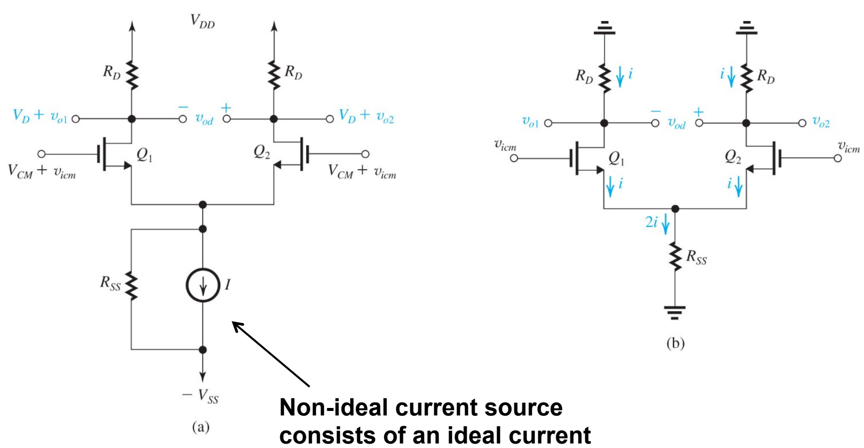
EE105 – Fall 2015
Microelectronic Devices and Circuits

Prof. Ming C. Wu

wu@eecs.berkeley.edu

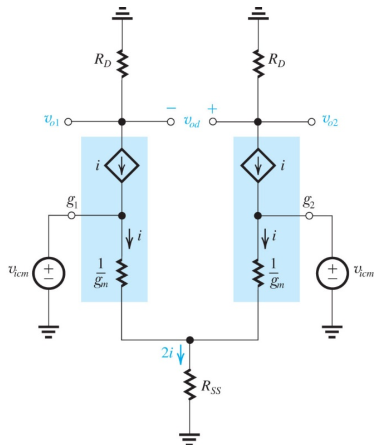
511 Sutardja Dai Hall (SDH)

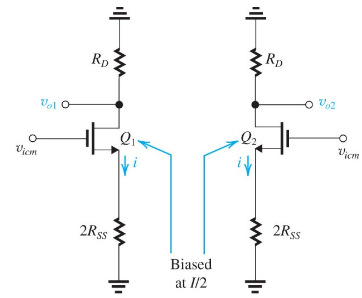
AC Equivalent Circuit for Common Mode Input



Non-ideal current source consists of an ideal current source shunted by a large resistance, R_{SS}

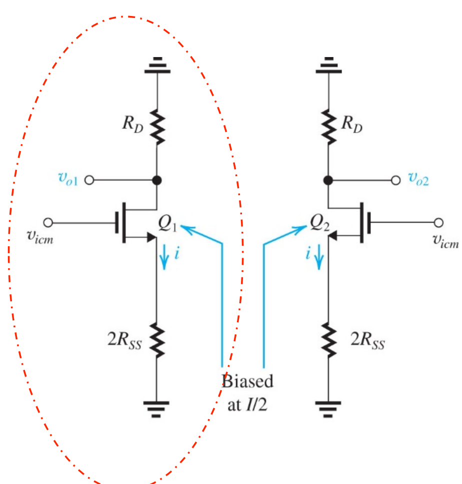
Common Mode "Half Circuit"





- For differential inputs, the two half circuits are anti-symmetric, and the joint (Source) is always at virtual ground
- For common-mode inputs, the two half circuits are symmetric. The Source is not virtual ground any more.
- R_{SS} can be considered as two parallel combination of $2R_{SS}$.
- Each CM half circuit has $2R_{SS}$ connected to the source

Ideal CM Output Voltage



Common-Source with degeneration

The common-mode half-circuit is basically a common-source amplifier with source degeneration.

The gain is

$$\frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} = \frac{-R_D}{1/g_m + 2R_{SS}}$$

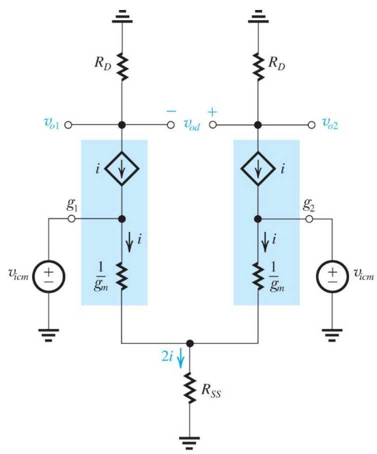
Since $2R_{SS} >> 1/g_m$,

$$\frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} \approx \frac{-R_D}{2R_{SS}}$$

$$v_{od} = v_{o2} - v_{o1} = 0$$

Output voltage is zero for ideal differential pair with perfectly matched transistors and resistors, and the CM voltage is small enough that Q_1 and Q_2 remains in Saturation

Common Mode Gain with Mismatched R_D



However, any mismatch in the half circuits will produce finite output voltage, e.g.,

$R_{D2} = R_D + \Delta R_D$

$$v_{od} = v_{o2} - v_{o1} = \frac{-\Delta R_D}{2R_{SS}} v_{icm}$$

Common mode gain:

$$A_{cm} = \frac{v_{od}}{v_{icm}} = \frac{-\Delta R_D}{2R_{SS}} = \left(\frac{-R_D}{2R_{SS}}\right)\left(\frac{\Delta R_D}{R_D}\right)$$

Common Mode Rejection Ratio (CMRR):

$$CMRR \equiv \frac{|A_d|}{|A_{cm}|}, \qquad \text{in dB: } CMRR(dB) \equiv 20\log\frac{|A_d|}{|A_{cm}|}$$

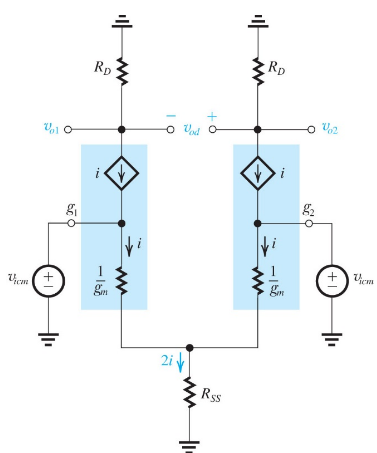
Corrected 12/15/15

CMRR should be as large as possible.

For the above case,

$$\text{CMRR} = \frac{g_m R_D}{\dfrac{\Delta R_D}{2R_{SS}}} = \frac{2g_m R_{SS}}{\left(\dfrac{\Delta R_D}{R_D}\right)}$$

Common Mode Gain with Mismatch of g_m



Mismatch in g_m :

$$g_{m1} = g_m + \frac{1}{2}\Delta g_m; \qquad g_{m2} = g_m - \frac{1}{2}\Delta g_m$$

$$g_{m1} - g_{m2} = \Delta g_m$$

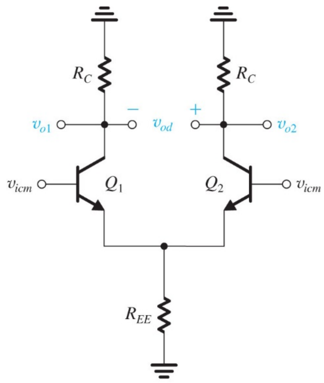
(Derivation skipped)

$$A_{cm} = \frac{v_{od}}{v_{icm}} = \frac{R_D}{2R_{SS}}\frac{\Delta g_m}{g_m}$$

$$\text{CMRR} = \frac{g_m R_D}{\dfrac{R_D}{2R_{SS}}\dfrac{\Delta g_m}{g_m}} = \frac{2g_m R_{SS}}{\left(\dfrac{\Delta g_m}{g_m}\right)}$$

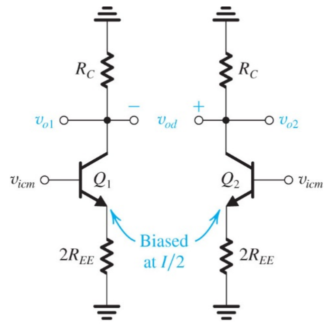
BJT Differential Amplifier

Differential Amplifier



(a)

Half Circuit



(b)

Similarly for BJT

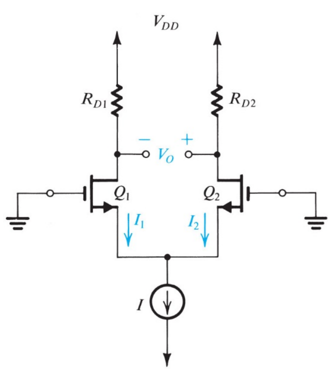
$A_d = g_m R_C$

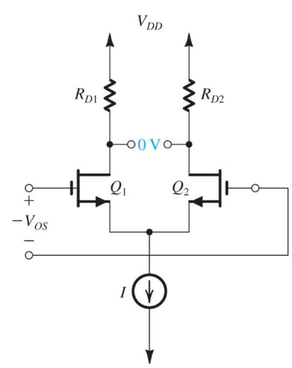
Common-mode gain due to mismatch of R_C :

$$A_{cm} = \frac{v_{od}}{v_{icm}} = \frac{-R_C}{2R_{EE}} \frac{\Delta R_C}{R_C}$$

$$\text{CMRR} = \frac{2g_m R_{EE}}{\left(\frac{\Delta R_C}{R_C}\right)}$$

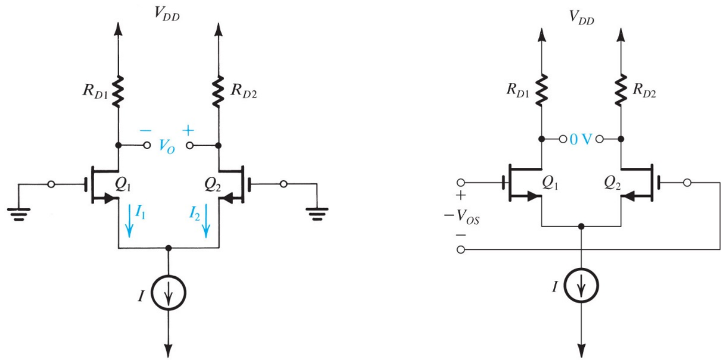
DC Offset





Due to mismatch in R_D, output voltage $V_O \neq 0$ even both inputs are grounded. To produce zero output, an input offset voltage $V_{OS} = \frac{V_O}{A_d}$, where A_d is differential gain, needs to be applied.

DC Offset

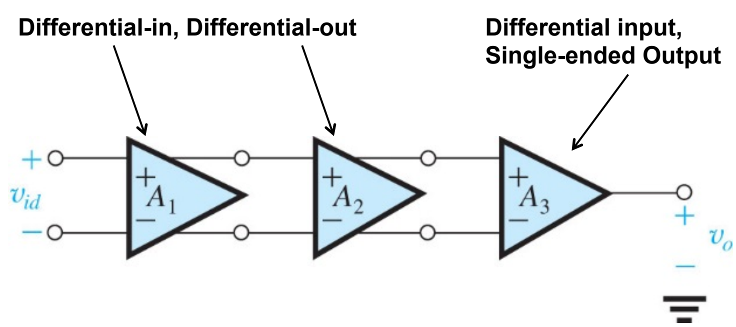


For example, DC offset caused by mismatch in R_D :

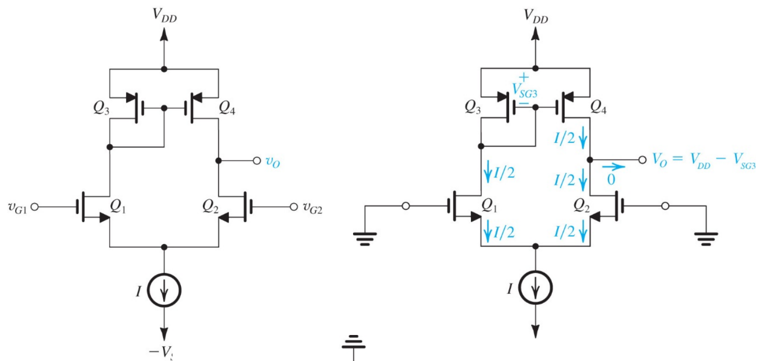
$$R_{D1} = R_D + \Delta R_D / 2; \quad R_{D2} = R_D - \Delta R_D / 2; \quad V_O = V_{D2} - V_{D1} = \frac{I}{2}\Delta R_D$$

$$V_{OS} = \frac{V_O}{A_d} = \frac{I\Delta R_D / 2}{g_m R_D} = \frac{I\Delta R_D / 2}{2\left(\frac{I/2}{V_{OV}}\right)R_D} = \frac{V_{OV}}{2}\frac{\Delta R_D}{R_D}$$

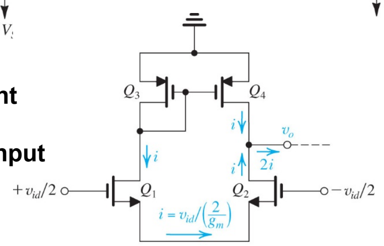
Differential Input, Single-End Output



MOS Differential Pair with Current Mirror Load

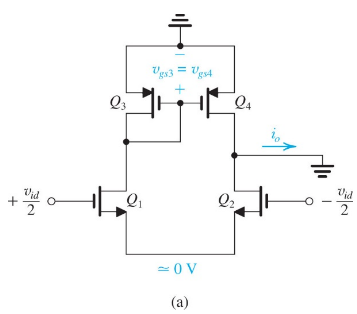


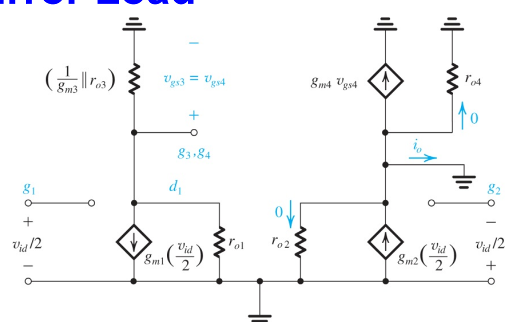
AC equivalent circuit for differential input



Current mirror forces small-signal currents through Q_3 and Q_4 to be the same
→ output currents = 2x that of half circuit

MOS Differential Pair with Current Mirror Load





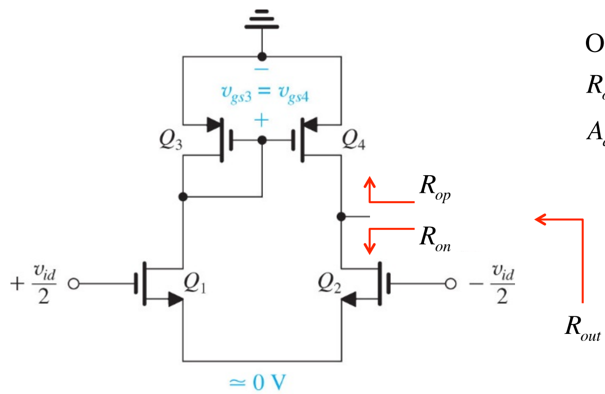
Short-circuit transconductance of differntial pair with current mirror load:

$G_m = \frac{i_o}{v_{id}}$; i_o : output current with short-circuit load

$$i_o = g_{m2}\left(\frac{v_{id}}{2}\right) - g_{m4}v_{gs4}; \quad v_{gs4} = v_{gs3} = -g_{m1}\left(\frac{v_{id}}{2}\right)\left(\frac{1}{g_{m3}} \| r_{o3} \| r_{o1}\right) \approx -\frac{g_{m1}}{g_{m3}}\frac{v_{id}}{2}$$

$$g_{m1} = g_{m2} = g_m; \quad g_{m3} = g_{m4} \quad \Rightarrow \quad i_o = g_m v_{id} \quad \Rightarrow \quad G_m = g_m$$

MOS Differential Pair with Current Mirror Load



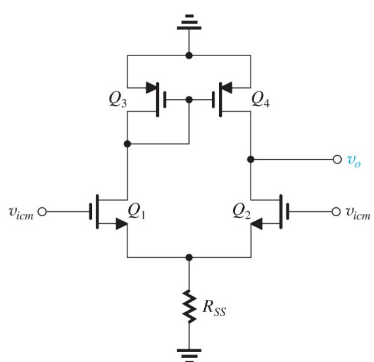
(a)

Output resistance,

$R_{out} = R_{op} \parallel R_{on} = r_{o4} \parallel r_{o2}$

$A_d = G_m R_{out} = g_m \left(r_{o4} \parallel r_{o2} \right)$

Common Mode Gain



(a)

Common mode gain (derivation skipped):

$$A_{cm} \approx -\frac{1}{2 g_{m3} R_{SS}}$$

$$CMRR = \frac{|A_d|}{|A_{cm}|} = g_m \left(r_{o4} \parallel r_{o3} \right) \cdot 2 g_{m3} R_{SS}$$

For $r_{o4} = r_{o3} = r_o$ and $g_m = g_{m3}$

$$CMRR = \left(g_m r_o \right) \left(g_m R_{SS} \right)$$