AC Equivalent Circuit for Common Mode Input

Non-ideal current source consists of an ideal current source shunted by a large resistance, $R_{SS}$
Common Mode “Half Circuit”

- For differential inputs, the two half circuits are anti-symmetric, and the joint (Source) is always at virtual ground.
- For common-mode inputs, the two half circuits are symmetric. The Source is not virtual ground any more.
- \( R_{SS} \) can be considered as two parallel combination of \( 2R_{SS} \).
- Each CM half circuit has \( 2R_{SS} \) connected to the source.

Ideal CM Output Voltage

The common-mode half-circuit is basically a common-source amplifier with source degeneration.

The gain is

\[
\frac{v_{s1}}{v_{cm}} = \frac{-R_D}{V_{cm}} = \frac{-R_D}{1/g_m + 2R_{SS}}
\]

Since \( 2R_{SS} \gg 1/g_m \),

\[
\frac{v_{s1}}{v_{cm}} = \frac{-R_D}{V_{cm}} = \frac{-R_D}{2R_{SS}}
\]

\( v_{cm} = v_{s2} - v_{s1} = 0 \)

Output voltage is zero for ideal differential pair with perfectly matched transistors and resistors, and the CM voltage is small enough that \( Q_1 \) and \( Q_2 \) remains in Saturation.
Common Mode Gain with Mismatch of $R_D$ 

However, any mismatch in the half circuits will produce finite output voltage, e.g.,

\[ R_D = R_D + \Delta R_D \]

\[ v_{od} = v_{o2} - v_{o1} = \frac{-\Delta R_D}{2R_{SS}} v_{xw} \]

Common mode gain:

\[ A_{cm} = \frac{v_{od}}{v_{xw}} = \frac{-\Delta R_D}{2R_{SS}} R_D \frac{\Delta R_D}{R_D} \]

Common Mode Rejection Ratio (CMRR):

\[ CMRR = \frac{|A_o|}{|A_m|} \text{ in dB: } CMRR(dB) = 20 \log \left( \frac{|A_o|}{|A_m|} \right) \]

CMRR should be as large as possible.

For the above case,

\[ CMRR = \frac{g_m R_D}{2 g_m R_{SS}} \frac{\Delta R_D}{R_D} \]

Corrected 12/15/15

Common Mode Gain with Mismatch of $g_m$ 

Mismatch in $g_m$ :

\[ g_{od} = g_m + \frac{1}{2} \Delta g_m; \quad g_{m2} = g_m - \frac{1}{2} \Delta g_m \]

\[ g_{od} - g_{m2} = \Delta g_m \]

(Derivation skipped)

\[ A_{cm} = \frac{v_{od}}{v_{xw}} = \frac{R_D}{2R_{SS}} \frac{\Delta g_m}{g_m} \]

\[ CMRR = \frac{g_m R_D}{2 g_m R_{SS}} \frac{\Delta g_m}{g_m} \]

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**BJT Differential Amplifier**

Similarly for BJT

\[ A_D = g_m R_C \]

Common-mode gain due to mismatch of \( R_C \):

\[ A_{CM} = \frac{v_{CM}}{v_{CM}} = \frac{-R_C \Delta R_C}{2R_{EX} R_C} \]

\[ CMRR = \frac{2g_m R_{EX}}{\Delta R_C} \]

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**DC Offset**

Due to mismatch in \( R_D \), output voltage \( V_O \neq 0 \) even both inputs are grounded. To produce zero output, an input offset voltage

\[ V_{OS} = \frac{V_O}{A_D} \]

where \( A_D \) is differential gain, needs to be applied.
For example, DC offset caused by mismatch in \( R_D \):  
\[
R_{D1} = R_D + \frac{\Delta R_D}{2}; \quad R_{D2} = R_D - \frac{\Delta R_D}{2}; \quad V_O = V_{D2} - V_{D1} = \frac{I}{2} \Delta R_D
\]

\[
V_{OS} = \frac{V_D}{A_d} = \frac{I \Delta R_D}{2} = \frac{1}{2} \frac{V_{OV}}{V_{OV}} \frac{\Delta R_D}{R_D}
\]
MOS Differential Pair with Current Mirror Load

AC equivalent circuit for differential input

Current mirror forces small-signal currents through Q₃ and Q₄ to be the same → output currents = 2x that of half circuit

MOS Differential Pair with Current Mirror Load

Short-circuit transconductance of differential pair with current mirror load:

\[ G_m = \frac{i_o}{V_{id}}; \quad i_o : \text{output current with short-circuit load} \]

\[ i_o = g_m \frac{V_{id}}{2} - g_m v_{g3}4; \quad v_{g3}4 = v_{g3} = -g_m \left( \frac{V_{id}}{2} \right) \left( \frac{1}{g_{m3}} \parallel r_{c3} \parallel r_{c4} \right) = -\frac{g_m}{g_{m3}} \frac{V_{id}}{2} \]

\[ g_m = g_m = g_m; \quad g_{m3} = g_{m4} \quad \Rightarrow \quad i_o = g_m v_{id} \quad \Rightarrow \quad G_m = g_m \]
MOS Differential Pair with Current Mirror Load

Output resistance,
\[ R_{out} = R_{op} \parallel R_{on} = r_{o4} \parallel r_{o2} \]
\[ A_d = G_m R_{out} = g_m (r_{o4} \parallel r_{o2}) \]

Common Mode Gain

Common mode gain (derivation skipped):
\[ A_{cm} \approx -\frac{1}{2g_{m3}R_{SS}} \]
\[ CMRR = \left| \frac{A_d}{A_{cm}} \right| = g_m (r_{o4} \parallel r_{o3}) \cdot 2g_{m3}R_{SS} \]
For \( r_{o4} = r_{o3} = r_o \) and \( g_m = g_{m3} \)
\[ CMRR = \left( g_{out} r_o \right) \left( g_m R_{SS} \right) \]