Lecture 20: Transistors As Amplifiers

- **Announcements:**
- HW#6 online and due Friday via Gradescope
- Those in the Wednesday lab section should finish their Lab#3 by going to the lab when the stations are free
- Per Piazza, Lab#3 is due on Friday at 8 p.m. for everyone
- Lab#4 this week
- Midterm 1 today, during class. (This is a recorded lecture.)
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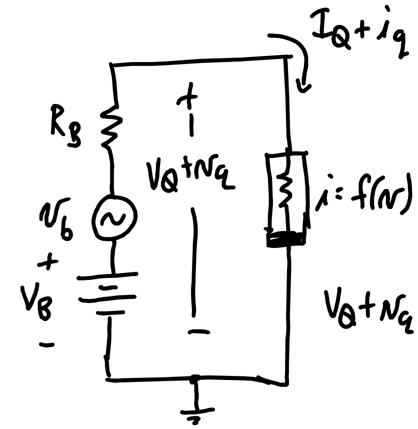
- Lecture Topics:
  - Small Signal Analysis
    - —Linearizing Non-Linear Elements
    - —DC and Small-Signal AC Components
    - —Taylor Series Approximation
  - Transistors As Amplifiers
  - Small-Signal Model for the BJT
- ------------------------------------
- **Last Time:**
- Discussing how Taylor series leads to the small-signal method for solving nonlinear problems
- Now, continue with this …

Using this notation:



$$V_Q + v_q = V_B + v_b - f[V_Q + v_q] R_B$$

For this: Use Taylor series approx. for $f(v)$ around the bias pt. $v = V_Q$.

Review Taylor Series:

$\Rightarrow$ function $f(x)$ can be approximated for points near $x = a$ by:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

1st derivative: $\left.\frac{df}{dv}\right|_{v=a}$

$$\approx \underbrace{f(a) + f'(a)(x-a)}_{\text{linear! easy to solve!}} \quad \text{for } \overbrace{(x-a)}^{\text{small signal}} = \text{small}$$

total signal (x), bias pt. (a)

$$V_Q + v_q = V_B + v_b - f[V_Q] R_B - f'[V_Q] v_q R_B$$

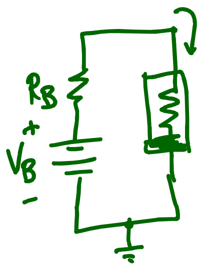
$$V_Q + v_q = V_B + v_b - I_Q R_B - \frac{R_B}{R_{s.s.}} v_q$$

Evaluate nonlinear fcn $I_Q = f(V_Q)$

$$R_{s.s.} = \frac{1}{\left.\frac{\partial f}{\partial v}\right|_{V_Q}} \triangleq \text{small-signal resistance}$$

$\Rightarrow$ Can split this into two equations $\rightarrow$ two ckts.

DC Components: $V_Q = V_B - I_Q R_B$

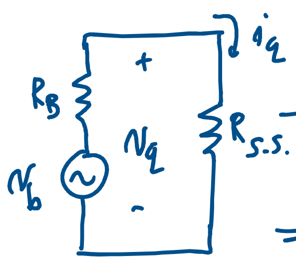


$I_Q = f(V_Q)$

must deal w/ a nonlinear calculation ... but only one!

want: $\underbrace{(V_Q, I_Q)}_{\text{operating pt.}}$

Small-Signal AC Components: $v_q = v_b - \frac{R_B}{R_{s.s.}} v_q \leftarrow$ linear!

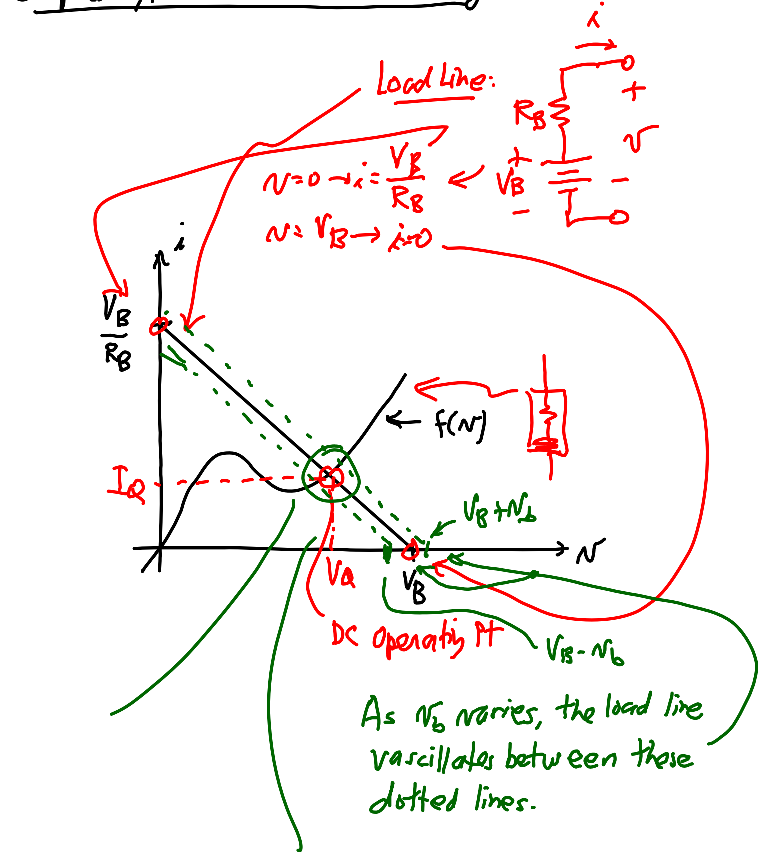


small-signal ckt.

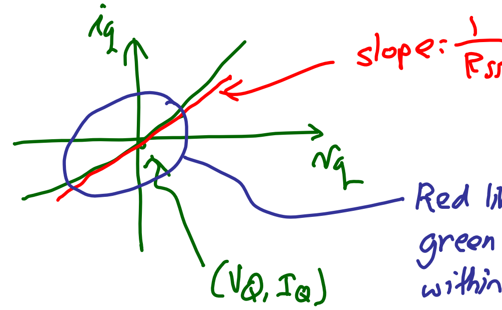
$R_{s.s.} = \frac{1}{\left.\frac{df}{dv}\right|_{V_Q}}$

$\Rightarrow$ all linear analysis! $\rightarrow$ Good!

Graphically, here's what we're doing:

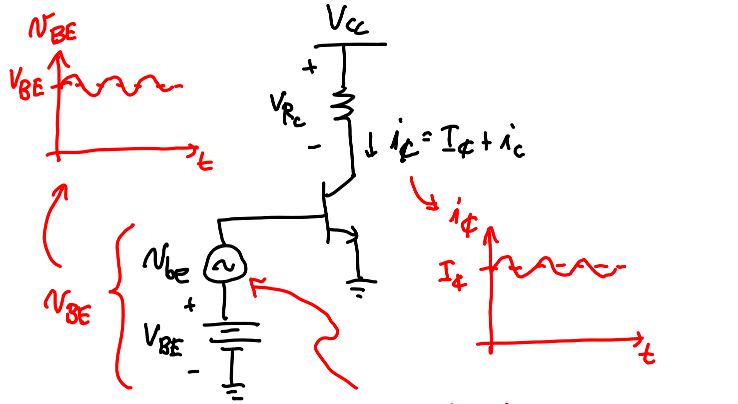


As v_b varies, the load line vascillates between these dotted lines.

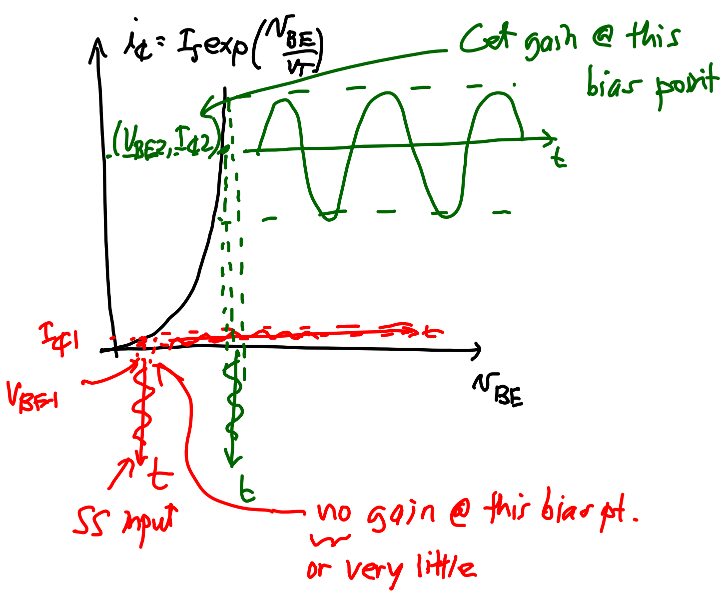


Red line approximates the green curve adequately within small excursions (inside the blue circle)

The Transistor As an Amplifier

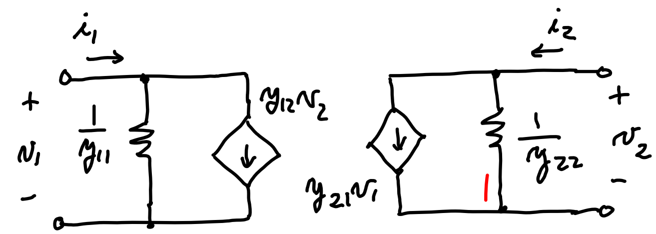


sinusoidal, because practically all signals of interest are AC signals



- So far, we have shown how to obtain the small-signal circuit model for a two-terminal nonlinear resistor
- How about for three-terminal transistors?
- Look back to our general amplifier networks
- Any of the networks apply … but the most popular one is the y-parameter model:

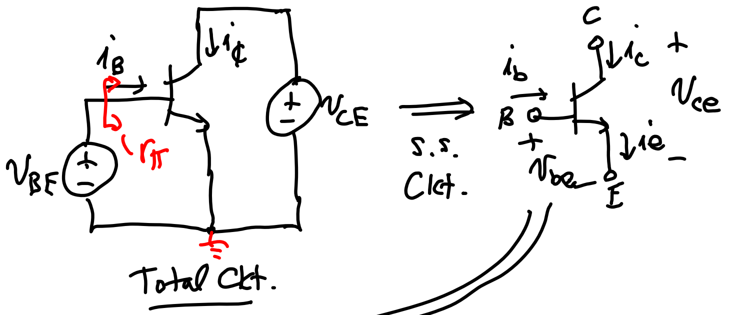
Y-Parameter Model



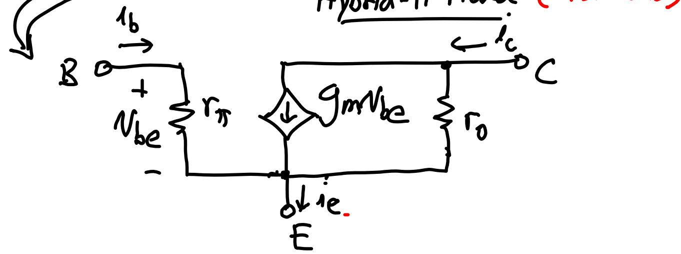
$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$$

$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

Small-Signal Models for Forward-Active BJTs



Hybrid-π Model (mid-band)



$v_{be}\uparrow \rightarrow i_b\uparrow \Rightarrow r_\pi = \frac{v_{be}}{i_b}$

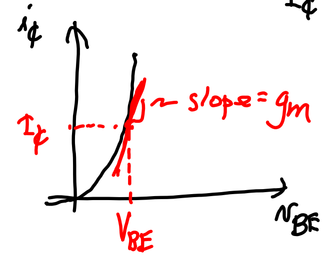
$v_{be}\uparrow \rightarrow i_c\uparrow \Rightarrow g_m = \frac{i_c}{v_{be}}$

$v_{ce}\uparrow \rightarrow i_c\uparrow \Rightarrow r_o = \frac{v_{ce}}{i_c}$

Each of these determined by the bias pt.

Determine the Small-Signal Elements

$g_m = \frac{i_c}{v_{be}}$:



$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$ ← $= V_{BE} + v_{be}$

$I_C + i_c = \underbrace{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}_{I_C} \exp\left(\frac{v_{be}}{V_T}\right)$

$= I_C \exp\left(\frac{v_{be}}{V_T}\right)$

↑ large signal DC collector current

For $v_{be} \ll V_T$ (25mV), can use Taylor series approximation:

$e^x \approx 1 + x$ for small x

$\cancel{I_C} + i_c = I_C\left(1 + \frac{v_{be}}{V_T}\right) = \cancel{I_C} + \frac{I_C}{V_T} v_{be}$

This is the condition that should be satisfied to have a "small-signal"

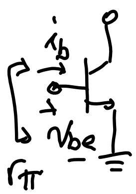
$\frac{i_c}{v_{be}} = \boxed{g_m = \frac{I_C}{V_T}}$ ← fcn. of I_C

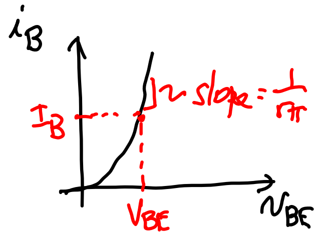
↑ comes from the DC operating pt. (Q-pt.) ↖ quiescent

Typical: $I_C = 1mA$

$g_m = \frac{1m}{25m} = 0.04$ ℧

$r_\pi = \dfrac{v_{be}}{i_b}$:





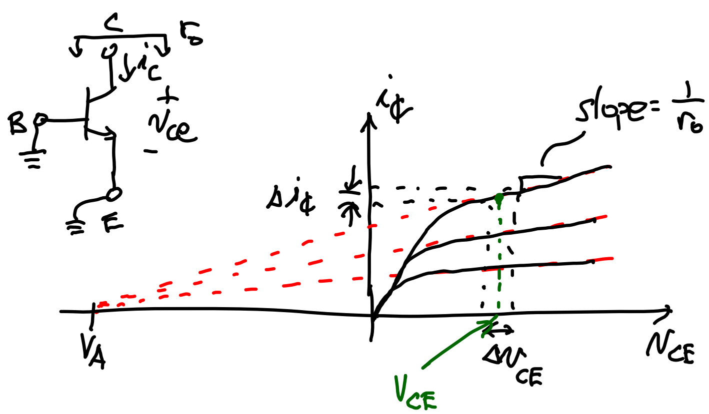
$$r_\pi = \frac{v_{be}}{i_b} = \frac{v_{be}}{\frac{i_c}{\beta}} = \frac{\beta}{\frac{i_c}{v_{be}}} = \boxed{\frac{\beta}{g_m} = \frac{V_T}{I_B} = r_\pi}$$

again, fn. of DC operating pt.

Typical: $\beta = 100$

$$r_\pi = \frac{100}{0.04} = 2.5k\Omega$$

$r_o = \dfrac{v_{ce}}{i_c}$:



$$i_C = I_s \exp\left(\frac{v_{BE}}{V_T}\right)\left(1 + \frac{v_{CE}}{V_A}\right)$$

$$r_o = \left[\frac{\partial i_C}{\partial v_{CE}}\bigg|_{Q\text{-pt.}}\right]^{-1} = \left[\underbrace{I_s \exp\left(\frac{v_{BE}}{V_T}\right)}_{= I_C}\frac{1}{V_A}\bigg|_{v_{BE}=V_{BE}}\right]^{-1}$$

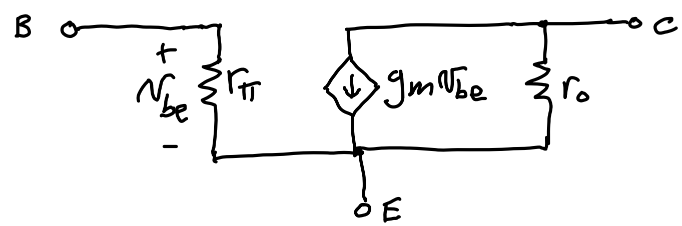
$$\boxed{r_o = \frac{V_A}{I_C}}$$

Typical: $V_A = 100V$

$$r_o = \frac{100}{1m} = 100k\Omega$$

Large, so often we neglect!

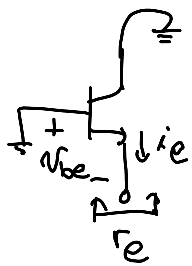
Hybrid-π Model Summary (for npn BJT)



$$\boxed{\begin{aligned} r_\pi &= \frac{\beta}{g_m} = \frac{V_T}{I_B} \\ g_m &= \frac{I_C}{V_T} \\ r_o &= \frac{V_A}{I_C} \end{aligned}}$$

- **Remarks**:
- g_m is independent of device specifics, i.e., β, I_s
- Depends only on temperature (via V_T) and biasing (I_C)
- Small-signal model valid for $v_{be} << V_T$

What about emitter resistance?

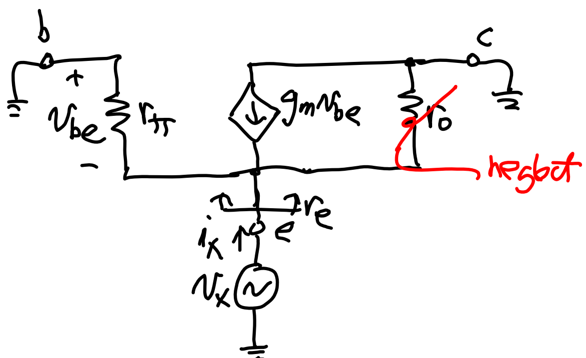


$v_{be}\uparrow \to i_e\uparrow \to r_e = \frac{v_{be}}{i_e}$

$$r_e = \frac{v_{be}}{i_e} = \frac{v_{be}}{\frac{i_c}{\alpha}} = \frac{\alpha}{g_m} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E}$$

$$\boxed{r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m} = \frac{V_T}{I_E}}$$

Why is this not included in the hybrid-π model?
↳ well... it is!



$$i_x = -\frac{v_{be}}{r_\pi} - g_m v_{be}$$

$$(v_x = -v_{be}) \Rightarrow i_x = v_x\left(\frac{1}{r_\pi} + g_m\right)$$

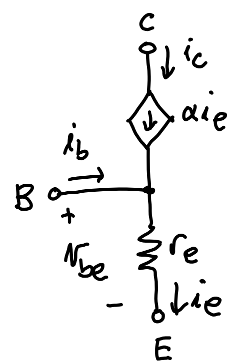
$$r_e = \frac{v_x}{i_x} = \frac{1}{\frac{1}{r_\pi} + g_m} = \frac{r_\pi}{1 + g_m r_\pi} = \frac{r_\pi}{1+\beta} = \frac{\beta}{g_m(1+\beta)}$$

$$\downarrow$$

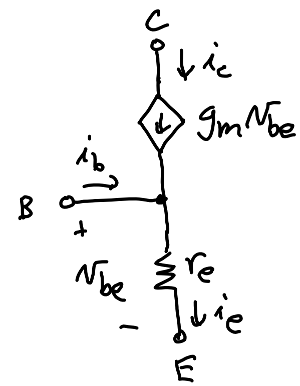
$$r_e = \frac{\alpha}{g_m} \checkmark$$

- To explicitly show the emitter resistance in the small-signal model, use the T-model:

$\boxed{\text{T-Model}}$ (Common Base Model)



or



where (as before): $g_m = \frac{I_C}{V_T}$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$$

