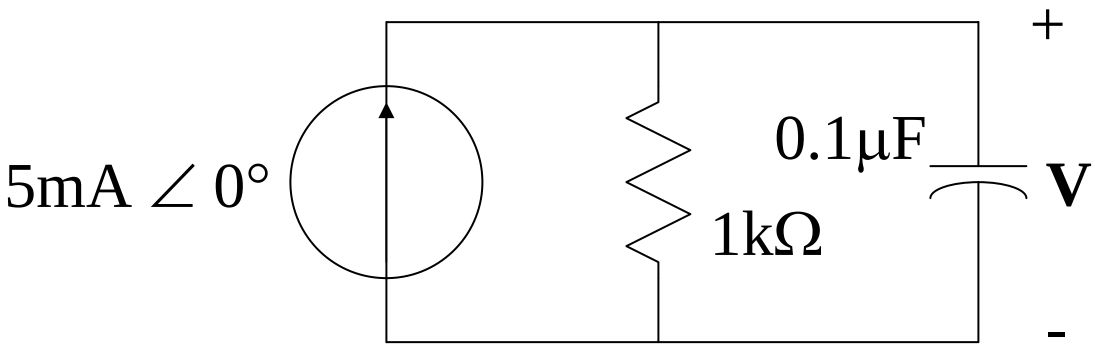
$\omega = 10$

Find $\mathbf{V}_C$



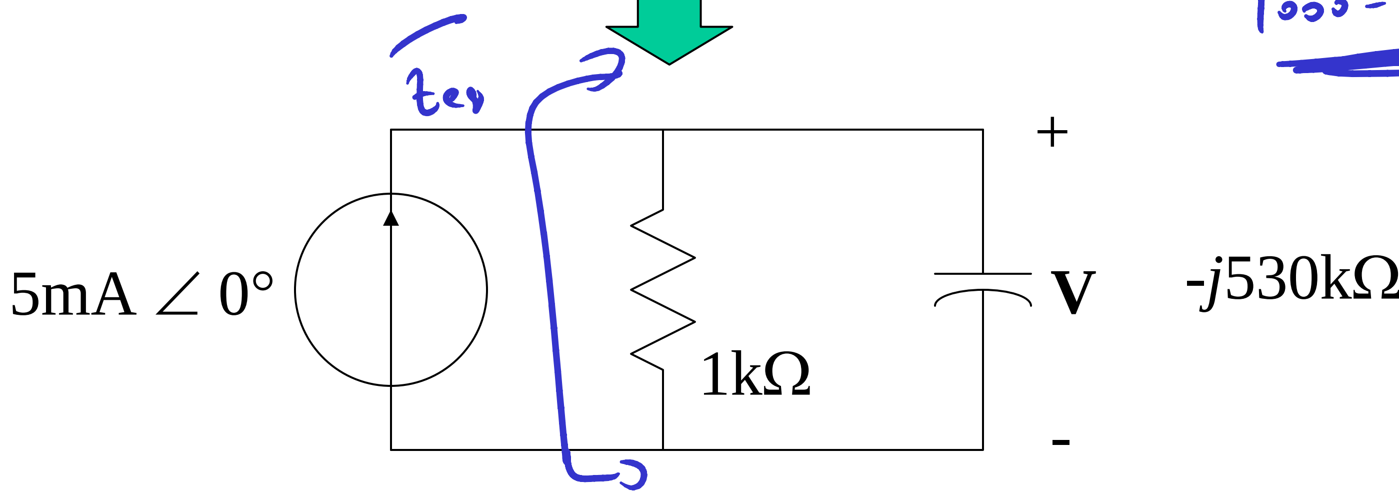


Steady-State AC Analysis

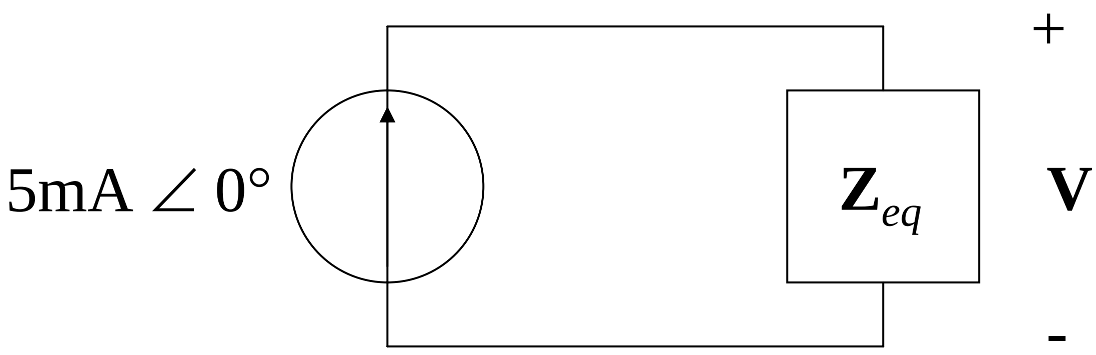


Find $v(t)$ for $\omega=2\pi$ 3000

$$Z_{eq} = \frac{(1000)(-j530k)}{1000 - j530k\Omega}$$



Find the Equivalent Impedance



$$\mathbf{Z}_{eq} = \frac{1000(-j530)}{1000 - j530} = \frac{10^3\angle 0° \times 530\angle -90°}{1132\angle -27.9°}$$

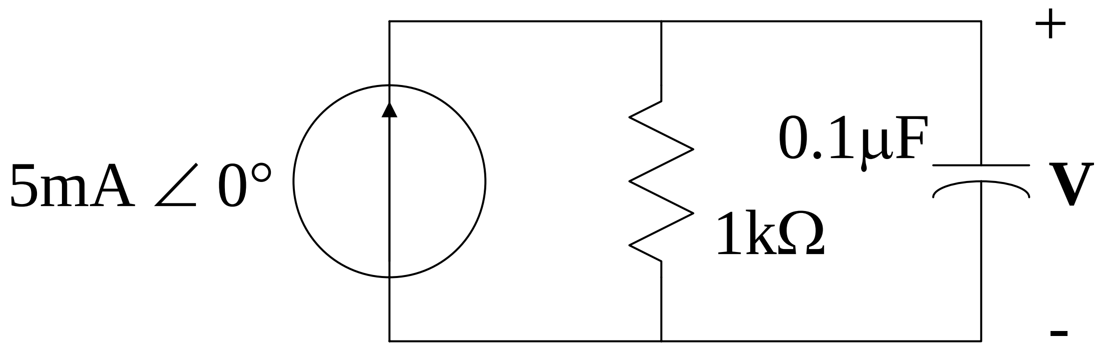
$$\mathbf{Z}_{eq} = 468.2\Omega\angle -62.1°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{eq} = 5\text{mA}\angle 0° \times 468.2\Omega\angle -62.1°$$

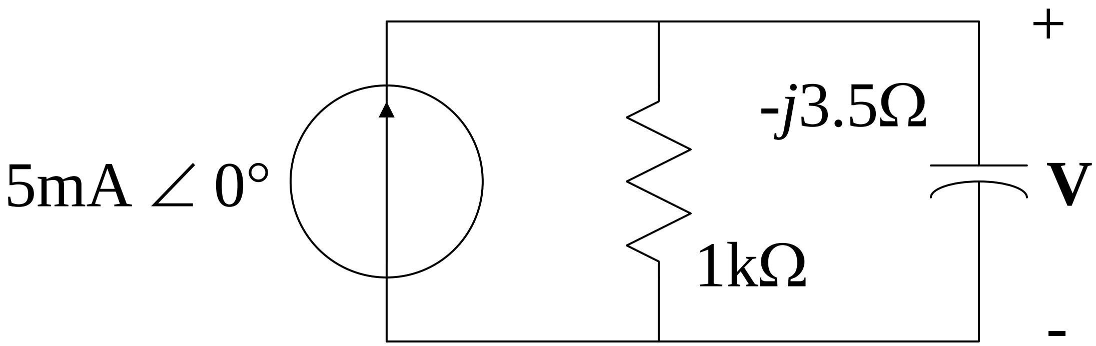
$$\mathbf{V} = 2.34\text{V}\angle -62.1°$$

$$v(t) = 2.34\text{V}\cos(2\pi 3000\text{t} - 62.1°)$$

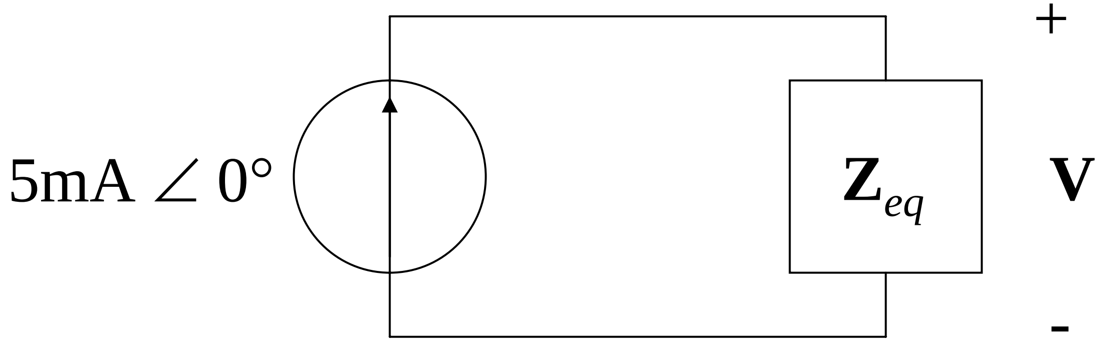
Change the Frequency



Find *v*(*t*) for ω=2π 455000



Find an Equivalent Impedance



$$\mathbf{Z}_{eq} = \frac{1000(-j3.5)}{1000 - j3.5} = \frac{10^3\angle 0° \times 3.5\angle -90°}{1000\angle -0.2°}$$

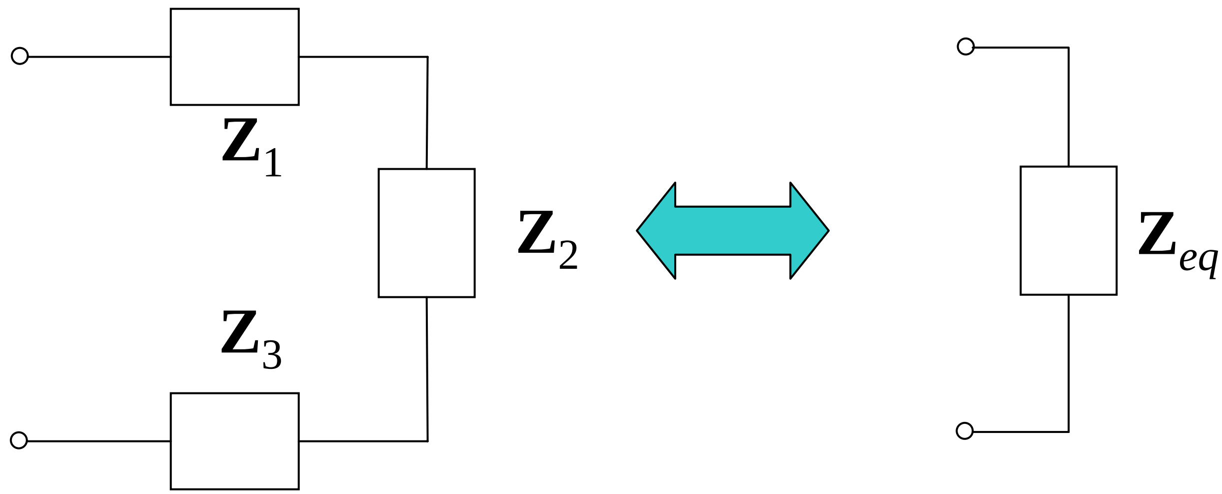
$$\mathbf{Z}_{eq} = 3.5\Omega\angle -89.8°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{eq} = 5\text{mA}\angle 0° \times 3.5\Omega\angle -89.8°$$

$$\mathbf{V} = 17.5\text{mV}\angle -89.8°$$

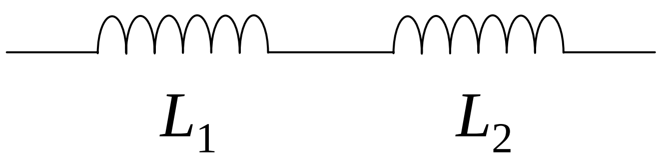
$$v(t) = 17.5\text{mV}\cos(2\pi 455000\text{t} - 89.8°)$$

Series Impedance

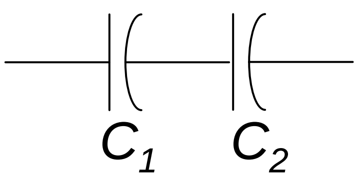


$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3$$

For example:

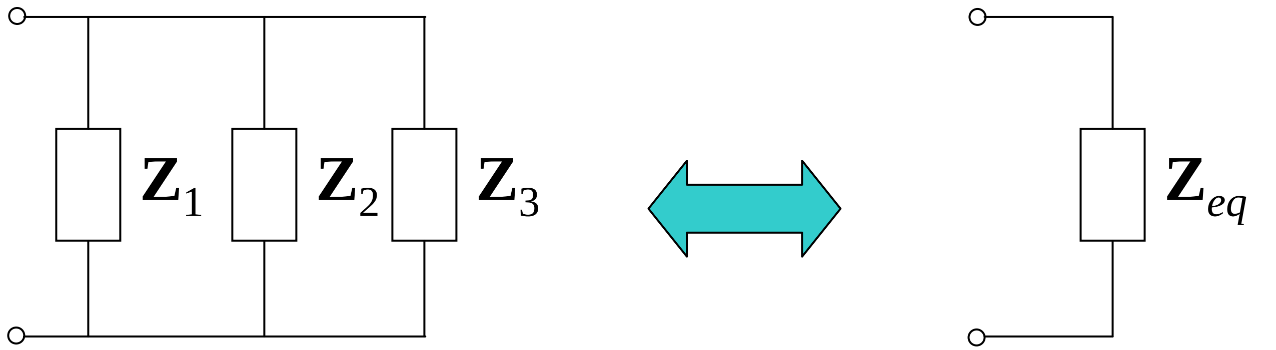


$$\mathbf{Z}_{eq} = j\omega(L_1 + L_2)$$



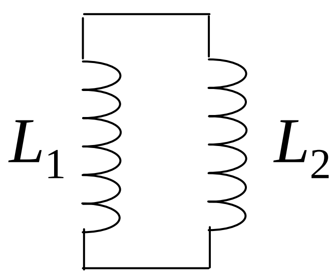
$$\mathbf{Z}_{eq} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}$$

Parallel Impedance

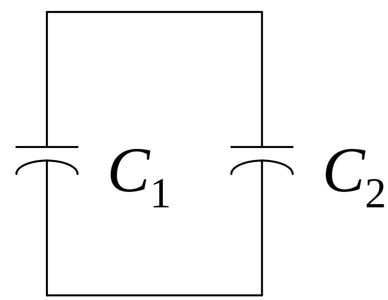


$$1/\mathbf{Z}_{eq} = 1/\mathbf{Z}_1 + 1/\mathbf{Z}_2 + 1/\mathbf{Z}_3$$

For example:

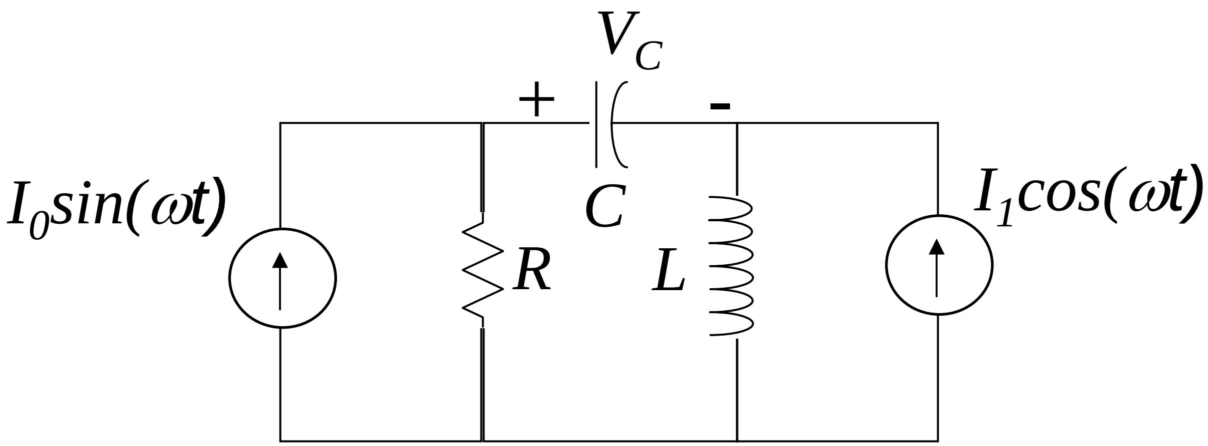


$$\mathbf{Z}_{eq} = j\omega \frac{L_1 L_2}{(L_1 + L_2)}$$



$$\mathbf{Z}_{eq} = \frac{1}{j\omega(C_1 + C_2)}$$

Steady-State AC Node-Voltage Analysis



- Try using Thevinin equivalent circuit.

- What happens if the sources are at different frequencies?

A: Superposition, write final answer in time domain.

Resistor I-V relationship

$v_R = i_R R$ .... $\mathbf{V}_R = \mathbf{I}_R R$ where R is the resistance in ohms,
$\mathbf{V}_R$ = phasor voltage, $\mathbf{I}_R$ = phasor current
(boldface indicates complex quantity)

Capacitor I-V relationship

$i_C = C dv_C/dt$ ....Phasor current $\mathbf{I}_C$ = phasor voltage $\mathbf{V}_C$ / capacitive impedance $\mathbf{Z}_C$: → $\mathbf{I}_C = \mathbf{V}_C/\mathbf{Z}_C$
where $\mathbf{Z}_C = 1/j\omega C$, $j = (-1)^{1/2}$ and boldface indicates complex quantity

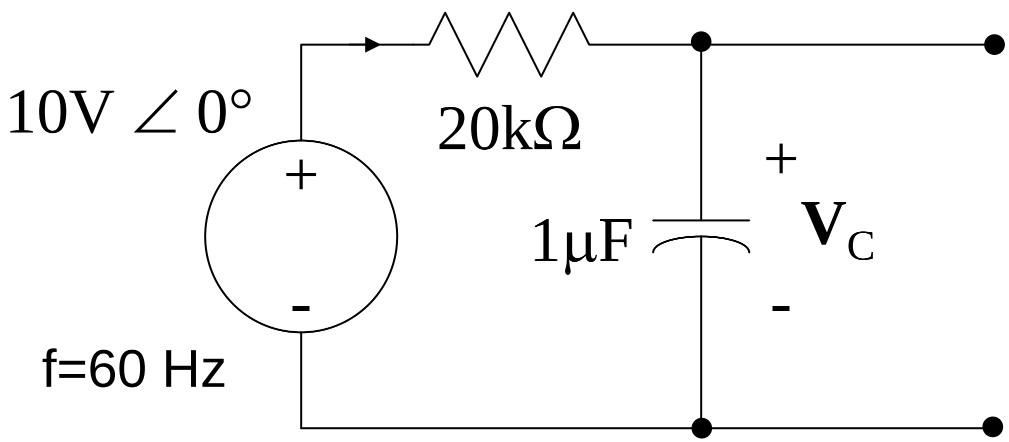
Inductor I-V relationship

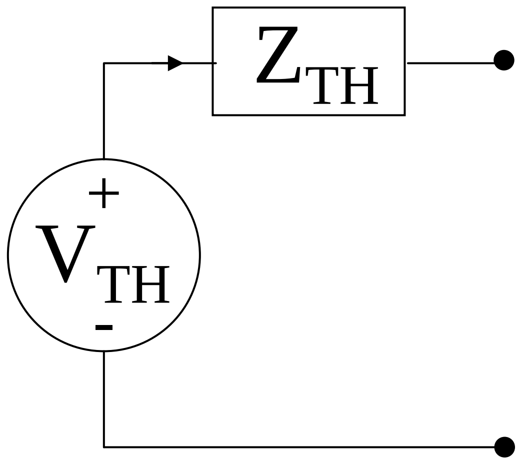
$v_L = L di_L/dt$ ....Phasor voltage $\mathbf{V}_L$ = phasor current $\mathbf{I}_L$/ inductive impedance $\mathbf{Z}_L$ → $\mathbf{V}_L = \mathbf{I}_L \mathbf{Z}_L$
where $\mathbf{Z}_L = j\omega L$, $j = (-1)^{1/2}$ and boldface indicates complex quantity

Summary of impedance relationships

R	C	L
$v_0(t) = V_0 \cos(\omega t)$	$v_0(t) = V_0 \cos(\omega t)$	$v_0(t) = V_0 \cos(\omega t)$
$\vec{V}_0 = V_0 \angle 0^\circ$	$\vec{V}_0 = V_0 \angle 0^\circ$	$\vec{V}_0 = V_0 \angle 0^\circ$
$i_0(t) = \frac{V_0}{R} \cos(\omega t)$	$i_0(t) = -\omega C V_0 \sin(\omega t)$	$i_0(t) = \frac{V_0}{\omega L} \sin(\omega t)$
$\vec{I}_0 = \frac{V_0}{R} \angle 0^\circ$	$\vec{I}_0 = \omega C V_0 \angle 90^\circ$	$\vec{I}_0 = \frac{V_0}{\omega L} \angle -90^\circ$

Thevenin Equivalent





$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°
$\mathbf{Z}_C$ = 1/*j* (2πf x 1μF) = 2.65kΩ ∠ -90°

$$\mathbf{V}_{TH} = \mathbf{V}_{OC} = 10\text{V} \angle 0° \left(\frac{2.65\text{k}\Omega \angle -90°}{2.65\text{k}\Omega \angle -90° + 20\text{k}\Omega \angle 0°} \right) = 1.31 \angle -82.4$$

$$\mathbf{Z}_{TH} = \mathbf{Z}_R \,\|\, \mathbf{Z}_C = ° \left(\frac{20\text{k}\Omega \angle 0° \cdot 2.65\text{k}\Omega \angle -90°}{2.65\text{k}\Omega \angle -90° + 20\text{k}\Omega \angle 0°} \right) = 2.62 \angle -82.4$$

Bode Plots (Appendix E, Chapter 1 in Reader)

- OUTLINE
  - dB scale
  - Frequency Response for Characterization
  - Asymptotic Frequency Behavior
  - Log magnitude vs log frequency plot
  - Phase vs log frequency plot
  - Transfer function example

Bel and Decibel (dB)

- A **bel** (symbol **B**) is a unit of measure of ratios of power levels, i.e. relative power levels.
  - The name was coined in the early 20th century in honor of Alexander Graham Bell, a telecommunications pioneer.
  - The bel is a logarithmic measure. The number of bels for a given ratio of power levels is calculated by taking the logarithm, to the base 10, of the ratio.
  - one bel corresponds to a ratio of 10:1.
  - $B = \log_{10}(P_1/P_2)$ where P_1 and P_2 are power levels.
- The bel is too large for everyday use, so the **decibel (dB)**, equal to 0.1B, is more commonly used.
  - $1\text{dB} = 10 \log_{10}(P_1/P_2)$
- dB are used to measure
  - Electric power, Gain or loss of amplifiers, Insertion loss of filters.

Logarithmic Measure for Power

- To express a power in terms of decibels, one starts by choosing a reference power, $P_{reference}$, and writing

$$\text{Power P in decibels} = 10 \log_{10}(P/P_{reference})$$

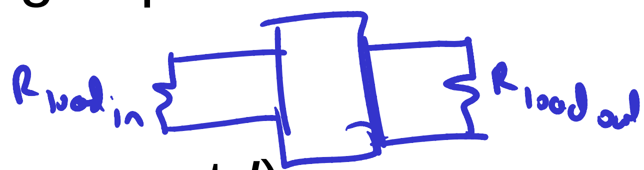
- Exercise:
  - Express a power of 50 mW in decibels relative to 1 watt.
  - $P\ (dB) = 10 \log_{10} (50 \times 10^{-3}) = -13\ dB$
- Exercise:
  - Express a power of 50 mW in decibels relative to 1 mW.
  - $P\ (dB) = 10 \log_{10} (50) = 17\ dB.$
- dBm to express absolute values of power relative to a milliwatt.
  - $dBm = 10 \log_{10}$ (power in milliwatts / 1 milliwatt)
  - 100 mW = 20 dBm
  - 10 mW = 10 dBm

P in dB (P = 1 watt)

$\Rightarrow P(dB) = (10 \log_{10} 1) = 0!$

Logarithmic Measures for Voltage or Current

From the expression for power ratios in decibels, we can readily derive the corresponding expressions for voltage or current ratios.



Suppose that the voltage V (or current I) appears across (or flows in) a resistor whose resistance is R. The corresponding power dissipated, P, is V^2/R (or I^2R). We can similarly relate the reference voltage or current to the reference power, as

$$P_{\text{reference}} = (V_{\text{reference}})^2/R \text{ or } P_{\text{reference}} = (I_{\text{reference}})^2 R.$$

Hence,

$$\text{Voltage, V in decibels} = 20\log_{10}(V/V_{\text{reference}})$$
$$\text{Current, I, in decibels} = 20\log_{10}(I/I_{\text{reference}})$$

Logarithmic Measures for Voltage or Current

Note that the voltage and current expressions are just like the power expression except that they have 20 as the multiplier instead of 10 because power is proportional to the square of the voltage or current.

Exercise: How many decibels larger is the voltage of a 9-volt transistor battery than that of a 1.5-volt AA battery? Let $V_{reference} = 1.5$.

The ratio in decibels is

$$20 \log_{10}(9/1.5) = 20 \log_{10}(6) = 16 \text{ dB.}$$

Logarithmic Measures for Voltage or Current

The gain produced by an amplifier or the loss of a filter is often specified in decibels.

The input voltage (current, or power) is taken as the reference value of voltage (current, or power) in the decibel defining expression:

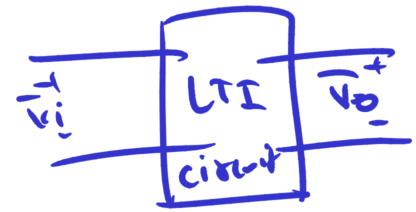
$$\text{Voltage gain in dB} = 20\log_{10}(V_{output}/V_{input})$$
$$\text{Current gain in dB} = 20\log_{10}(I_{output}/I_{input}$$
$$\text{Power gain in dB} = 10\log_{10}(P_{output}/P_{input})$$

Example: The voltage gain of an amplifier whose input is 0.2 mV and whose output is 0.5 V is

$$20\log_{10}(0.5/0.2\text{x}10^{-3}) = 68\text{ dB.}$$

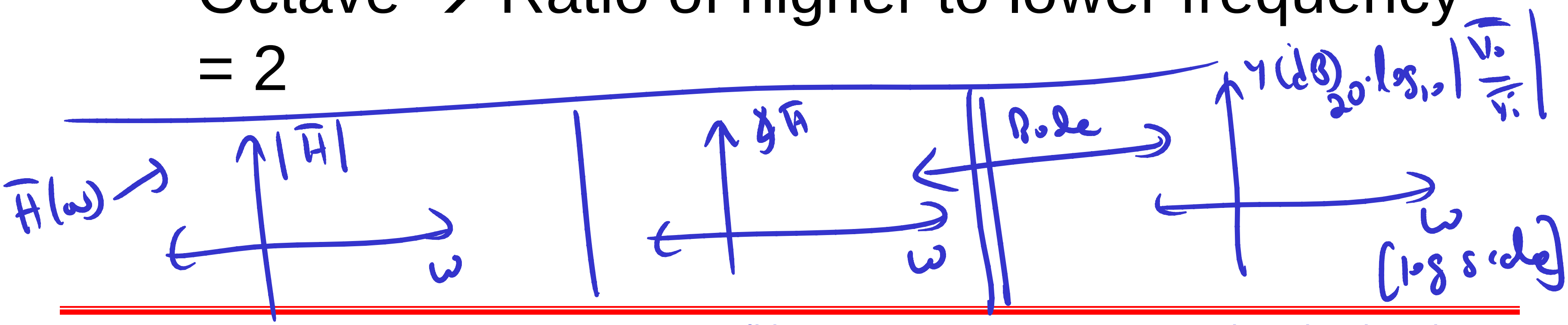
Bode Plot

- Plot of magnitude of transfer function vs. frequency
  - Both x and y scale are in log scale
  - Y scale in dB

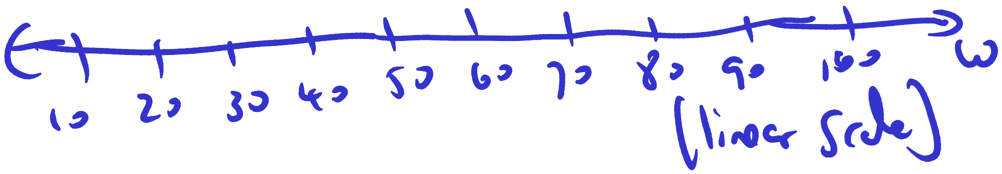


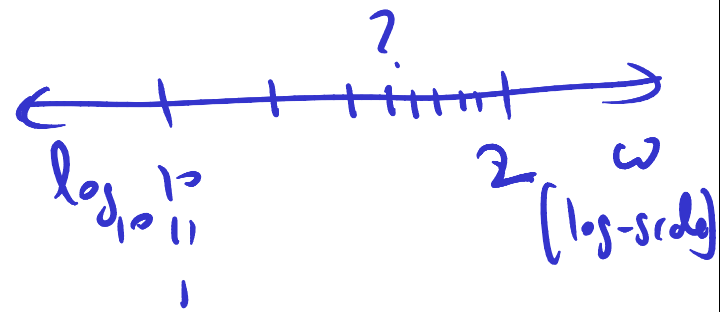
Transfer function $= \bar{H}(\omega) = \frac{\bar{V}_o}{\bar{V}_i}$

- Log Frequency Scale
  - Decade → Ratio of higher to lower frequency = 10
  - Octave → Ratio of higher to lower frequency = 2



Note: Converting linear scale → log-scale [Useful for x-axis of Bode plot]





Note:

(1) $\log_a x \stackrel{?}{=} \dfrac{\ln_e x}{\ln_e a}$

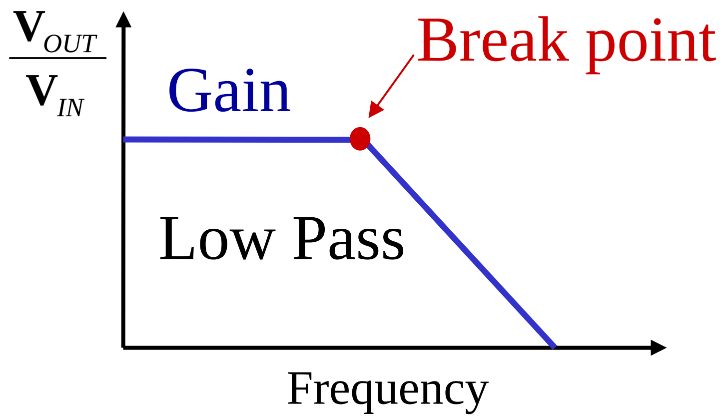
exact. But if incorrect

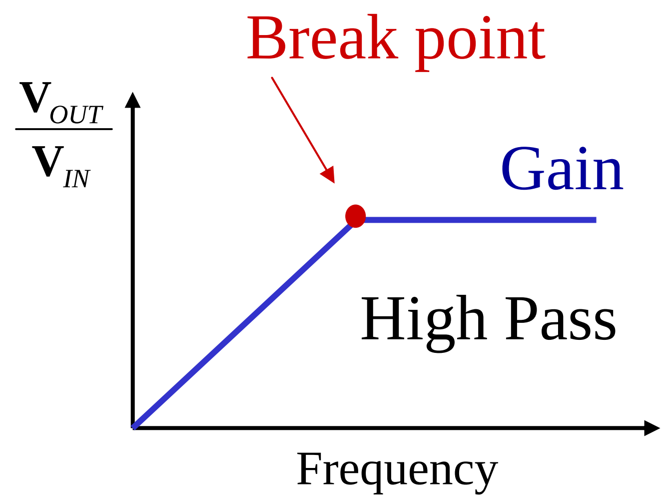
(2) log is NONLINEAR!

i.e. $\log(x_1 + x_2) \neq \log x_1 + \log x_2 = \log(x_1 x_2)$

Frequency Response: Why?

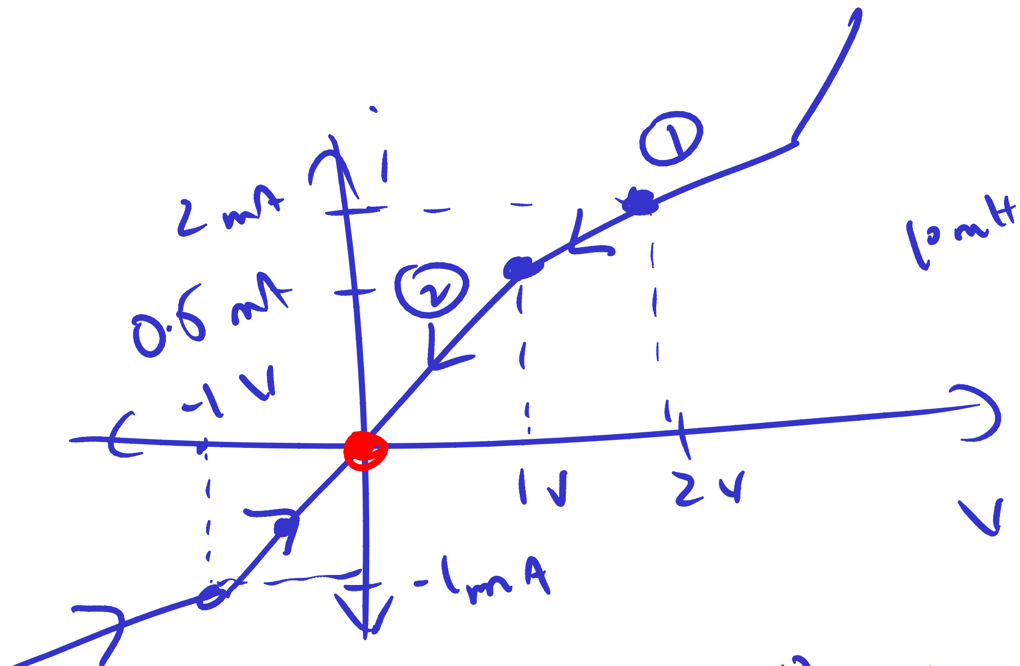
- The shape of the frequency response of the complex ratio of phasors $\mathbf{V}_{OUT}/\mathbf{V}_{IN}$ is a convenient means of classifying a circuit behavior and identifying key parameters.

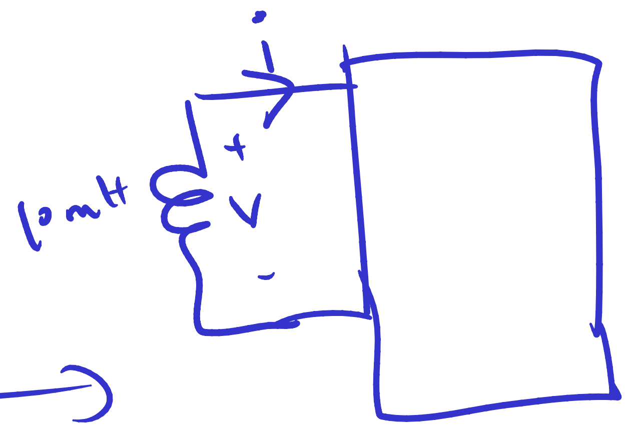




FYI: These are log ratio vs log frequency plots

H.W question on nonlinear circuit analysis (1st order)





$$V = -L\frac{di}{dt}$$

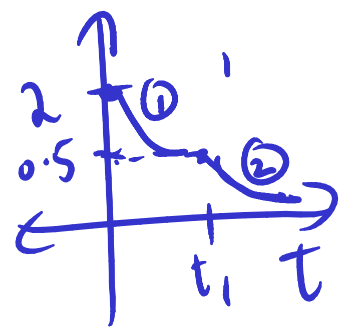
eq. point: $i' = 0 \Rightarrow V = 0$

[Stable eq. point]

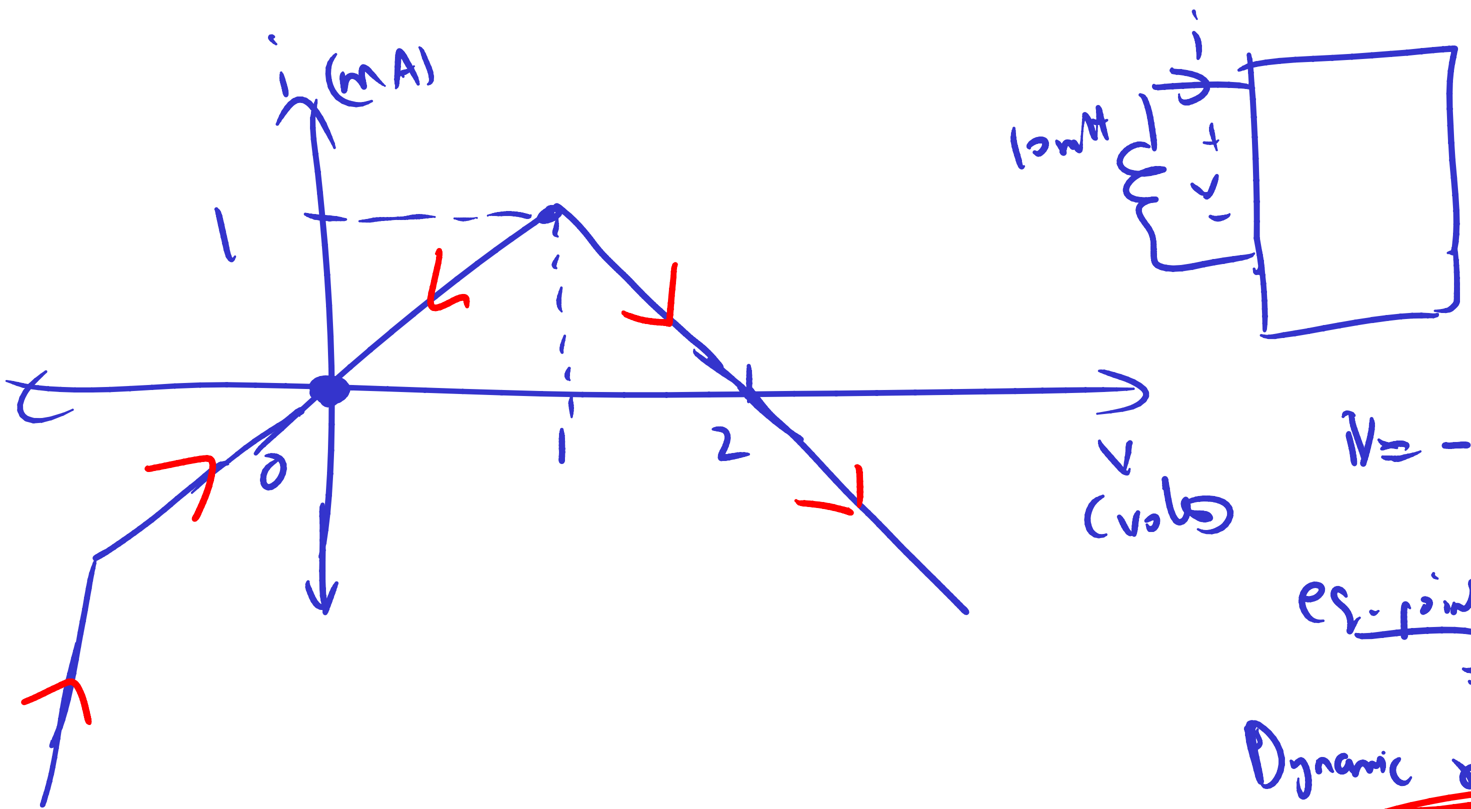
(Q:) What happens, $V(0) = 2V$?

Dynamic route

$V>0$,	$V<0$,
$i'<0$	$i'>0$



More interesting case:



$v = -L\frac{di}{dt}$

eq. points: $i' = 0$

$\Rightarrow v = 0$

Dynamic route & stability:

$v > 0,\ i' < 0$	$v < 0,\ i' > 0$