# EE105 – Fall 2015
# Microelectronic Devices and Circuits

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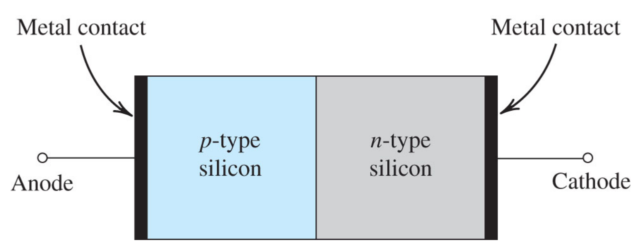
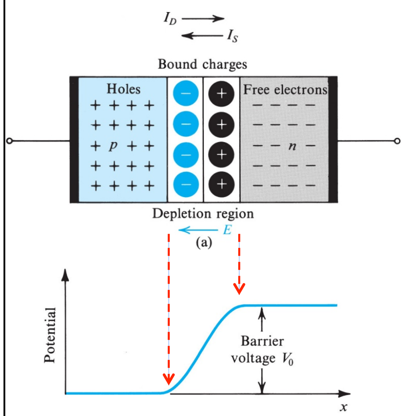
## pn Junction

- **p-type semiconductor in contact with n-type**
- **Basic building blocks of semiconductor devices**
  - **Diodes,**
  - **Bipolar junction transistors (BJT),**
  - **Metal-oxide-semiconductor field effect transistors (MOSFET)**



# Built-in Voltage




- At pn junction, free electrons from n-side "recombine" with free holes from p-side.
- "Depletion region" has no electrons or holes, but has fixed (immobile) charges from donor and acceptor ions
- The fixed charges establish an electric field, and create a potential difference between p- and n-sides.
- This potential is called "built-in potential"

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$V_T$ : thermal voltage (= 26 mV at room temp)
$N_A$ : acceptor concentration on p-side
$N_D$ : donor concentration on n-side
$n_i$ : intrinsic carrier concentration
(=$1.5\times10^{10}$ cm$^{-3}$ at room temp)

Example: A pn junction with n-doping of $10^{17}$ cm$^{-3}$ and p-doping of $10^{18}$ cm$^{-3}$

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.88 \text{ V}$$

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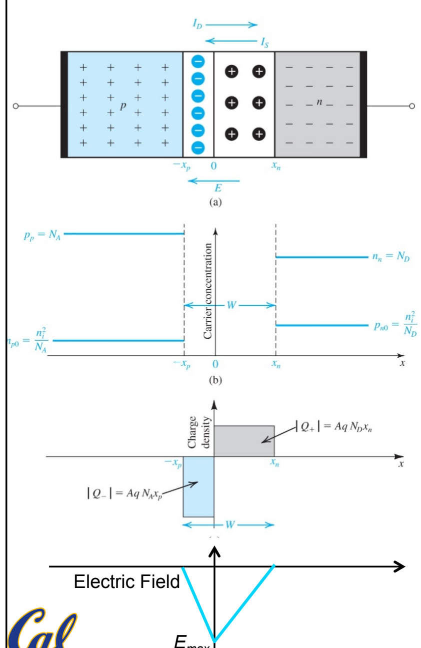
# Electrostatic Analysis of pn Junction (1)




Charge density: $q(x) = \begin{cases} -qAN_A, & -x_p < x < 0 \\ qAN_D, & 0 < x < x_n \end{cases}$

Gauss Law: $\oint \vec{E}\cdot\vec{dA} = \frac{Q}{\varepsilon_S}$, $\quad \varepsilon_S = 11.7\varepsilon_0$, $\varepsilon_0 = 8.854\times10^{-14}$ F/cm

In one dimension:

$$E(x)A - E(-x_p)A = \frac{-qAN_A(x+x_p)}{\varepsilon_S}$$

$E(-x_p) = 0$ (in charge neutral region, $N_A = p$)

$$E(x) = \begin{cases} \frac{-qN_A(x+x_p)}{\varepsilon_S}, & -x_p < x < 0 \\ \frac{qN_D(x-x_n)}{\varepsilon_S}, & 0 < x < x_n \end{cases}$$

Note: $N_A x_p = N_D x_n$ (charge equality)

Maximum electrical field occurs at $x = 0$ (junction)

$$E_{\max} = -\frac{qN_A x_p}{\varepsilon_S} = -\frac{qN_D x_n}{\varepsilon_S}$$

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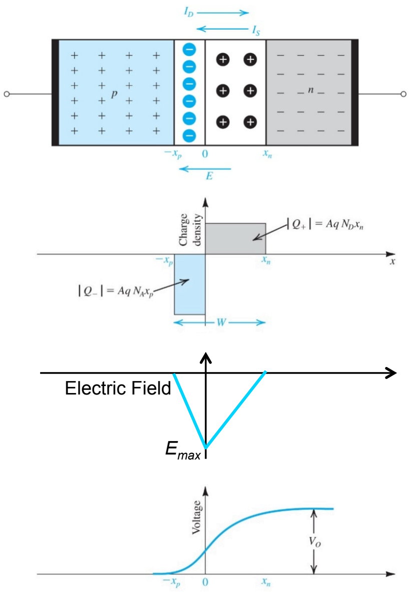
## Electrostatic Analysis of pn Junction (2)




$$E(x) = \begin{cases} \dfrac{-qN_A(x+x_p)}{\varepsilon_S}, & -x_p < x < 0 \\ \dfrac{qN_D(x-x_n)}{\varepsilon_S}, & 0 < x < x_n \end{cases}$$

$$V(x) = -\int_{-x_p}^{x} E(x')dx'$$

$$V(x) = \begin{cases} \dfrac{qN_A}{2\varepsilon_S}\left(x+x_p\right)^2, & -x_p < x < 0 \\ \dfrac{q}{2\varepsilon_S}\left(N_A x_p^2 + N_D x_n^2\right) - \dfrac{qN_D}{2\varepsilon_S}\left(x-x_n\right)^2, & 0 < x < x_n \end{cases}$$

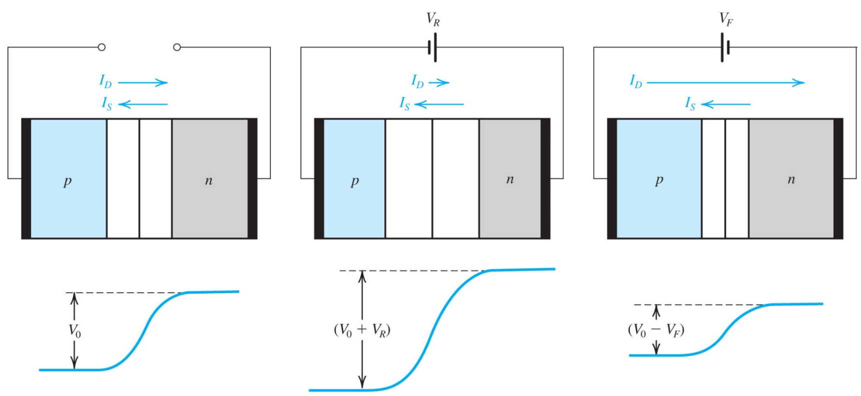
$$V(x_n) = \frac{q}{2\varepsilon_S}\left(N_A x_p^2 + N_D x_n^2\right) = V_0$$

$$x_p = \frac{N_D}{N_A+N_D}W, \qquad x_n = \frac{N_A}{N_A+N_D}W$$

$$W = \sqrt{\frac{2e_S}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_0} \quad : \text{ Depletion Width}$$



## Depletion Width Under Bias




(a) Open-circuit (equilibrium) (b) Reverse bias (c) Forward bias

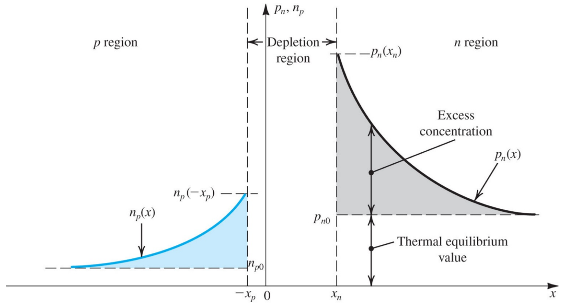
$$W = \sqrt{\frac{2e_S}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_0 - V)}$$

$V$ is the applied voltage to the pn junction,

it's positive for forward bias and negative for reverse bias.

Depletion width is widened in reverse bias

# Current-Voltage (I-V) Characteristics




Under forward bias, minority carriers at the edge of depletion region is boosted up by $(e^{V/V_T}-1)$:

$$p_n(x) = p_{n0} + p_{n0}\left(e^{V/V_T}-1\right)\cdot e^{-\frac{x-x_n}{L_p}}$$

$L_p$ : hole diffusion length in n-type

Hole diffusion current density on n-side

$$J_p = -qD_p \left.\frac{dp_n(x)}{dx}\right|_{x=x_n} = q\frac{D_p}{L_p}p_{n0}\left(e^{V/V_T}-1\right)$$

Similarly, electron diffusion current density in p-side

$$J_n = qD_n \left.\frac{dn_p(x)}{dx}\right|_{x=-x_p} = q\frac{D_n}{L_n}n_{p0}\left(e^{V/V_T}-1\right)$$

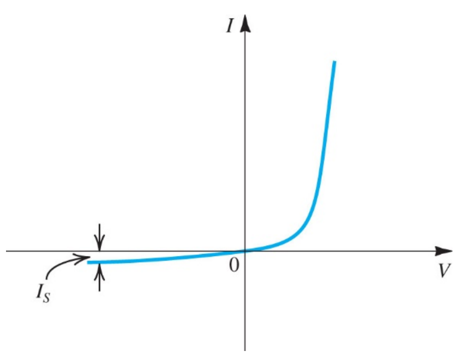
Total current :

$$I = \left(J_p + J_n\right)A = A\left(q\frac{D_p}{L_p}p_{n0} + q\frac{D_n}{L_n}n_{p0}\right)\left(e^{V/V_T}-1\right)$$

$I = I_S\left(e^{V/V_T}-1\right)$ where $I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D}+\frac{D_n}{L_nN_A}\right)$



# I-V Curve

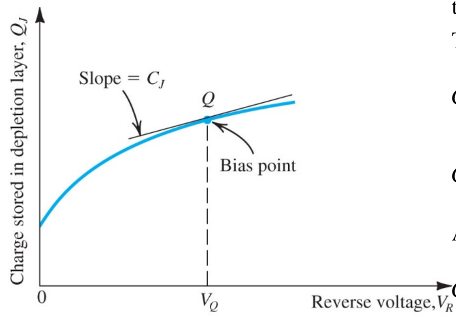



$$I = I_S\left(e^{V/V_T}-1\right)$$

where

$$I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D}+\frac{D_n}{L_nN_A}\right)$$

## Capacitance in pn Junction (1): Depletion Capacitance (Mainly Reverse Bias)




In comparision, for a "linear" (normal) capacitor:

$C = \dfrac{Q}{V}$ is a constant

Total charge Ain depletion width at $V = -V_R$

$$Q_J = AqN_D x_n = AqN_D \frac{N_A}{N_A + N_D} W$$

$$W = \sqrt{\frac{2e_S}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$$

As bias voltage change, the amount of charge in the junction change. This is a "nonlinear" capacitor. The capacitance value is

$$C_j = \frac{dQ_J}{dV_R} = Aq\frac{N_D N_A}{N_A + N_D}\sqrt{\frac{2e_S}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}\frac{d}{dV}\sqrt{(V_0 + V_R)}$$

$$C_j = A\sqrt{\frac{\varepsilon_S q}{2}\left(\frac{N_A N_D}{N_A + N_D}\right)}\frac{1}{\sqrt{(V_0 + V_R)}}$$

Note: $C_j = \dfrac{\varepsilon_S A}{W}$

At zero bias, $V_R = 0$

$$C_{j0} = A\sqrt{\frac{\varepsilon_S q}{2}\left(\frac{N_A N_D}{N_A + N_D}\right)}\frac{1}{\sqrt{V_0}}$$

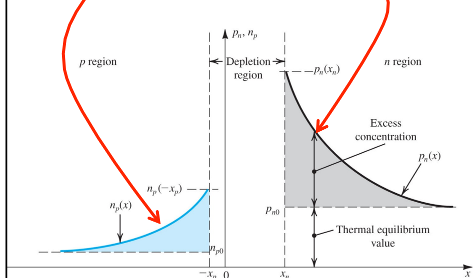
Therefore at $V = -V_R$ , $C_j = \dfrac{C_{j0}}{\sqrt{1 + \dfrac{V_R}{V_0}}}$

This is a variable capacitor, controllable by voltage !



## Capacitance in pn Junction (2): Diffusion Capacitance (Forward Bias)

**Extra minority carriers stored outside junction under forward bias**




$Q_p = Aq \times$ shaded area under $p_n(x)$

$$Q_p = Aq\int_{x_n}^{\infty} p_{n0}\left(e^{V/V_T} - 1\right)\cdot e^{-\frac{x - x_n}{L_p}}\,dx$$

$$Q_p = AqL_p p_{n0}\left(e^{V/V_T} - 1\right) = \frac{L_p^2}{D_p} I_p = \tau_p I_p$$

$\dfrac{L_p^2}{D_p}$ has unit of time, its physical meaning is

minority carrier lifetime: $\tau_p = \dfrac{L_p^2}{D_p}$

Similarly, $Q_n = \tau_n I_n$

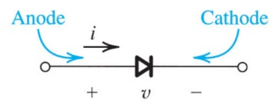
Total charge stored : $Q = \tau_p I_p + \tau_n I_n = \tau_T I$

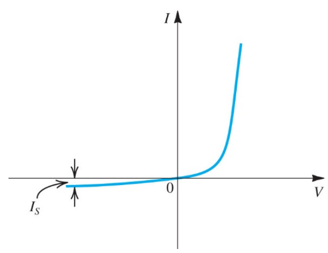
$\tau_T$ is mean transit time

These stored charges correspond to another nonlinear capacitor call "diffusion capacitance":

$$C_d = \frac{dQ}{dV} = \frac{d(\tau_T I)}{dV} = \tau_T \frac{dI}{dV} \Rightarrow C_d = \left(\frac{\tau_T}{V_T}\right) I$$

# Summary of pn Junction




Built-in potential : $V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$

<u>Under forward bias</u> :

I-V curve : $I = I_S\left(e^{V/V_T} - 1\right)$

Diffusion capacitance : $C_d = \left(\frac{\tau_T}{V_T}\right) I$

<u>Under revserse bias</u> :

Negligible current, $I = -I_S$

Depletion capacitance : $C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$

Other important parameter :

Depletion Width: $W = \sqrt{\frac{2e_S}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 - V)}$



# Detailed Derivation of pn Junction Potential

$$E(x) = \begin{cases} \frac{-qN_A(x + x_p)}{\varepsilon_S}, & -x_p < x < 0 \\ \frac{qN_D(x - x_n)}{\varepsilon_S}, & 0 < x < x_n \end{cases}$$

$$V(x) = -\int_{-x_p}^{x} E(x')dx'$$

<u>(1) for $-x_p < x < 0$</u> : $V(x) = -\int_{-x_p}^{x} E(x')dx' = \int_{-x_p}^{x} \frac{qN_A(x' + x_p)}{\varepsilon_S} dx' = \frac{qN_A}{2\varepsilon_S}\left(x' + x_p\right)^2 \Big|_{x'=-x_p}^{x'=x} = \frac{qN_A}{2\varepsilon_S}\left(x + x_p\right)^2$

<u>(2) for $0 < x < x_n$</u> : Because $E(x)$ has different expression for $x < 0$ and $x > 0$, the integration should be performed in two separate ranges, first from $-x_p$ to 0, and then from 0 to $x$. We can use $V(x = 0)$ from the above equation for the first intgration. Therefore,

$$V(x) = \frac{qN_A}{2\varepsilon_S} x_p^2 - \int_0^x \frac{qN_D(x' - x_n)}{\varepsilon_S} dx' = \frac{qN_A}{2\varepsilon_S} x_p^2 - \frac{qN_D(x' - x_n)^2}{2\varepsilon_S}\Bigg|_0^x$$

$$= \frac{qN_A}{2\varepsilon_S} x_p^2 - \left(\frac{qN_D(x - x_n)^2}{2\varepsilon_S} - \frac{qN_D x_n^2}{2\varepsilon_S}\right) = \frac{qN_A}{2\varepsilon_S} x_p^2 + \frac{qN_D x_n^2}{2\varepsilon_S} - \frac{qN_D(x - x_n)^2}{2\varepsilon_S}$$

Built-in potential : $V_0 = V(x_n) = \frac{q}{2\varepsilon_S}\left(N_A x_p^2 + N_D x_n^2\right)$