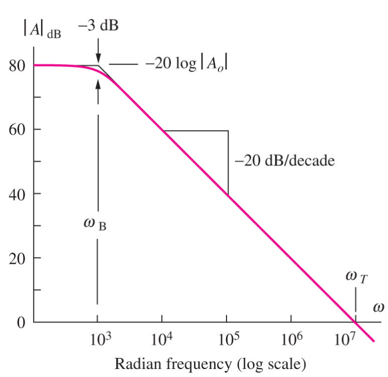
EE105 – Fall 2014
Microelectronic Devices and Circuits

Prof. Ming C. Wu

wu@eecs.berkeley.edu

511 Sutardja Dai Hall (SDH)

Op Amp Frequency Response
Single-pole Amplifiers



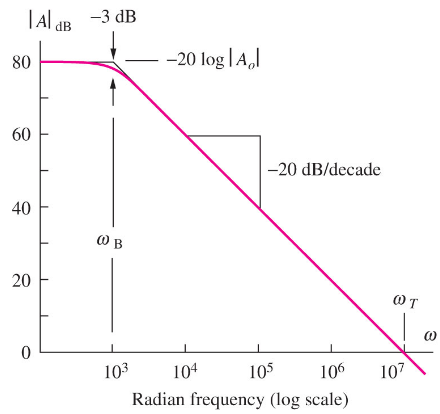
General purpose op amps are typically low-pass amplifiers with high gain at dc and a single-pole frequency response.

$$A_v(s) = \frac{A_o \omega_B}{s+\omega_B} = \frac{\omega_T}{s+\omega_B}$$

$$\left|A(j\omega)\right| = \frac{A_o \omega_B}{\sqrt{\omega^2+\omega_B^2}} = \frac{A_o}{\sqrt{1+\frac{\omega^2}{\omega_B^2}}}$$

ω_B = open loop bandwidth of the op amp. ω_T = unity gain frequency or gain bandwidth product (frequency at which magnitude of gain is unity).

Op Amp Frequency Response
Single-pole Amplifiers (cont.)



For $\omega >> \omega_B, \quad \left|A(j\omega)\right| = \frac{A_o \omega_B}{\omega} = \frac{\omega_T}{\omega}$

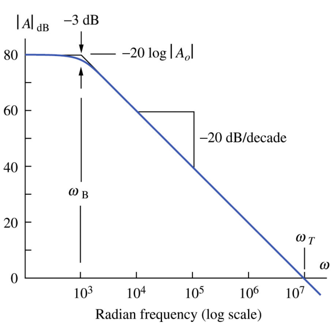
and $\quad \left|A(j\omega)\right| \cdot \omega = \omega_T$

For $\omega = \omega_T, \quad \left|A(j\omega)\right| = \frac{\omega_T}{\omega_T} = 1$

For ω >> ω_B, the product of magnitude of amplifier gain and frequency is a constant value equal to the unity-gain frequency.

Hence, ω_T is also called the gain-bandwidth product.

Op Amp Frequency Response
Single-Pole Amplifier Example



- **Problem:**
  Find transfer function describing frequency-dependent amplifier voltage gain.

Single-pole amplifier response:

$$A_v(s) = \frac{A_o \omega_B}{s + \omega_B} = \frac{\omega_T}{s + \omega_B}$$

$$A_o = 10^{\frac{80dB}{20dB}} = 10^4 \qquad \omega_B = 10^3 \text{ rad/s}$$

$$\omega_T = A_o \omega_B = 10^7 \text{ rad/s}$$

$$A_v(s) = \frac{10^7}{s + 10^3}$$

Frequency values are often expressed in Hz:

$$f_B = \frac{\omega_B}{2\pi} = 159 \text{ Hz} \qquad f_T = \frac{\omega_T}{2\pi} = 1.59 \text{ MHz}$$

Frequency Response of Noninverting Amplifier

For a closed-loop feedback amplifier:

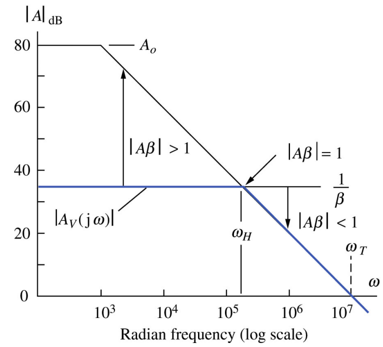
$$A_v(s) = \frac{A(s)}{1 + A(s)\beta} = \frac{A_o\omega_B}{s + \omega_B(1 + A_o\beta)}$$

$$A_v(s) = \frac{\dfrac{A_o}{1 + A_o\beta}}{\dfrac{s}{\omega_B(1 + A_o\beta)} + 1} = \frac{A_v(0)}{\dfrac{s}{\omega_H} + 1}$$

$$\omega_H = (1 + A_o\beta)\omega_B = \frac{\omega_T}{A_v(0)}$$

For $A_o\beta >> 1$,

$$A_v(0) \cong \frac{1}{\beta} \quad \text{and} \quad \omega_H \cong \beta\omega_T$$



At low frequencies, gain is set by the feedback, but at high frequencies, it follows the gain of the amplifier.

Frequency Response of Noninverting Amplifier

- **Given data: A_o= 10^5 = 100 dB, f_T = 10 MHz, desired A_v = 1000 or 60 dB**
- **Assumptions: Amplifier is described by a single-pole transfer function.**
- **Analysis:**

$$f_B = \frac{f_T}{A_o} = \frac{10^7\,Hz}{10^5} = 100\ Hz$$

$$f_H = f_B(1 + A_o\beta) = f_B\left(1 + \frac{A_o}{A_v(0)}\right) = 100\left(1 + \frac{10^5}{10^3}\right) = 10.1\ kHz$$

Op amp transfer function: $A_v(s) = \dfrac{\omega_T}{s + \omega_B} = \dfrac{2x10^7\pi}{s + 200\pi}$

Close-loop amplifier transfer function: $A_v(s) = \dfrac{\omega_T}{s + \omega_B(1 + A_o\beta)} = \dfrac{2x10^7\pi}{s + 2.01x10^4\pi}$

Frequency Response of Inverting Amplifier

$$A_v(s) = A_v^{Ideal}\frac{T}{1+T} = A_v^{Ideal}\frac{A(s)\beta}{1+A(s)\beta} = \left(-\frac{R_2}{R_1}\right)\frac{\dfrac{A_o\beta}{1+A_o\beta}}{\dfrac{s}{(1+A_o\beta)\omega_B}+1}$$

For $A_o\beta >> 1$,

$$A_v(s) \cong \left(-\frac{R_2}{R_1}\right)\frac{1}{\dfrac{s}{\omega_H}+1} \qquad \omega_H = \frac{\omega_T}{\dfrac{A_o}{1+A_o\beta}} \cong \beta\omega_T$$

Frequency Response of Inverting Amplifier

- **Given data: A_o = 2x10^5, f$_T$ = 5x10^5 Hz, desired A_v = -100 or 40 dB**
- **Assumptions: Amplifier is described by a single-pole transfer function.**
- **Analysis:**

$$f_B = \frac{f_T}{A_o} = \frac{5x10^5 Hz}{2x10^5} = 2.5\ Hz \qquad \beta = \frac{1}{1+|A_v(0)|} = \frac{1}{101}$$

$$f_H = f_B(1+A_o\beta) = 2.5Hz\left(1+\frac{2x10^5}{101}\right) = 4.95\ kHz$$

Op amp transfer function: $A_v(s) = \dfrac{\omega_T}{s+\omega_B} = \dfrac{10^6\pi}{s+5\pi}$

Inverting amplifier transfer function:

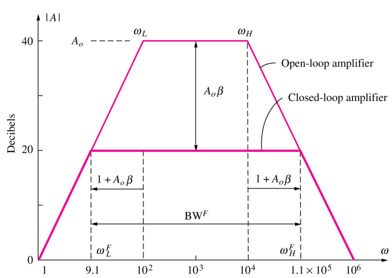
$$A_v(s) = A_v(0)\frac{\beta\omega_T}{s+\omega_B(1+A_o\beta)} = -\frac{9.9x10^5\pi}{s+9.91x10^3\pi}$$

Op Amp Frequency Response Summary

TABLE 11.3
Inverting and Noninverting Amplifier Frequency Response Comparison

$\beta = \frac{R_1}{R_1+R_2}$	NONINVERTING AMPLIFIER	INVERTING AMPLIFIER
dc gain	$A_v(0) = 1 + \frac{R_2}{R_1}$	$A_v(0) = -\frac{R_2}{R_1}$
Feedback factor	$\beta = \frac{1}{A_v(0)}$	$\beta = \frac{1}{1 + \lvert A_v(0)\rvert}$
Bandwidth	$f_B = \beta f_T$	$f_B = \beta f_T$
Input resistance	$R_{ic} \\| R_{id}(1 + A\beta)$	$R_1 + \left(R_{ID} \\| \frac{R_2}{1+A}\right)$
Output resistance	$\frac{R_o}{1 + A\beta}$	$\frac{R_o}{1 + A\beta}$

Feedback Control of Frequency Response



$$A_v(s) = A_o \frac{\omega_H s}{s^2 + \left[\omega_L + (1 + A_o\beta)\omega_H\right]s + \omega_L\omega_H}$$

$$For \quad \omega_H(1 + A_o\beta) >> \omega_L :$$

$$\omega_L^F \cong \frac{\omega_L}{1 + A_o\beta} \qquad \omega_H^F \cong \omega_H(1 + A_o\beta)$$

$$BW_F \cong \omega_H(1 + A_o\beta)$$

Upper and lower cutoff frequencies as well as bandwidth of amplifier are improved, gain is stabilized at

$$A_v(s) = \frac{A(s)}{1 + A(s)\beta}$$

where $A(s) = A_o \frac{s}{(s+\omega_L)\left(1 + \frac{s}{\omega_H}\right)} = \frac{A_o\omega_H s}{(s+\omega_L)(s+\omega_H)}$

$$A_{mid} = \frac{A_o}{1 + A_o\beta}$$

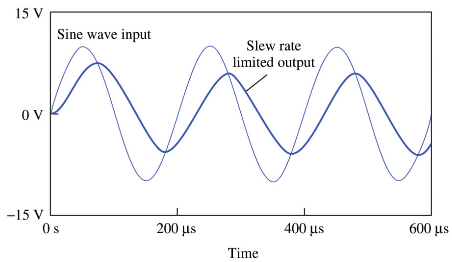
$$GBW = A_{mid} \cdot BW_F = A_o\omega_H$$

Large Signal Limitations
Slew Rate and Full-Power Bandwidth

- **Slew rate: Maximum rate of change of voltage at the output of an op amp. Typical values range from 0.1V/μs to 10V/μs.**

$$v_O = V_M \sin \omega t$$

$$\left.\frac{dv_O}{dt}\right|_{\max} = V_M \omega \cdot \cos \omega t\Big|_{\max} = V_M \omega$$



- **For given frequency, slew rate limits the maximum signal amplitude that can be amplified without distortion.**

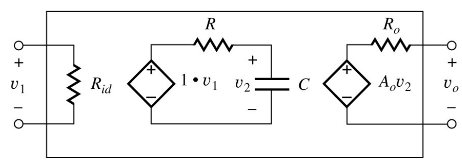
For no signal distortion,

$$V_M \omega \le SR \quad \rightarrow \quad V_M \le \frac{SR}{\omega}$$

Full-power bandwidth is highest frequency at which a full-scale signal can be developed.

$$f_M \le \frac{SR}{2\pi V_{FS}}$$

Operational Amplifier
Macro Model for Frequency Response



- **Simplified circuit representations are available in most simulators to model the terminal behavior of op amps that include all nonideal limitations of op amps and a large number of parameters that can be adjusted to model op amp behavior.**
- **To model a single-pole roll-off, an auxiliary "dummy" loop (voltage controlled voltage source v_1 in series with R and C) is added to the original two-port model.**

$$A_v(s) = \frac{V_o(s)}{V_1(s)} = \frac{A_o \omega_B}{s + \omega_B} = \frac{A_o}{\frac{s}{\omega_B} + 1} \qquad \omega_B = \frac{1}{RC}$$

- **The RC product is chosen to give the desired -3dB point for the open-loop amplifier.**

General Purpose Op Amp Parameters Sample Values

TABLE 11.4
Typical Op Amp Macro Model Parameter Set

PARAMETER	TYPICAL VALUE
Differential-mode gain (dc)	106 dB
Differential-mode input resistance	2 MΩ
Input capacitance	1.5 pF
Common-mode rejection ratio	90 dB
Common-mode input resistance	2 GΩ
Output resistance	50 Ω
Input offset voltage	1 mV
Input bias current	80 nA
Input offset current	20 nA
Positive slew rate	0.5 V/μs
Negative slew rate	0.5 V/μs
Maximum output source current	25 mA
Maximum output sink current	25 mA
Input type (*n*- or *p*-type)	*n*-type
Frequency of first pole	5 Hz
Frequency of zero	5 MHz
Frequency of second pole	2 MHz
Frequency of third pole	20 MHz
Frequency of fourth pole	100 MHz
Power supply voltage (3-pin model)	15 V

Note the four-pole frequency response