## Lecture 21: MOS Model and Common Emitter Amp

- **Announcements**:
- HW#7 online and due Friday via Gradescope
- Lab#5 online (this is your first project)
  - ↳ Due Tuesday, Nov. 12, 5 p.m.
- Graded Midterm coming back today
- ------------------------------------
- Lecture Topics:
  - ↳ T Model
  - ↳ Pnp Transistor Hybrid-π Model
  - ↳ Saturated NMOS Hybrid-π Model
  - ↳ Example: Common Emitter Amplifier
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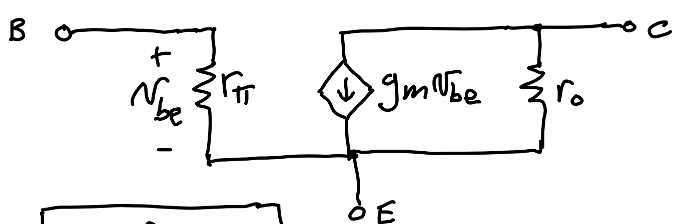
- **Last Time**:
- Derived the BJT Hybrid-π Model
- Now, continue with models …

**Hybrid-π Model Summary** (for npn BJT)



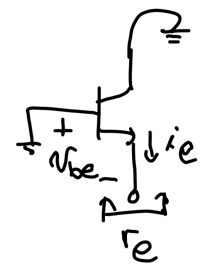

$$r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

$$g_m = \frac{I_C}{V_T}$$

$$r_o = \frac{V_A}{I_C}$$

- **Remarks**:
- $g_m$ is independent of device specifics, i.e., β, $I_s$
- Depends only on temperature (via $V_T$) and biasing ($I_C$)
- Small-signal model valid for $v_{be} << V_T$

**What about emitter resistance?**

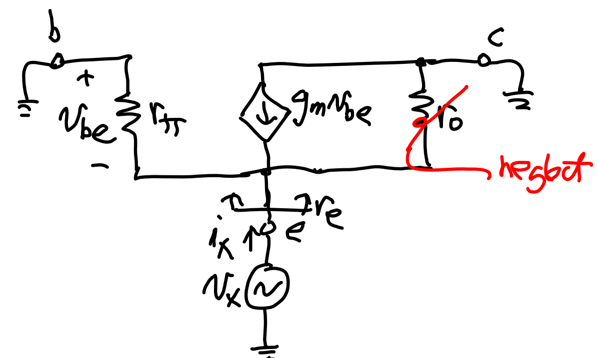



$v_{be}\uparrow \rightarrow i_e\uparrow \rightarrow r_e = \frac{v_{be}}{i_e}$

$$r_e = \frac{v_{be}}{i_e} = \frac{v_{be}}{\frac{i_c}{\alpha}} = \frac{\alpha}{g_m} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E}$$

$$\boxed{r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m} = \frac{V_T}{I_E}}$$

Why is this not included in the hybrid-π model?
↳ well... it is!



$$i_x = -\frac{v_{be}}{r_\pi} - g_m v_{be}$$

$$(v_x = -v_{be}) \Rightarrow i_x = v_x\left(\frac{1}{r_\pi} + g_m\right)$$

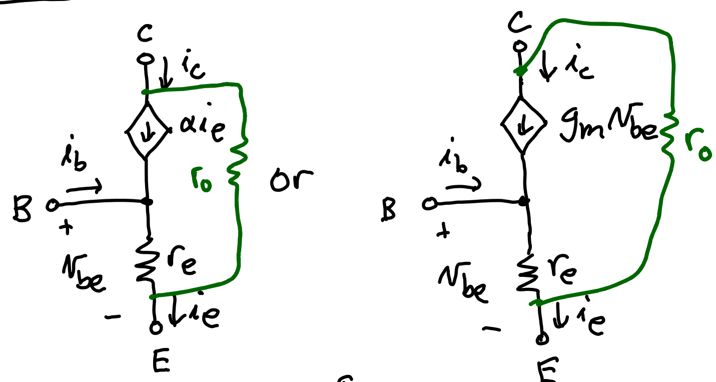
$$r_e = \frac{v_x}{i_x} = \frac{1}{\frac{1}{r_\pi} + g_m} = \frac{r_\pi}{1 + g_m r_\pi} = \frac{r_\pi}{1+\beta} = \frac{\beta}{g_m(1+\beta)}$$

$$\downarrow$$

$$r_e = \frac{\alpha}{g_m} \quad \checkmark$$

- To explicitly show the emitter resistance in the small-signal model, use the T-model:

**T-Model** (Common Base Model)

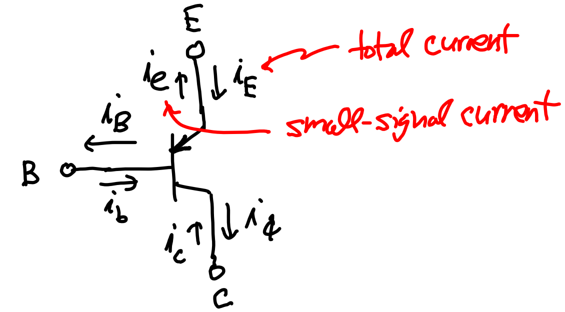



where (as before): $g_m = \frac{I_C}{V_T}$

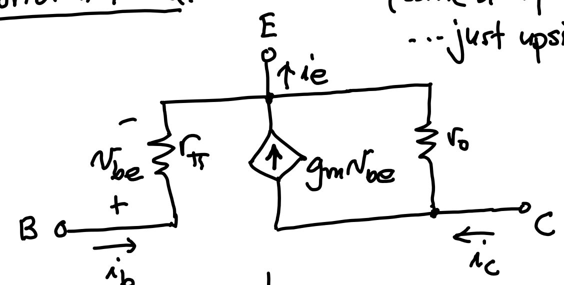
$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$$

**Small-Signal Models for Forward-Active pnp Xsistors**

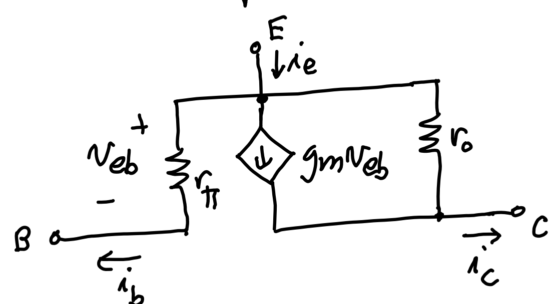
- For pnp transistors, use the same small-signal models as npn with NO change in polarities




**Hybrid-π Model:** (same as npn model ...just upside down)

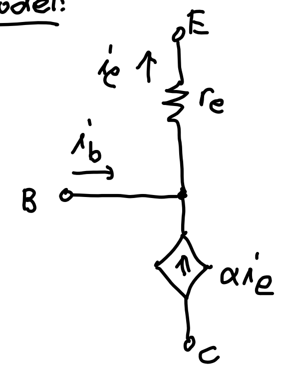



↓ or




(again, same as npn model, but upside down)

T-Model:




- Note that in the above small-signal models, the current directions are the same as used for npn, i.e., no change in polarities
- Large signal directions, however, are as before

Need Proof?

$$i_C = I_S \exp\left(\frac{v_{EB}}{V_T}\right)$$

$$I_C - i_c = I_S \exp\left(\frac{V_{EB} - v_{be}}{V_T}\right)$$

$$= I_S \exp\left(\frac{V_{EB}}{V_T}\right)\exp\left(-\frac{v_{be}}{V_T}\right) = I_C \exp\left(-\frac{v_{be}}{V_T}\right)$$

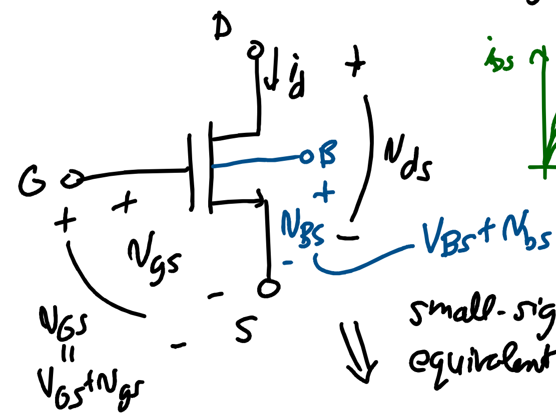
[2 terms of Taylor expansion] $\Rightarrow$ $= I_C\left(1 - \frac{v_{be}}{V_T}\right) = I_C - \frac{I_C}{V_T} v_{be}$

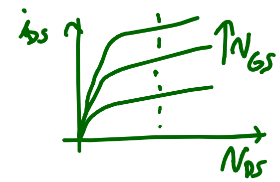
$$i_c = \frac{I_C}{V_T} v_{be} \quad \therefore \quad g_m = \frac{i_c}{v_{be}} = \frac{I_C}{V_T}$$

(as with npn, and with the same directions as npn)

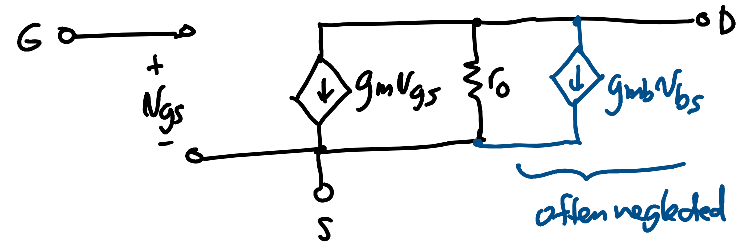
## Small Signal Model for Saturated NMOS Transistor

↳ for analog applications




small-signal equivalent ckt.




Where:

$$g_m = \frac{i_d}{v_{gs}} = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q\text{-}pt} = \left.\frac{\partial}{\partial v_{GS}}\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS} - V_{TN})^2\right)\right|_{Q\text{-}pt}$$

still $C_{ox}''$ →

$$= \mu_n C_{ox}\frac{W}{L}(v_{GS} - V_{TN})\Big|_{v_{GS} = V_{GS}}$$

$$\therefore \boxed{g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TN}) = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TN})^2$$

$$(V_{GS}-V_{TN}) = \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} = V_{OV} \triangleq \text{Overdrive Voltage}$$

$$r_o = \frac{v_{ds}}{i_d} = \left[\frac{\partial i_D}{\partial v_{DS}}\bigg|_{Q\text{-pt}}\right]^{-1}$$

$$= \left[\frac{\partial}{\partial v_{DS}}\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS}-V_{TN})^2(1+\lambda v_{DS})\right)\bigg|_{Q\text{-pt}}\right]^{-1}$$

$$= \left[i_D\lambda\big|_{i_D=I_D}\right]^{-1} = [\lambda I_D]^{-1}$$

$$\Rightarrow \boxed{r_o = \frac{1}{\lambda I_D}}$$

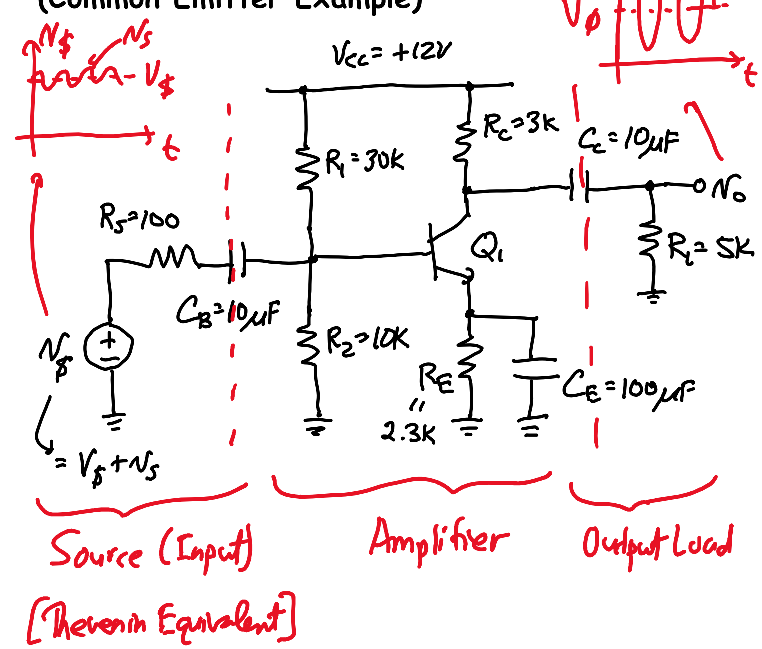
**Threshold Voltage $V_t = f(v_{BS})$:**

$\Rightarrow$ gives rise to a Body Effect Transconductance:

$$g_{mb} = \frac{\partial i_D}{\partial v_{BS}}\bigg|_{Q\text{-pt}} = \boxed{\frac{g_m\gamma}{2\sqrt{2\phi_f + V_{SB}}} = g_{mb}}$$

**Procedure for Small-Signal Analysis**

- (Common Emitter Example)




For Q: $\beta = 100$, $V_A = 100V$

Find the voltage gain, $\frac{v_o}{v_s}$.

Draw the collector voltage waveform for

$$v_\$ = \underbrace{(0.014)\cos\omega_o t}_{\text{AC small-signal component}} + \underbrace{1V}_{\text{DC component}}$$

Graphically, here's what we're looking to do: