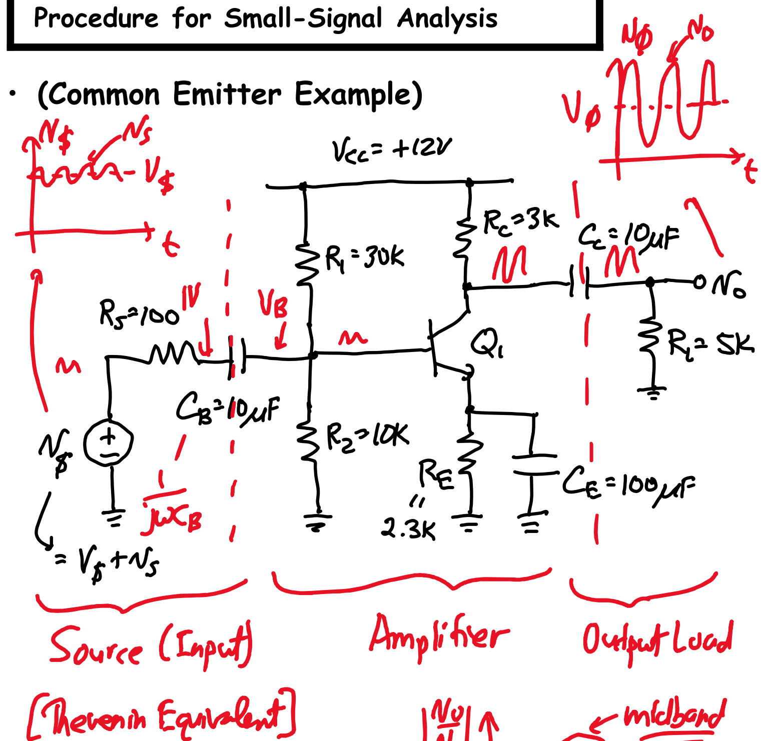
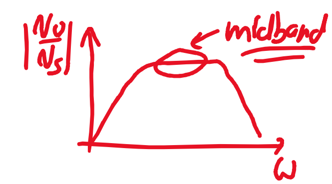
Lecture 23: Small-Signal CE Amplifier Analysis

- Announcements:
- HW#7 online and due Friday via Gradescope
- Lab#5 due Tuesday, Nov. 12, 5 p.m.
- Gave Z scores last time
- ------------------------------------
- Lecture Topics:
  - Finish Common Emitter Amplifier
  - Frequency Response
  - High Frequency Model for BJT
- ------------------------------------
- Last Time:
- Going through a Common Emitter Amplifier small-signal analysis example
- Now, continue with this …

Procedure for Small-Signal Analysis

- (Common Emitter Example)





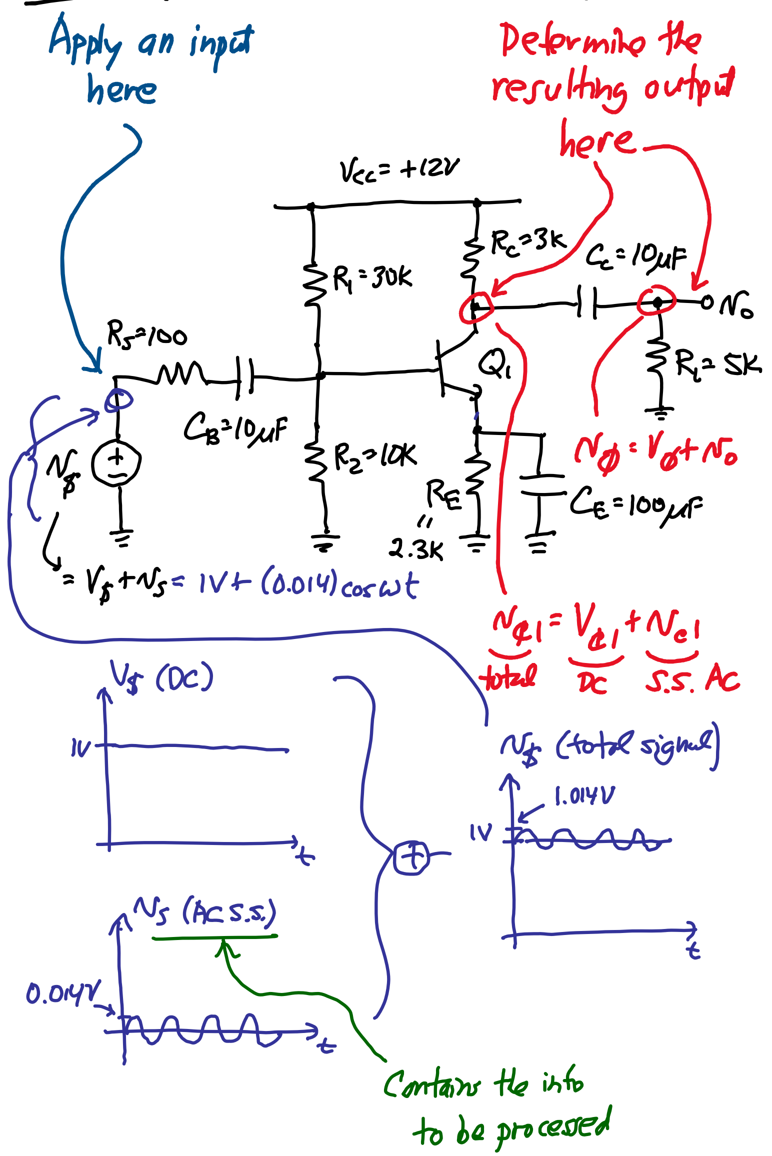
For Q: $\beta = 100$, $V_A = 100V$

Find the voltage gain, $\frac{v_o}{v_s}$.

Draw the collector voltage waveform for

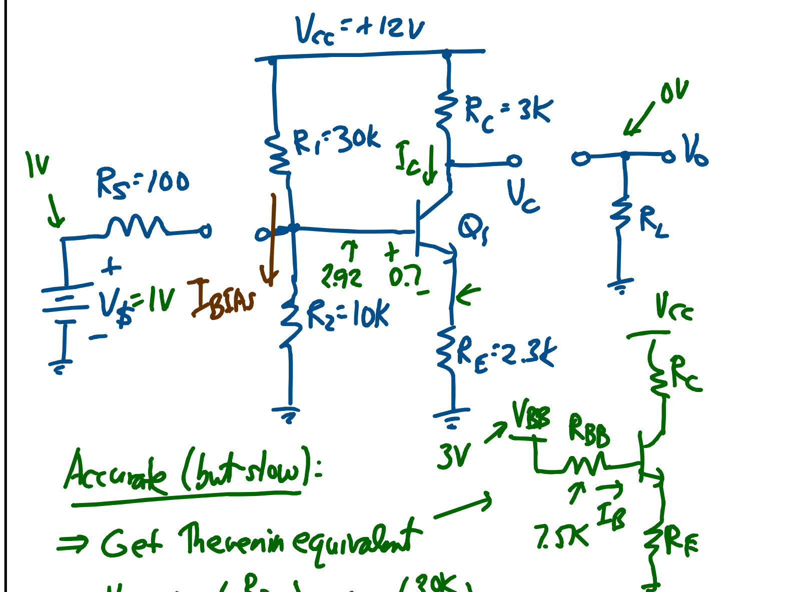
$$v_\$ = \underbrace{(0.014)\cos\omega_o t}_{\text{AC small-signal component}} + \underbrace{1V}_{\text{DC component}}$$

Graphically, here's what we're looking to do:



① Determine the DC operating point

- i.e., find the relevant DC voltages at all nodes and DC currents through all branches
- Draw the DC circuit
  - Eliminate independent AC small-signal sources
    - Short AC voltage sources
    - Open AC current sources
  - Open all capacitors (in particular, open the bypass/coupling capacitors)
  - Use DC transistor models
    - this might entail nonlinearity in some cases, but approximations can alleviate



Accurate (but slow):

⇒ Get Thevenin equivalent

$$V_{BB} = V_{CC}\left(\frac{R_2}{R_1+R_2}\right) = (12)\left(\frac{30k}{40k}\right) = 3V$$

$$R_{BB} = R_1 \| R_2 = (10k)\|(30k) = 7.5K$$

$$I_C = \frac{V_{BB}-V_{BE}}{\frac{R_E}{\alpha}+\frac{R_{BB}}{\beta}} = \frac{3-0.7}{\frac{2.3K}{1}+\frac{7.5K}{100}} = 0.97mA \approx 1mA$$

$$I_B = \frac{I_C}{\beta} = 0.01mA$$

$\therefore I_E \approx 0.97mA \approx 1mA$

$$V_B = V_{BB} - I_B R_{BB} = 3-(0.01m)(7.5K) = 2.92V$$

$\therefore V_E = 2.92-0.7 = 2.22V$

$$V_C = V_{CC} - I_C R_C = 12-(1m)(3K) = 9V$$

≈ ✓

Faster Way:

Ignore I_B → $V_B = V_{CC}\left(\frac{R_2}{R_1+R_2}\right) = 3V$

$V_E = V_B - V_{BE(on)} = 3-0.7 = 2.3V$

$\therefore I_E = \frac{V_E}{R_E} = \frac{2.3}{2.3K} = 1mA = I_C$

$I_B = \frac{I_C}{\beta} = \frac{1m}{100} = 0.01mA$

$V_C = V_{CC} - I_C R_C = 9V$

$I_{BIAS} = \frac{V_{CC}}{R_1+R_2} = \frac{12}{40K} = 0.3mA > 10 I_B$ ✓

For a stable bias pt.

② Determine the elements in the small-signal transistor model(s)

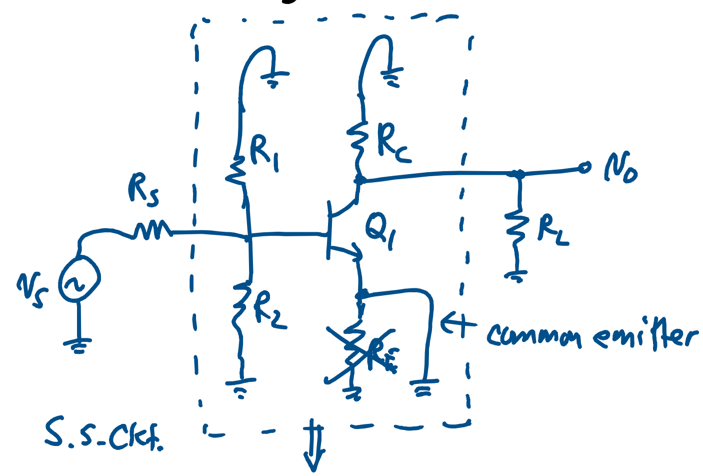
- If more than one transistor, might need to determine SS element values for several of them

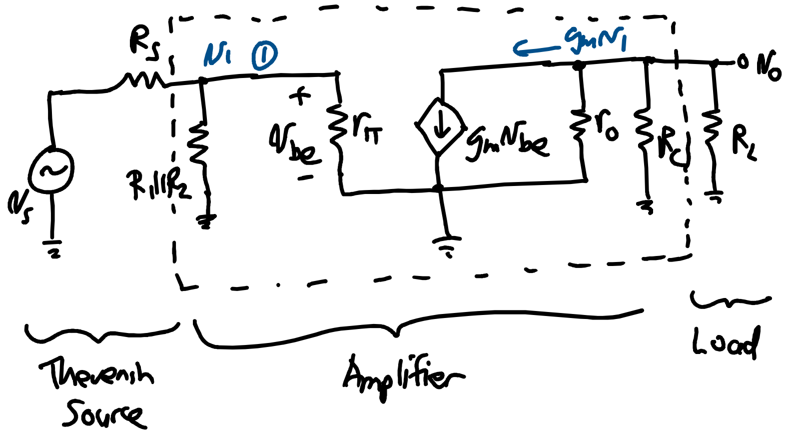
$g_m = \frac{I_C}{V_T} = \frac{1m}{25m} = 0.04℧$ $\quad r_o = \frac{V_A}{I_C} = \frac{100}{1m} = 100k\Omega$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.04} = 2.5k\Omega$ $\quad r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m} = 25\Omega$

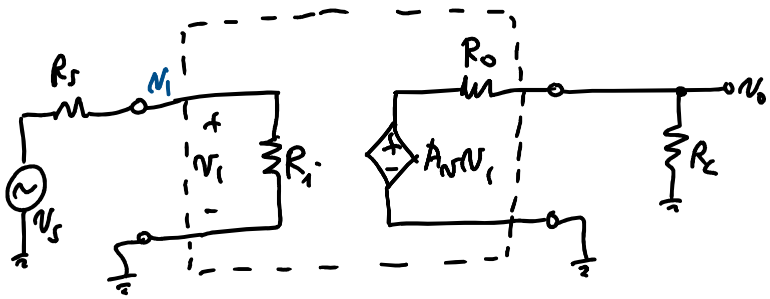
③ Obtain the small-signal circuit

- Eliminate independent DC sources
  - Short DC voltage sources
  - Open DC current sources
  - Short large coupling capacitors (C's > 10nF)
  - Use small-signal transistor models





⇒ would like to get this in the form of our simple amplifier model:



$$\frac{R_i}{R_i+R_S} \times A_v \times \frac{R_L}{R_o+R_L} = A_{tot}$$

need these to describe the amplifier

④ Use standard circuit analysis (i.e., KCL or KVL with superposition) to determine the parameters of interest

- Usually, the parameters of interest include
  - ↳ Gain, A_v
  - ↳ Input Resistance, R_i
  - ↳ Output Resistance, R_o
  - ↳ Low Frequency Cut-off, ω_b
  - ↳ High Frequency Cut-off, ω_h
- Determine all of these during small-signal analysis
- The total gain of the simplified amplifier circuit takes the form

$$\frac{v_o}{v_s} = \frac{R_i}{R_i+R_s} A_v \frac{R_L}{R_L+R_o}$$

For ideal want: $R_i = \infty$ ($R_i \gg R_S$), $R_o = 0$ ($R_o \ll R_L$)

Amplifier Gain –

$$A_v = \frac{v_o}{v_1}\Big|_{R_L=\infty} \ (\text{or } i_o = 0)$$

$$v_o = -(g_m v_1)(r_o \| R_C) \rightarrow A_v = \frac{v_o}{v_1}\Big|_{R_L=\infty} = -g_m(r_o \| R_C) = A_v$$

(100kΩ, 3kΩ)

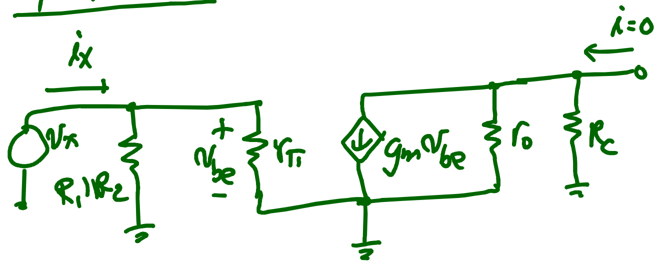
$$A_v \approx -g_m R_C = -120$$

$r_o \gg R_C$

⇒ max gain occurs when $R_C = \infty$:

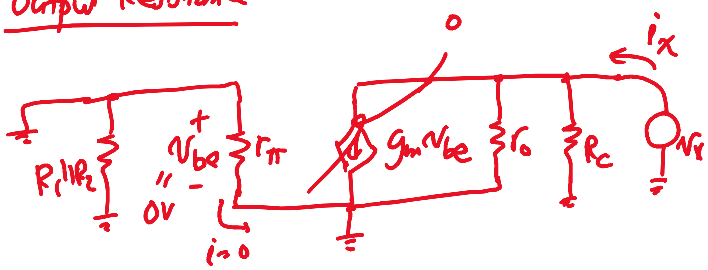
$$\text{Max Gain} = -g_m r_o = -\frac{I_C}{V_T}\frac{V_A}{I_C} = \boxed{-\frac{V_A}{V_T} = \text{Max Gain}}$$

Input Resistance -



$$\boxed{R_i = \frac{v_x}{i_x} = r_\pi \| R_1 \| R_2} = 1875\Omega$$

Output Resistance -



$$\boxed{R_o = \frac{v_x}{i_x} = r_o \| R_C \cong R_C} = 3k\Omega$$

$[r_o \gg R_C]$

For the total ckt:

$$\text{Gain} = \frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} A_v \frac{R_L}{R_L + R_o}$$

$$= \frac{r_\pi \| R_1 \| R_2}{r_\pi \| R_1 \| R_2 + R_s}(-g_m R_C)\left(\frac{R_L}{R_L + R_C}\right) \qquad = R_C \| R_L$$

$$= -\frac{r_\pi \| R_1 \| R_2}{r_\pi \| R_1 \| R_2 + R_s} g_m (R_C \| R_L)$$

$$= -\frac{1875}{1875 + 100}(0.04)(3K \| 15K)$$

$$= \boxed{-71.2 = \frac{v_o}{v_s}} \quad \text{(midband gain)}$$

⇒ just for kicks... for the ckt. in the box

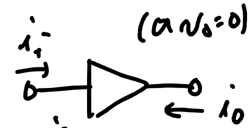
Short Ckt. Current Gain for the C.E. Amp -

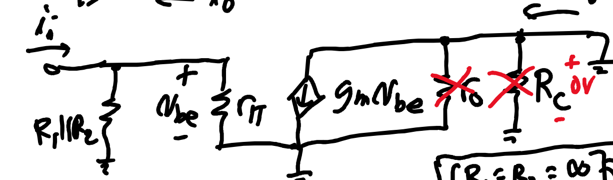
$$A_{is} = \frac{i_o}{i_i}\Big|_{R_L=0} \quad (\text{or } v_o = 0)$$

$i_o|_{R_L=0} = g_m v_{be}$

$v_{be} = i_i (r_\pi \| R_1 \| R_2)$

$i_o|_{R_L=0} = g_m (r_\pi \| R_1 \| R_2) i_i$





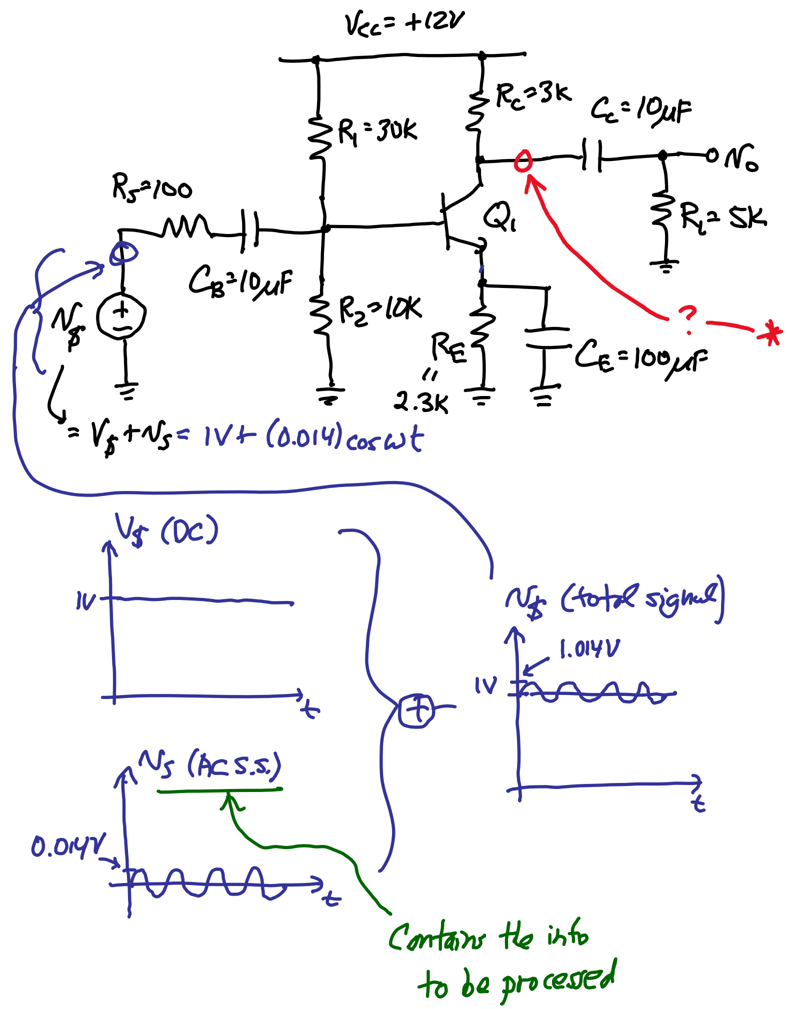
$$\boxed{A_{is} = g_m (r_\pi \| R_1 \| R_2) = g_m R_i}$$

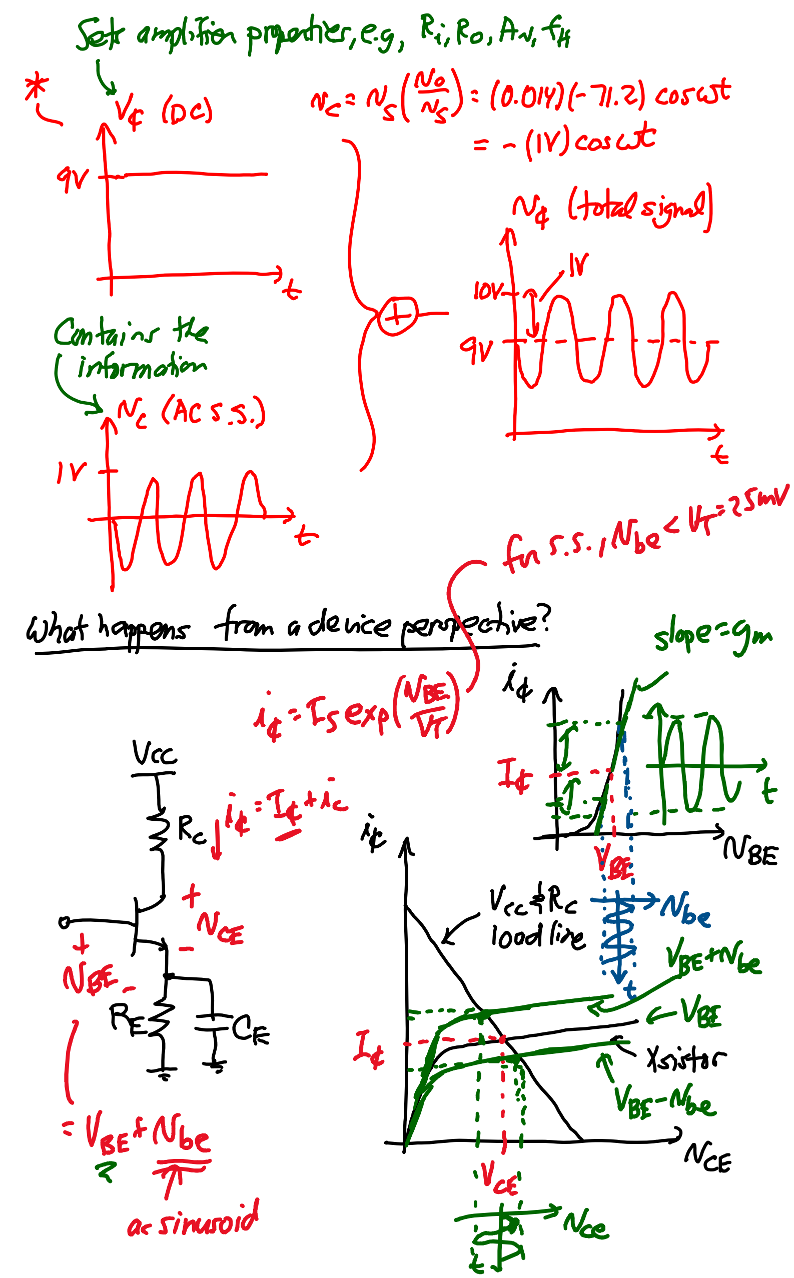
$[R_1 = R_2 = \infty]$

$$\boxed{A_{is} = g_m r_\pi = \beta}$$

⑤ Use the gain and resistances determined above to obtain the small-signal voltages and currents at each node/branch of the circuit

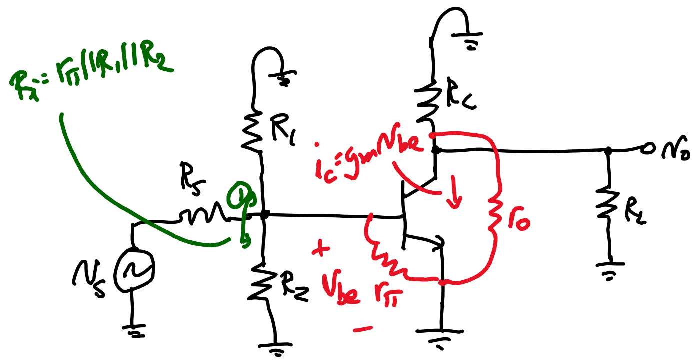
- To obtain the actual node & branch signals, superpose the DC and small-signal AC solutions





An Inkling of Inspection Analysis

⇒ for fast s.s. ckt analysis:



$$\frac{v_o}{v_s} = \frac{v_①}{v_s} \cdot \frac{v_o}{v_①} = \frac{r_\pi || R_1 || R_2}{r_\pi || R_1 || R_2}\left(-g_m(r_o || R_C || R_L)\right)$$

...but more later...