EE105 – Fall 2015
Microelectronic Devices and Circuits
Frequency Response

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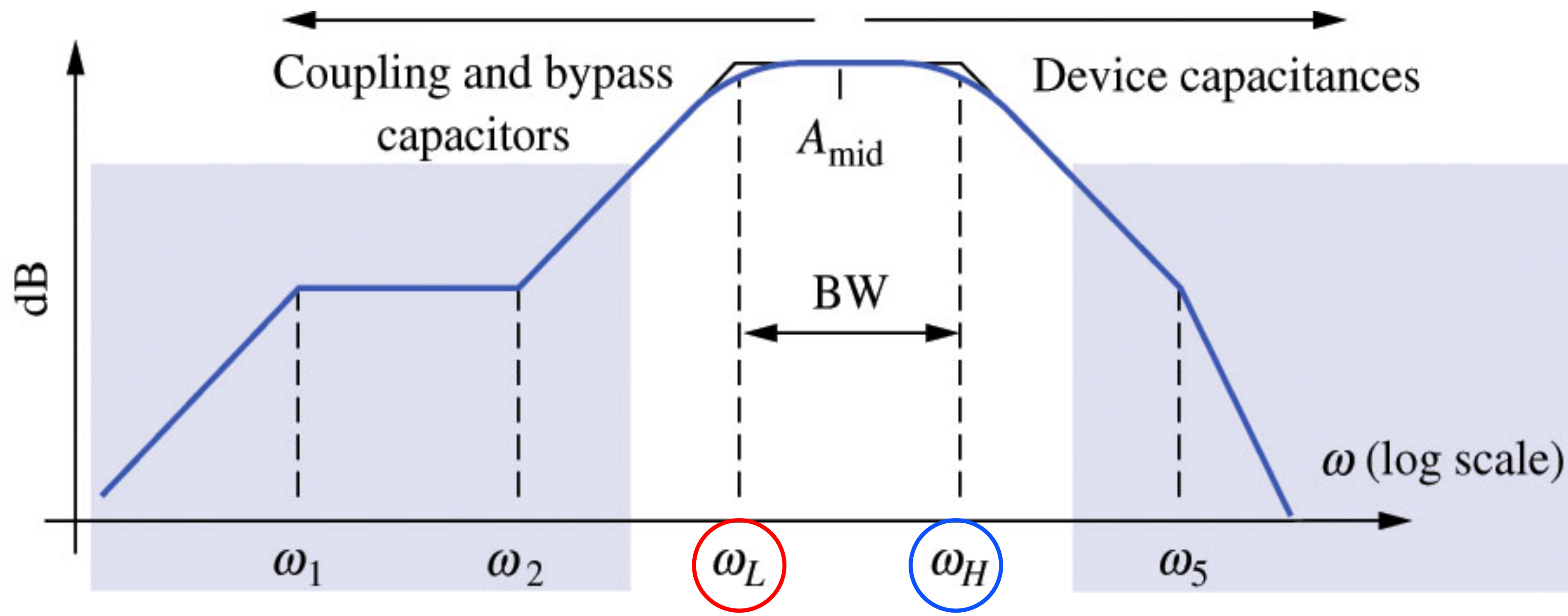
Amplifier Frequency Response: Lower and Upper Cutoff Frequency



- **Midband gain A_{mid} and upper and lower cutoff frequencies ω_H and ω_L that define bandwidth of an amplifier are often of more interest than the complete transfer function**
- **Coupling and bypass capacitors (~ μF) determine ω_L**
- **Transistor (and stray) capacitances (~ pF) determine ω_H**

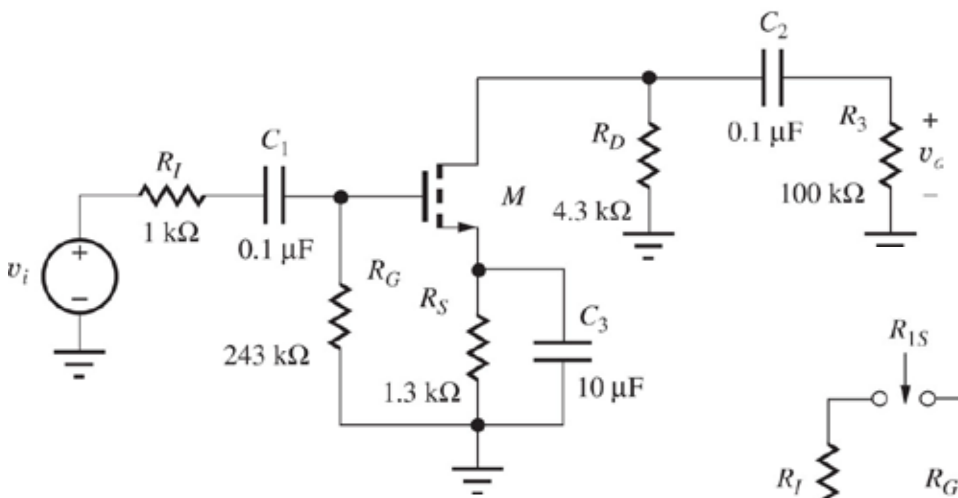
Lower Cutoff Frequency (ω_L) Approximation: Short-Circuit Time Constant (SCTC) Method

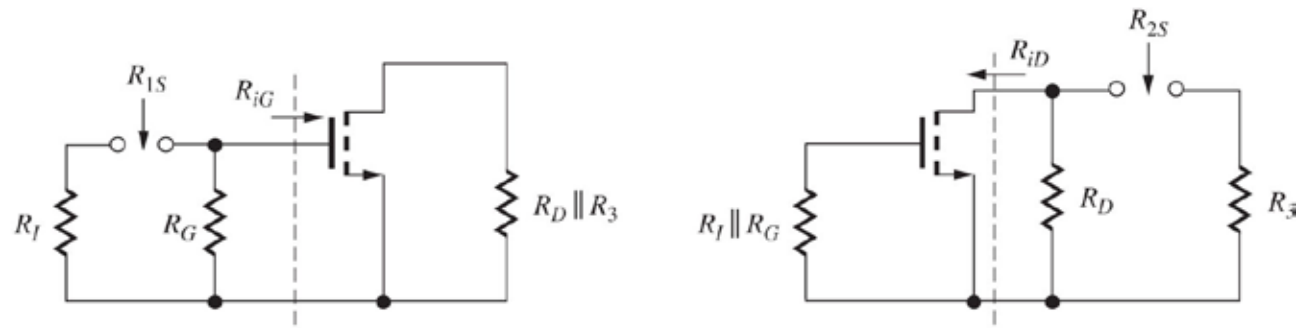
1. Identify all coupling and bypass capacitors
2. Pick one capacitor (C_i) at a time, replace all others with short circuits
3. Replace independent voltage source with *short*, and independent current source with *open*
4. Calculate the resistance (R_{iS}) in parallel with C_i
5. Calculate the time constant, $\frac{1}{R_{iS}C_i}$
6. Repeat this for each of the n capacitor
7. The low cut-off frequency can be approximated by

$$\omega_L \cong \sum_{i=1}^{n} \frac{1}{R_{iS}C_i}$$

Note: this is an approximation. The real low cut-off is slightly lower

Lower Cutoff Frequency (ω_L) Using SCTC Method for CS Amplifier





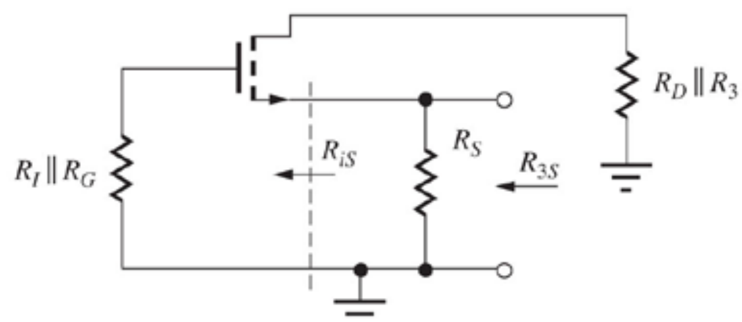
(a) (b)

SCTC Method:

$$f_L \cong \frac{1}{2\pi}\sum_{i=1}^{n}\frac{1}{R_{iS}C_i}$$

For the Common-Source Amplifier:

$$f_L \cong \frac{1}{2\pi}\left(\frac{1}{R_{1S}C_1}+\frac{1}{R_{2S}C_2}+\frac{1}{R_{3S}C_3}\right)$$



Lower Cutoff Frequency (ω_L) Using SCTC Method for CS Amplifier

Using the SCTC method:

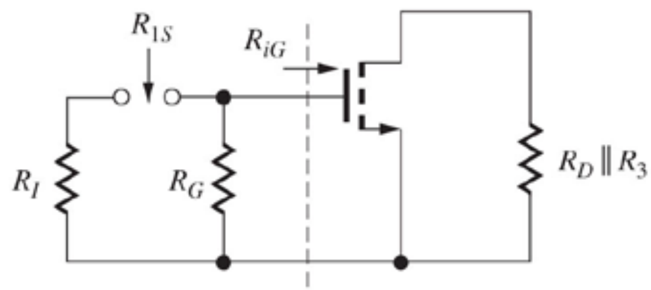
$$f_L \cong \frac{1}{2\pi}\left(\frac{1}{R_{1S}C_1}+\frac{1}{R_{2S}C_2}+\frac{1}{R_{3S}C_3}\right)$$

For C_2 :

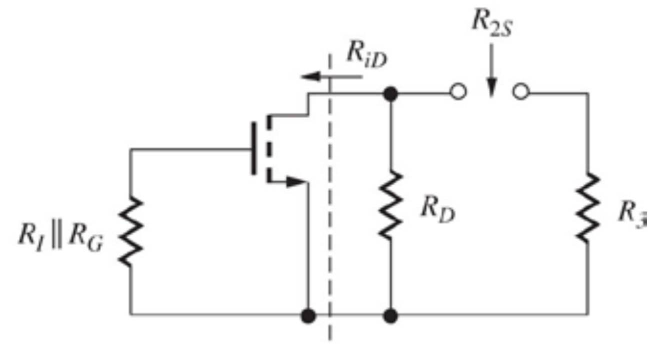
$$R_{3S} = R_3 + \left(R_D \| R_{iD}\right) = R_3 + \left(R_D \| r_o\right)$$

For C_1:

$$R_{1S} = R_I + \left(R_G \| R_{iG}\right) = R_I + R_G$$



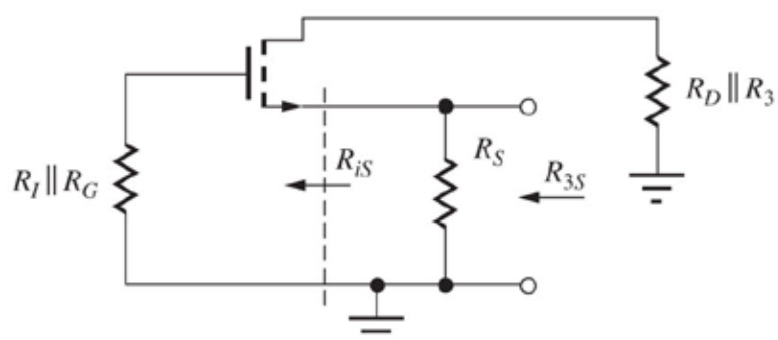
(a)



(b)

For C_3 :

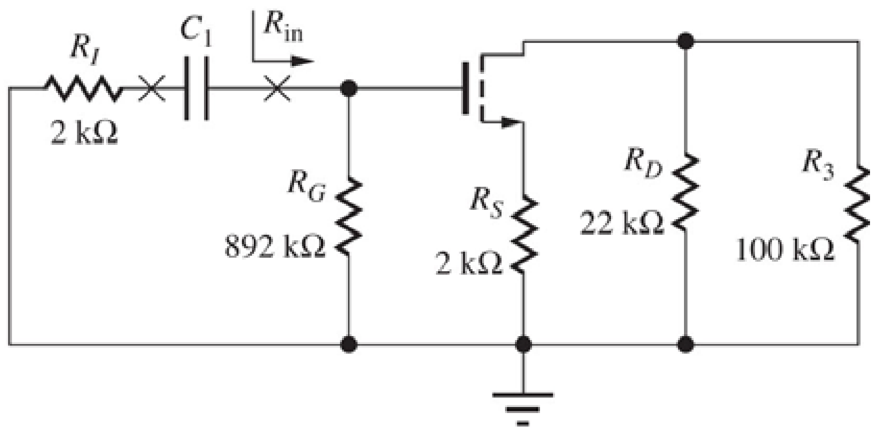
$$R_{2S} = R_S \| R_{iS} = R_S \| \frac{1}{g_m}$$



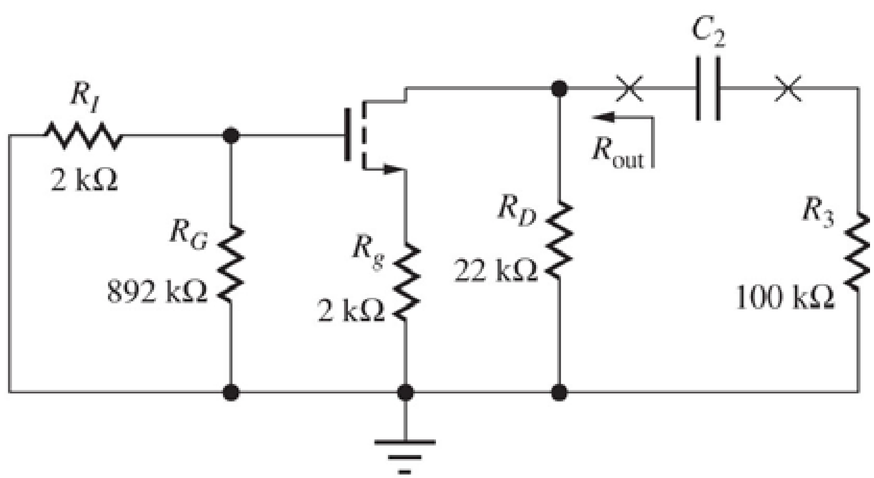
Design: How Do We Choose the Coupling and Bypass Capacitor Values?

- Since the impedance of a capacitor increases with decreasing frequency, coupling/bypass capacitors reduce amplifier gain at low frequencies.
- To choose capacitor values, the short-circuit time constant (SCTC) method is used: each capacitor is considered separately with all other capacitors replaced by short circuits.
- To be able to neglect a capacitor at a given frequency, the magnitude of the capacitor's impedance must be much smaller than the equivalent resistance appearing at its terminals at that frequency

Coupling and Bypass Capacitor Design Common-Source Amplifiers



(b)



For the C-S Amplifier:

$$R_{in} = R_G \| R_{in}^{CS} \qquad R_{out} = R_D \| R_{out}^{CS}$$

For coupling capacitor C_1 :

$$C_1 >> \frac{1}{\omega\left(R_I + R_{in}\right)}$$

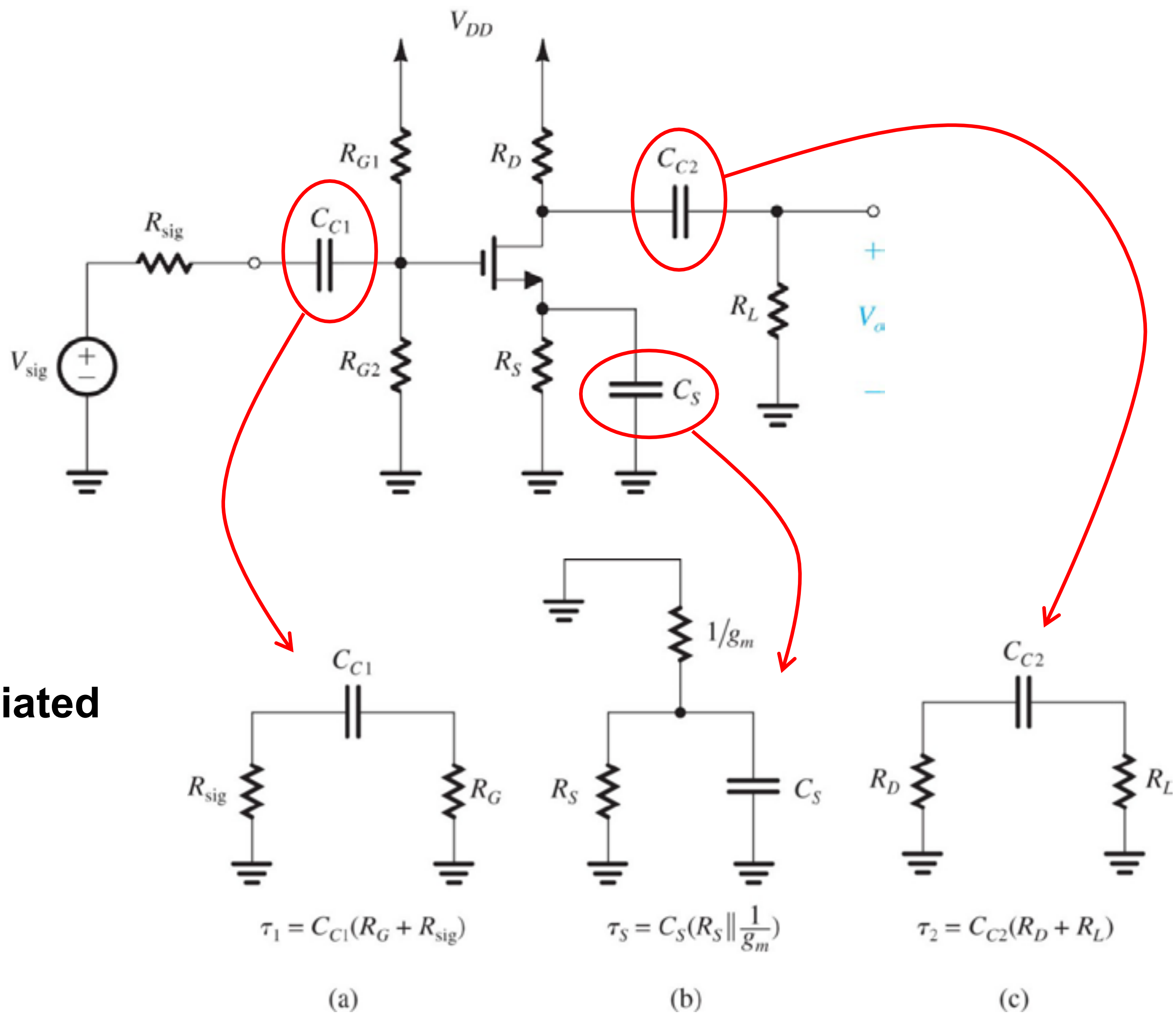
For coupling capacitor C_3 :

$$C_2 >> \frac{1}{\omega\left(R_3 + R_{out}\right)}$$

Lower Cutoff Frequency f_L
Dominant Pole Design

- Instead of having the lower cutoff frequency set by the interaction of several poles, it can be set by the pole associated with just one of the capacitors. The other capacitors are then chosen to have their pole frequencies much below f_L.
- The capacitor associated with the emitter or source part of the circuit tends to be the largest due to low resistance presented by emitter or source terminal of transistor and is commonly used to set f_L.
- Values of other capacitors are increased by a factor of 10 to push their corresponding poles to much lower frequencies.

Capacitively Coupled CS Amplifier



Find the pole frequency associated with each coupling/bypass capacitor:

Low Frequency Response

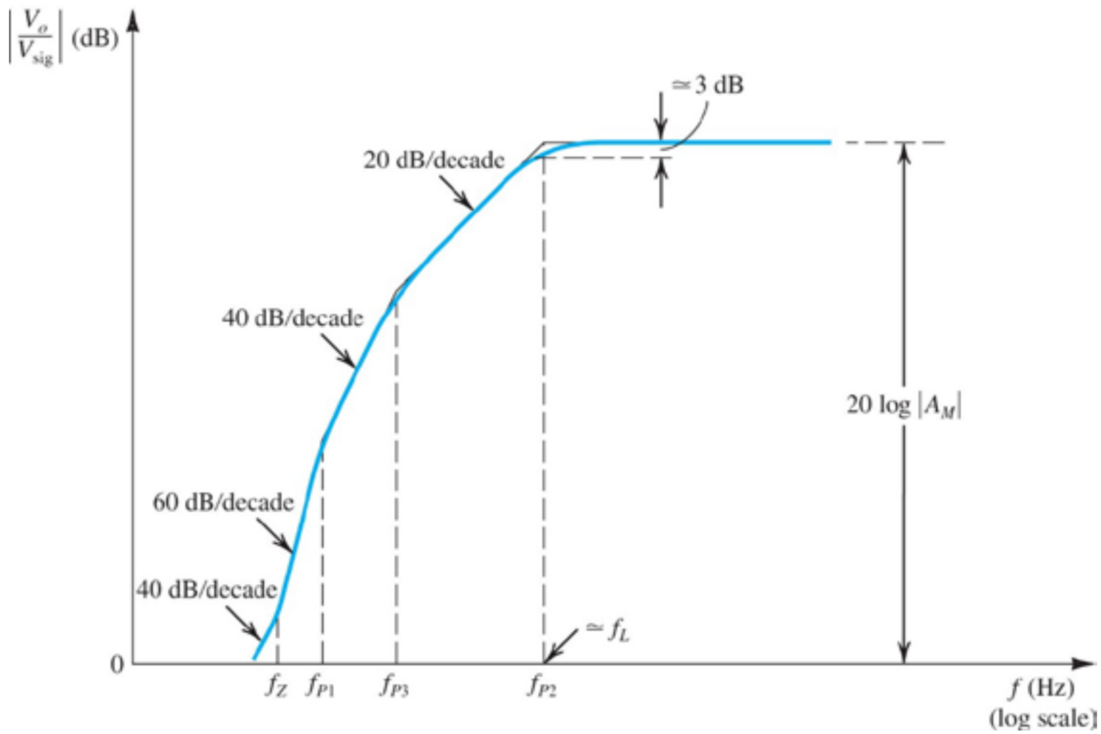
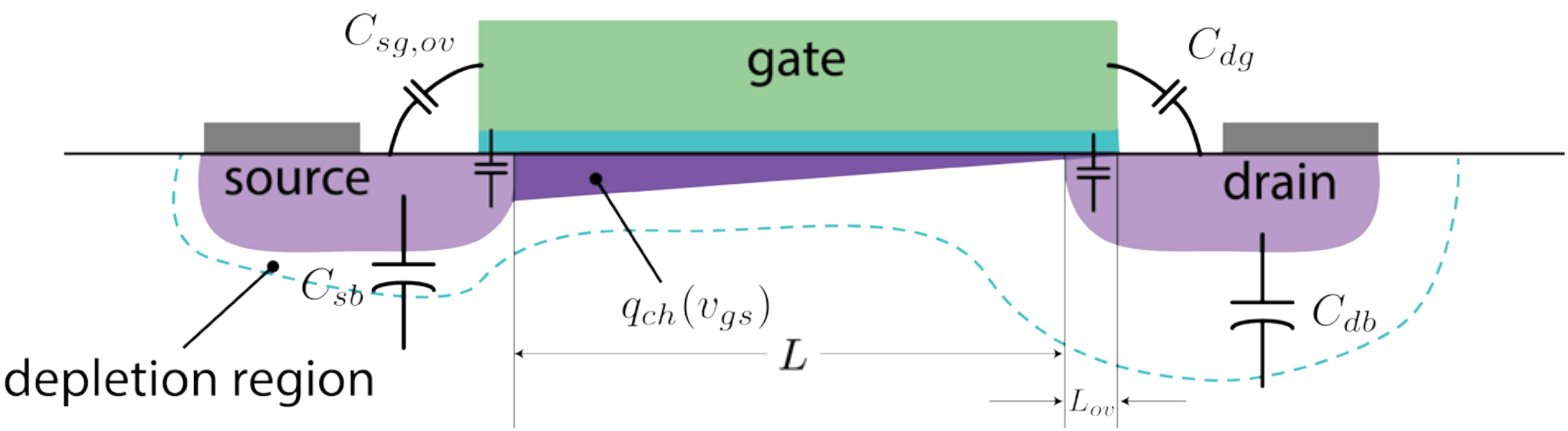


Figure 10.7 Sketch of the low-frequency magnitude response of a CS amplifier for which the three pole frequencies are sufficiently separated for their effects to appear distinct.

High Frequency Response

Capacitors in MOS Device



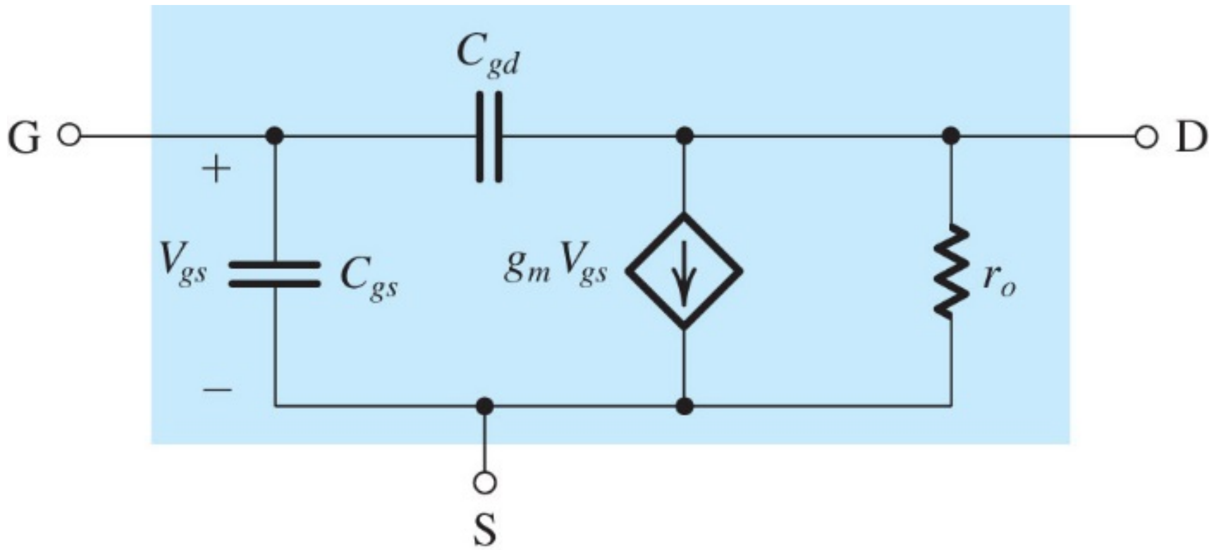
$$C_{gs} = (2/3)WLC_{ox} + C_{ov}$$

$$C_{gd} = C_{ov}$$

$$C_{sb} = C_{jsb}(area + perimeter)\,junction$$

$$C_{db} = C_{jdb}(area + perimeter)\,junction$$

(Simplified) High-Frequency Equivalent-Circuit Model for MOSFET

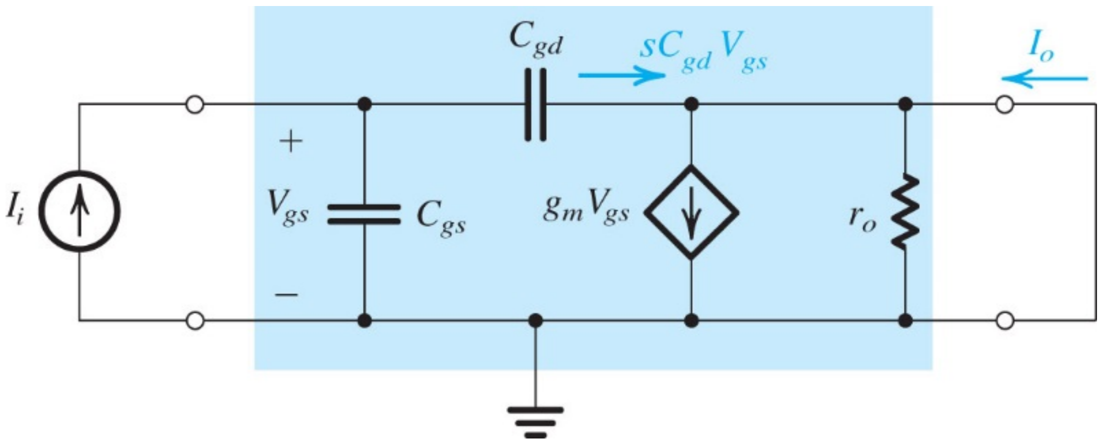


Capacitances between source/body, C_{sb}, and between drain/body, C_{db}, are neglected

Intrinsic Response of FET: Unity-Gain Frequency, f_T

f_T : defined as frequency at which short-circuit current gain = 1

f_T : a figure-of-merit for transistor speed



Drain is grounded (short-circuit load)

$$V_{gs} = \frac{I_i}{sC_{gs} + sC_{gd}}$$

$$\text{KCL:} \quad I_o + \frac{V_{gs}}{1/sC_{gd}} = g_m V_{gs}$$

$$I_o = g_m V_{gs} - sC_{gd}V_{gs} \approx g_m V_{gs}$$

$$= g_m \frac{I_i}{sC_{gs} + sC_{gd}}$$

$$A_I = \frac{I_o}{I_i} = \frac{g_m}{sC_{gs} + sC_{gd}}$$

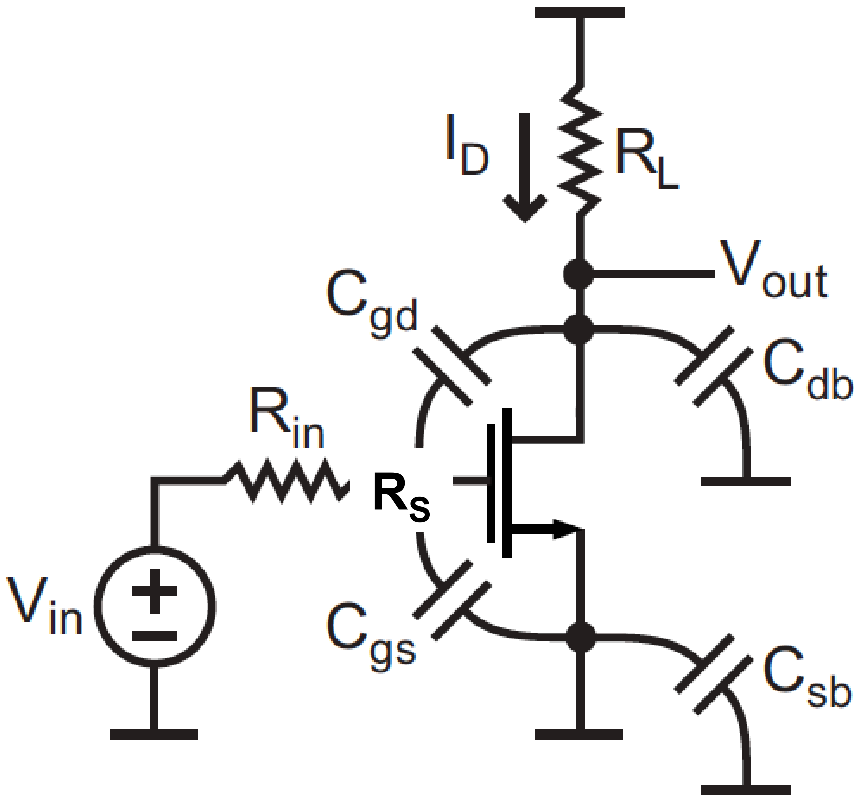
$$s = j\omega$$

$$\left|\frac{I_o}{I_i}\right| = \frac{g_m}{\omega(C_{gs} + C_{gd})}$$

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}}$$

As gate length reduces in advanced technology node, C_{gs} reduces and f_T increases

Common-Source Voltage Amplifier

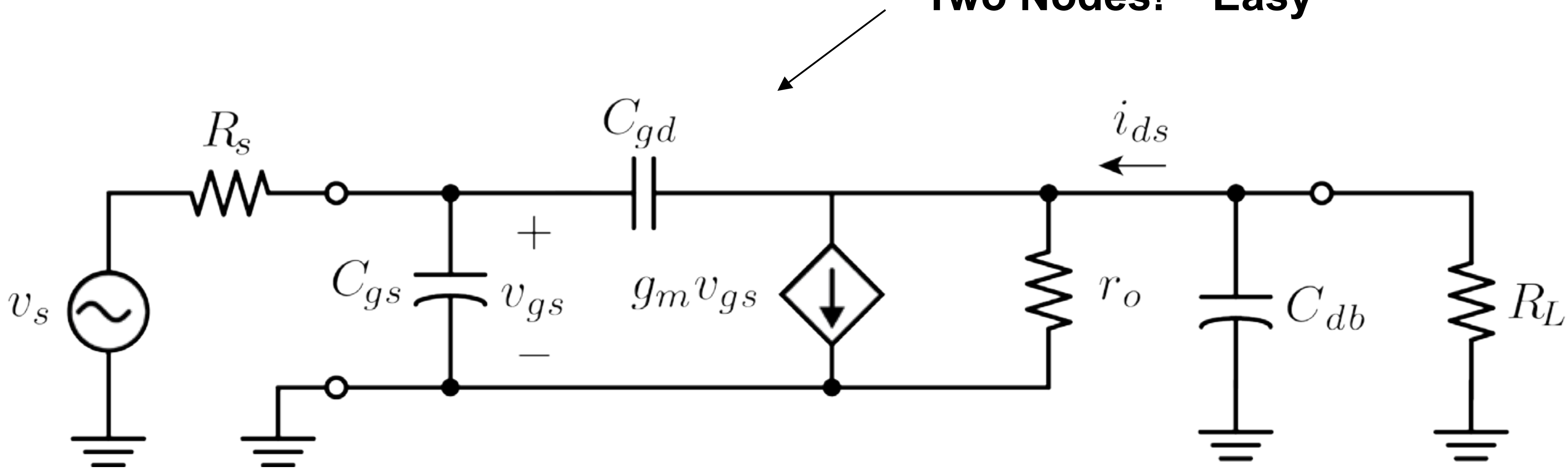


- **Small-signal model:**
- **C_{sb} is connected to ground on both sides, therefore can be ignored**
- **Can solve problem directly by nodal analysis or using 2-port models of transistor**
- **OK if circuit is "small" (1-2 nodes)**

We can find the complete transfer function of the circuit, but in many cases it's good enough to get an estimate of the -3dB bandwidth

CS Voltage Amp Small-Signal Model

Two Nodes! “Easy”



For now we will ignore C_{db} to simplify the math

Frequency Response

KCL at input and output nodes; analysis is made complicated

Zero

$$\frac{V_{out}}{V_{in}} = \frac{-g_m\left[r_o \parallel R_L\right]\left(1 - j\omega/\omega_z\right)}{\left(1 + j\omega/\omega_{p1}\right)\left(1 + j\omega/\omega_{p2}\right)}$$

Two Poles

Low-frequency gain:

$$\frac{V_{out}}{V_{in}} = \frac{-g_m\left[r_o \parallel R_L\right]\left(1 - j0\right)}{\left(1 + j0\right)\left(1 + j0\right)} \rightarrow -g_m\left[r_o \parallel R_L\right]$$

Zero: $$\omega_z = \frac{g_m}{C_{gs} + C_{gd}}$$

Calculating the Poles

$$\omega_{p1} \approx \frac{1}{R_s\left\{C_{gs} + \left(1 + g_m R'_{out}\right)C_{gd}\right\} + R'_{out}C_{gd}}$$

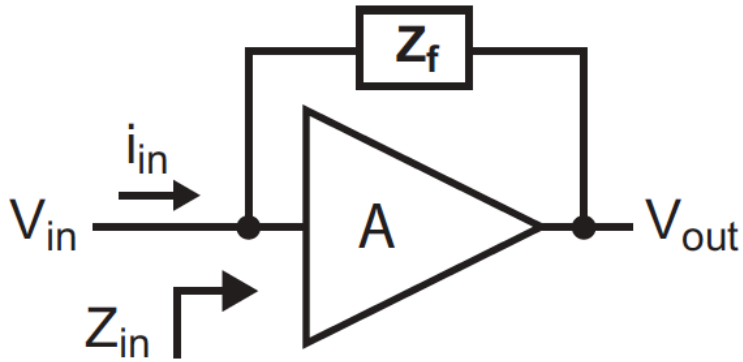
Usually >> 1

$$\omega_{p2} \approx \frac{R'_{out} / R_S}{R_S\left\{C_{gs} + \left(1 + g_m R'_{out}\right)C_{gd}\right\} + R'_{out}C_{gd}}$$

Results of complete analysis: not exact and little insight

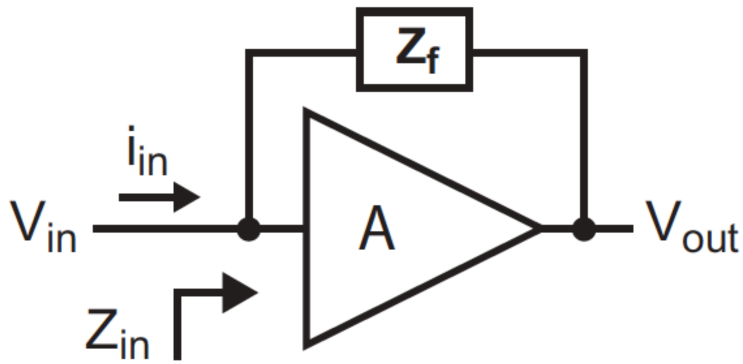
These poles are calculated after doing some algebraic manipulations on the circuit. It's hard to get any intuition from the above expressions.
There must be an easier way!

Method: The Miller Effect



- **Derive input impedance (assume gain of amplifier = *A*):**

The Miller Effect



- Derive input impedance (assume gain of amplifier = *A*):

$$Z_{in} = \frac{V_{in}}{i_{in}} = \frac{V_{in}}{(V_{in} - V_{out})/Z_f} = \frac{V_{in} Z_f}{V_{in} - AV_{in}} = \frac{Z_f}{1 - A}$$

- Consider the case where Z_f is a capacitor

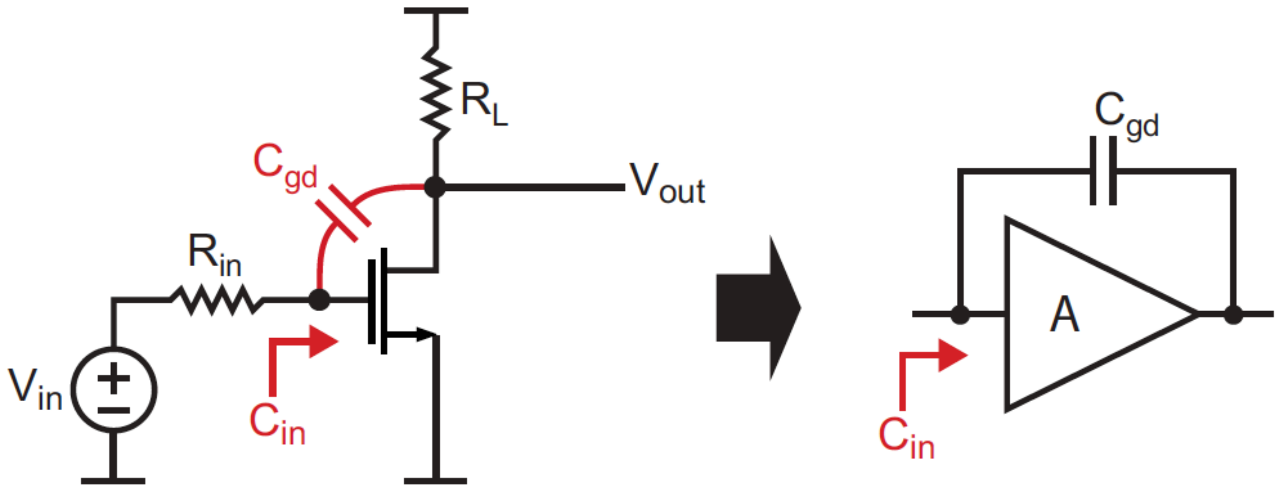
$$Z_f = \frac{1}{sC} \Rightarrow Z_{in} = \frac{1}{s(1-A)C}$$

  - For negative *A*, input impedance sees increased cap value
  - For *A* = 1, input impedance sees no influence from cap
  - For *A* > 1, input impedance sees negative capacitance!

Cal

BSAC

Using The Miller Effect



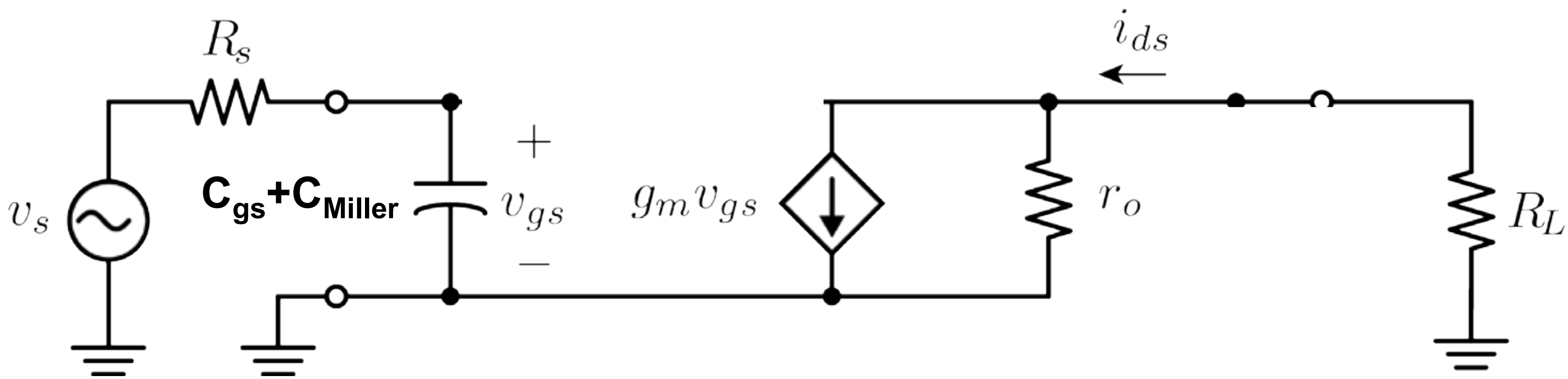
- Notice that C_{gd} is in the feedback path of the common source amplifier
  - Recall Miller effect calculation: $C_{in} = (1 - A)C_{gd}$

Effective input capacitance:

$$C_{in} = \frac{1}{j\omega C_{Miller}} = \left(\frac{1}{1 - A_{v,Cgd}}\right)\left(\frac{1}{j\omega C_{gd}}\right) = \frac{1}{j\omega\left[\left(1 - A_{vCgd}\right)C_{gd}\right]}$$

CS Voltage Amp Small-Signal Model

Modified Small-Signal Model with Miller Effect:



- **We can approximate the first pole by using Miller capacitance**
- **This gives us a good approximation of the -3dB bandwidth**

Comparison with "Exact Analysis"

Miller result (calculate RC time constant of input pole):

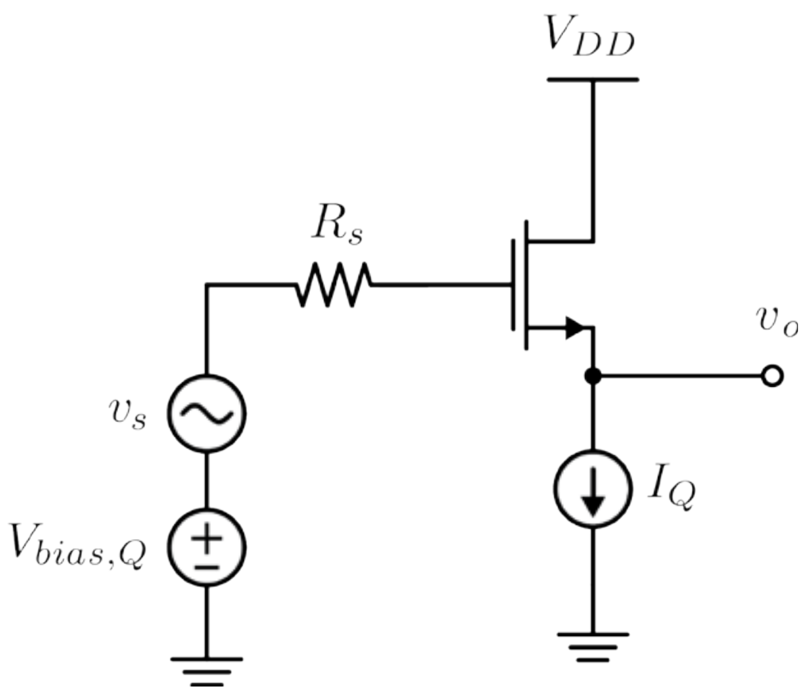
$$\omega_{p1}^{-1} = R_S\left[C_{gs} + \left(1 + g_m R'_{out}\right) C_{gd}\right]$$

Exact result:

$$\omega_{p1}^{-1} = R_S\left[C_{gs} + \left(1 + g_m R'_{out}\right) C_{gd}\right] + R'_{out} C_{gd}$$

As a result of the Miller effect there is a fundamental gain-bandwidth tradeoff

Common Drain Amplifier

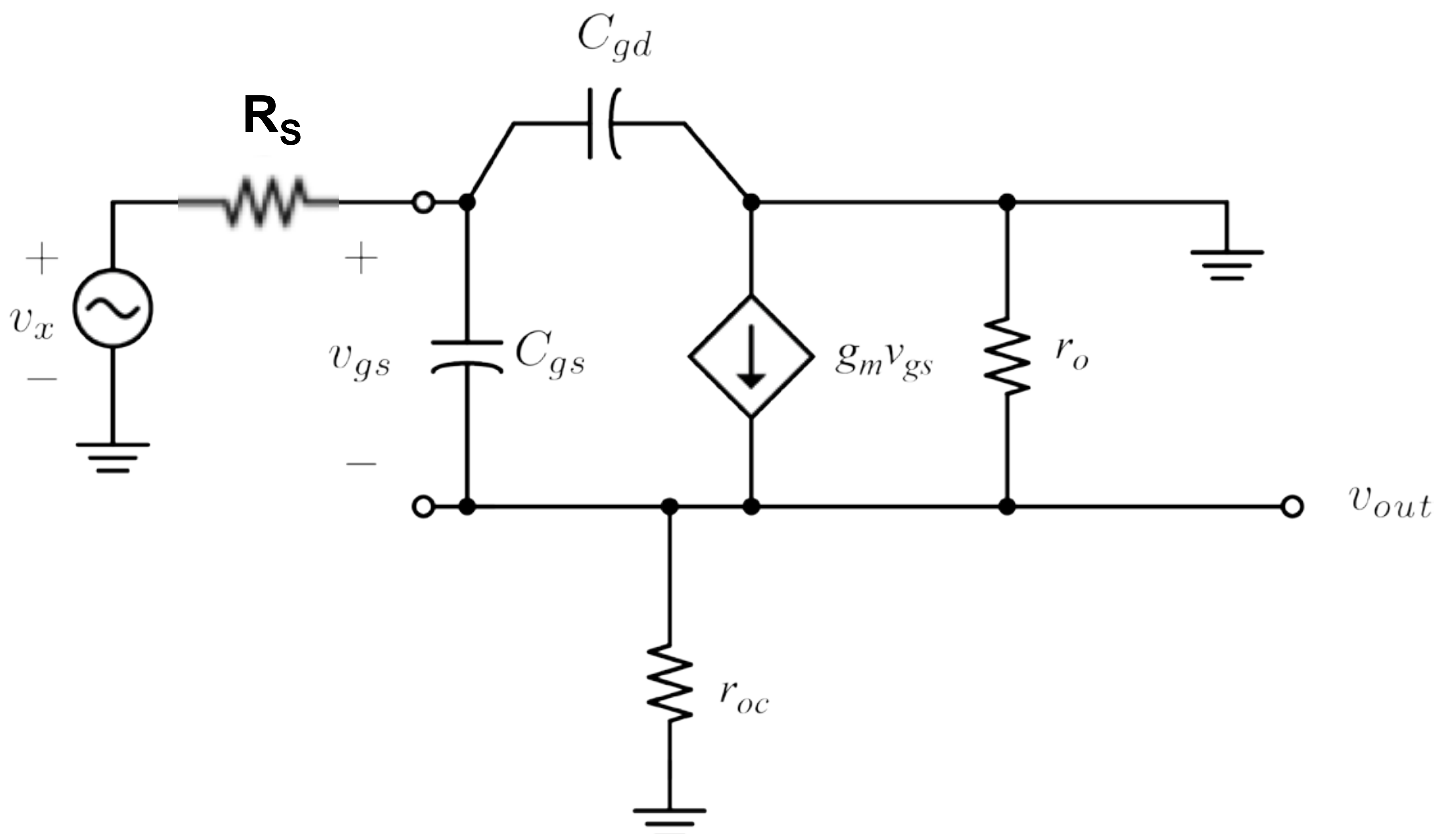


Calculate Bandwidth of the Common Drain (Source-Follower)

Procedure:

1. **Replace current source with MOSFET-based current mirror**
2. **Draw small-signal model with capacitors (for simplicity, we will focus on C_{gd} and C_{gs})**
3. **Find the DC small-signal gain**
4. **Use the Miller effect to calculate the input capacitance**
5. **Calculate the dominant pole**

Small-Signal Model with Capacitors



- **Find DC Gain**
- **Find Miller capacitor for C_{gs} -- *note that the gate-source capacitor is between the input and output!***

Voltage Gain Across C_{gs}

Write KCL at output node:

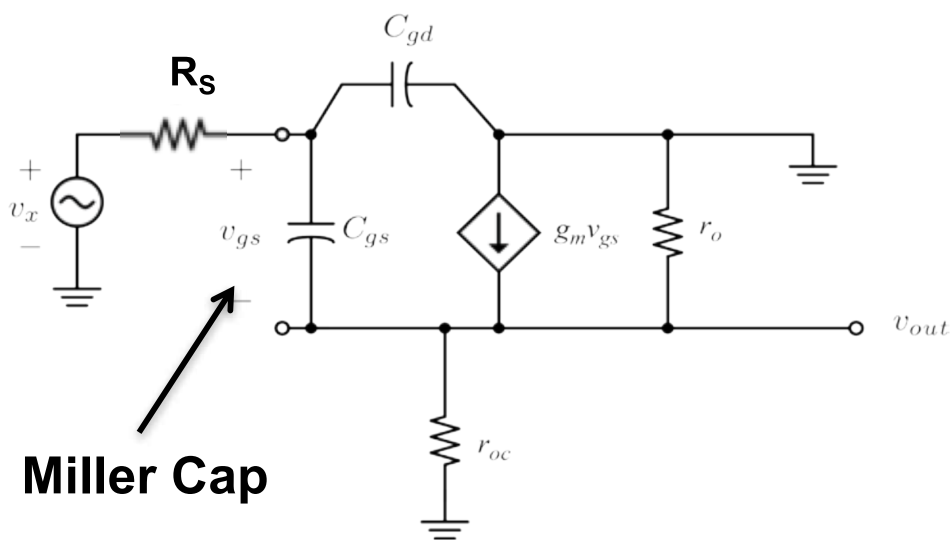
$$\frac{v_{out}}{r_o \| r_{oc}} = g_m v_{gs} = g_m (v_{in} - v_{out})$$

$$v_{out}\left(\frac{1}{r_o \| r_{oc}} + g_m\right) = g_m v_{in}$$

$$\frac{v_{out}}{v_{in}} = \frac{g_m}{\left(\frac{1}{r_o \| r_{oc}} + g_m\right)} = \frac{g_m (r_o \| r_{oc})}{1 + g_m (r_o \| r_{oc})} = A_{vCgs}$$

Compute Miller Effected Capacitance

Now use the Miller Effect to compute C_{in}:
Remember that C_{gs} is the capacitor from the input to the output



$$C_{in} = C_{gd} + C_M$$

$$C_{in} = C_{gd} + (1 - A_{vC_{gs}})C_{gs}$$

$$C_{in} = C_{gd} + \left(1 - \frac{g_m(r_o \| r_{oc})}{1 + g_m(r_o \| r_{oc})}\right)C_{gs}$$

$$C_{in} = C_{gd} + \left(\frac{1}{1 + g_m(r_o \| r_{oc})}\right)C_{gs}$$

$$C_{in} \approx C_{gd}$$ **(for large $g_m(r_o // r_{oc})$)**

Bandwidth of Source Follower

Input low-pass filter's –3 dB frequency:

$$\omega_p^{-1} = R_S\left(C_{gd} + \frac{C_{gs}}{1 + g_m(r_o \| r_{oc})}\right)$$

Substitute favorable values of R_S, r_o:

$$R_S \approx 1/g_m \qquad r_o >> 1/g_m$$

$$\omega_p^{-1} \approx (1/g_m)\left(C_{gd} + \frac{C_{gs}}{1 + BIG}\right) \approx C_{gd}/g_m$$

$$\omega_p \approx g_m / C_{gd}$$

Very high frequency! Model not valid at these high frequencies

Some Examples

***Common Source* Amplifier:**

A_{vCgd} = Negative, large number (-100)

$$C_{Miller} = (1 - A_{V,C_{gd}})C_{gd} \approx 100C_{gd}$$

Miller Multiplied Cap has detrimental impact on bandwidth

***Common Drain* Amplifier:**

A_{vCgs} = Slightly less than 1

$$C_{Miller} = (1 - A_{V,Cgs})C_{gs} \simeq 0$$

"Bootstrapped" cap has negligible impact on bandwidth!