Lecture 24: High Frequency Small-Signal Models

- **Announcements**:
- HW#8 online and due Friday via Gradescope
- Lab#5 due Tuesday, Nov. 12, 5 p.m.
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- Lecture Topics:
  - Frequency Response
  - High Frequency Model for BJT
  - High Frequency Model for MOSFET
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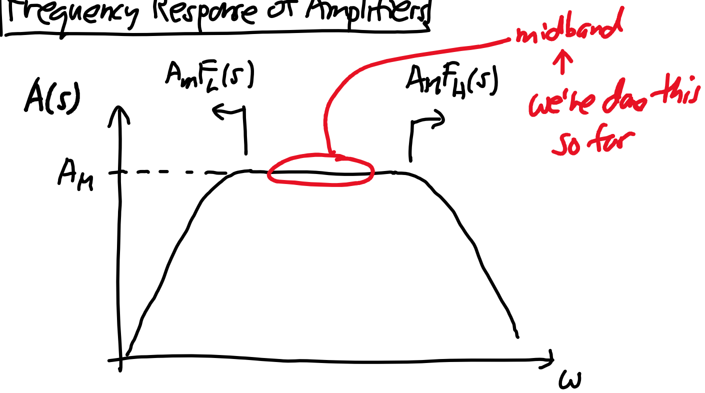
- **Last Time**:
- Finished Common Emitter Amplifier small-signal analysis example
- Now, start on Frequency Response …

$\boxed{\text{Frequency Response of Amplifiers}}$



General Form: $A(s) = A_M F_L(s) F_H(s)$

Where A_M = midband gain (constant w/ freq.)

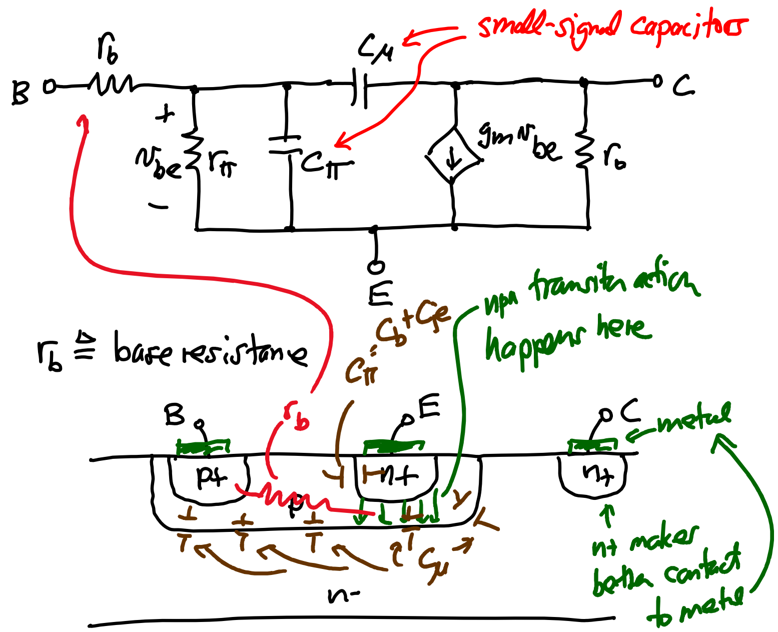
$F_L(s)$ models the low frequency behavior generally governed by coupling capacitors:

For $\omega \to 0$: $F_L(s) \to 0$
$\omega \to \infty$: $F_L(s) \to 1$
} It's a HPF

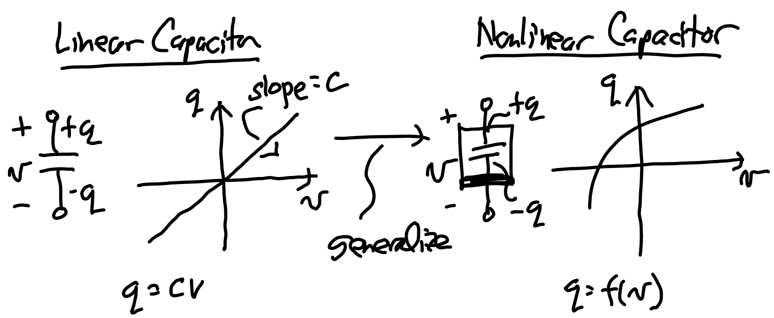
$F_H(s)$ models the high frequency behavior governed by parasitic capacitors (often inside the transistor)

For $\omega \to 0$: $F_H(s) \to 1$
$\omega \to \infty$: $F_H(s) \to 0$
} It's a LPF

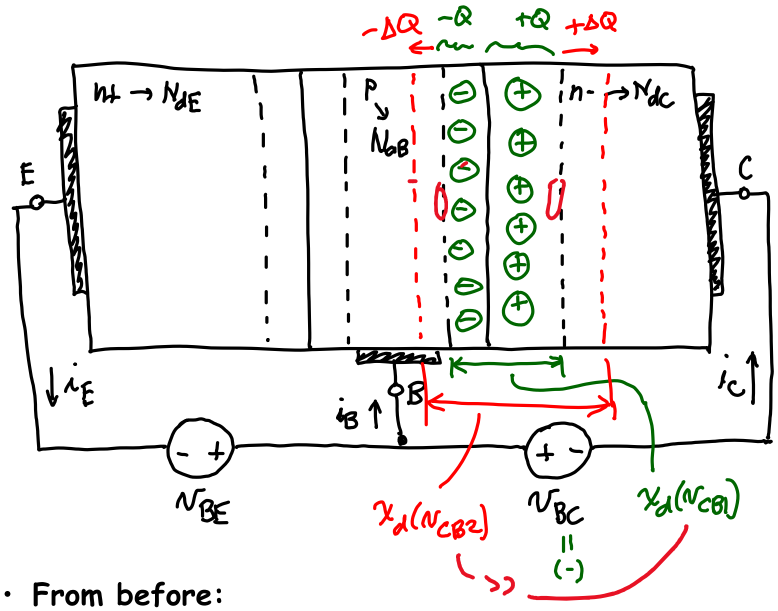
High Frequency Hybrid-π Model



Capacitor



C_μ – Base-to-Collector Capacitance



- From before:
- The depletion region width x_d is a function of the applied reverse-bias voltage v_{CB}: $x_d=f(v_{CB})$
- $Q=qN_ax_dA$, where A=cross sectional area
- Since Q is a function of x_d, it is also a function of v_{CB}: $Q=g(x_d)=g[f(v_{CB})]$

In general, for a pn junction:

$$C_j = \frac{dQ}{dv_R} = \frac{C_{jo}}{\sqrt{1+\frac{v_R}{\phi_j}}} \quad ; \quad C_{jo} = \frac{\epsilon_s A}{x_{do}} \leftarrow @\ v_R = 0V$$

C_μ is a junction capacitance:

$$C_\mu = \frac{C_{\mu o}}{\sqrt{1+\frac{V_{CB}}{\phi_j}}} \quad \text{(for an abrupt junction)}$$

where $C_{\mu o}$ = capacitance for $V_{CB}=0V$

ϕ_j = built-in potential between p & n type semiconductors

$$= \frac{kT}{q}\ln\left(\frac{N_{aB}N_{dc}}{n_i^2}\right) \; ; \; n_i = 1.45\times10^{10}\,cm^{-3}$$

More generally: $C_\mu = \frac{C_{\mu o}}{\left(1+\frac{V_{CB}}{\phi_j}\right)^m}$; m = fcn of junction interface ($=\frac{1}{2}$ for abrupt)

CJC in SPICE

MJC in SPICE

VJC in SPICE

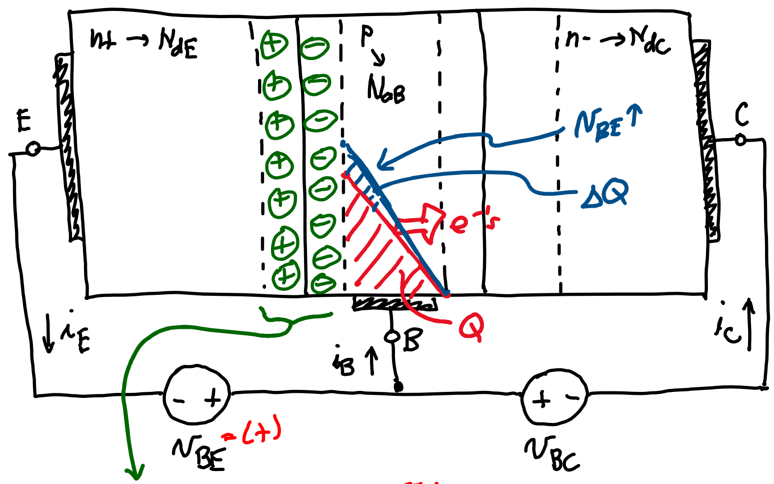
We assume this

$\boxed{C_\pi \text{ — Base-to-Emitter Capacitance}}$

⇒ two components comprise C_π:

① Junction capacitance, C_{je}

② Diffusion capacitance, C_b

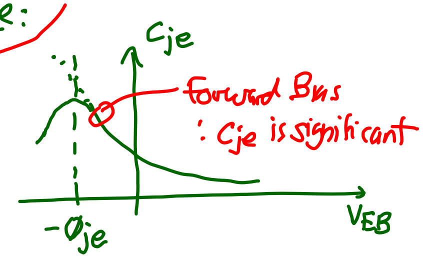


Junction Capacitance:

CJE in SPICE

$$C_{je} = \frac{C_{jeo}}{\sqrt{1+\frac{V_{EB}}{\phi_{je}}}}$$

VJE in SPICE



Diffusion Capacitance:

⇒ define base transit time: average time a carrier takes to cross the base

$$\tau_F = \frac{Q}{I_c}$$ ← think of I_c as the rate of transfer of charge through the base

$Q = \tau_F I_c$

$\Delta Q = \tau_F \Delta I_c$

⇒ switch to small-signal parameters:

$q = \tau_F i_c$ small-signal variable → "dq" $= q$

$C_b = \frac{q}{v_{be}} = \tau_F \frac{i_c}{v_{be}} = \boxed{\tau_F g_m = \tau_F \frac{I_C}{V_T} = C_b}$

$\frac{dq}{dv_{be}}$

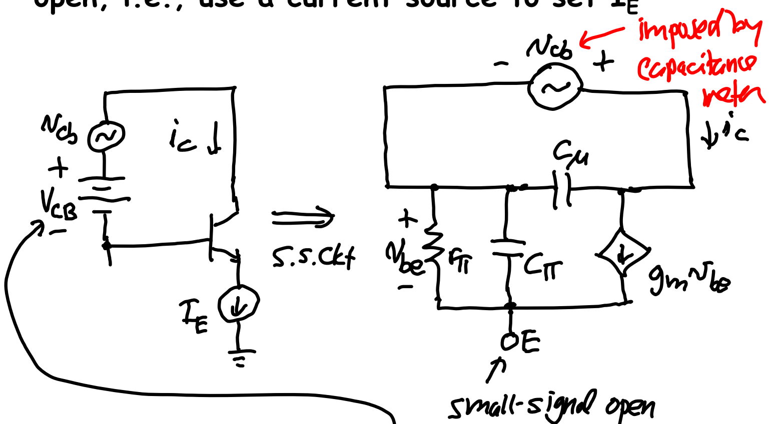
$\therefore C_b \propto I_C$

Putting it all together:

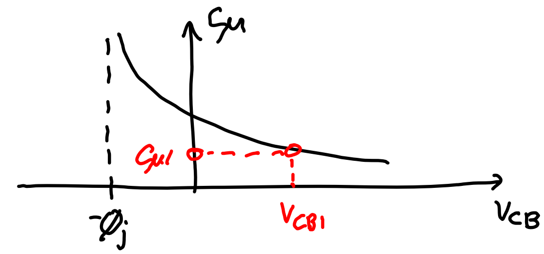
$$\boxed{C_\pi = C_b + C_{je} = \tau_F g_m + \frac{C_{je0}}{\sqrt{1 + \frac{V_{EB}}{\phi_{je}}}}}$$

Determining C_π and C_μ

- Can experimentally determine C_μ by measuring the small-signal capacitance between the base and collector terminals with the emitter incrementally open, i.e., use a current source to set I_E

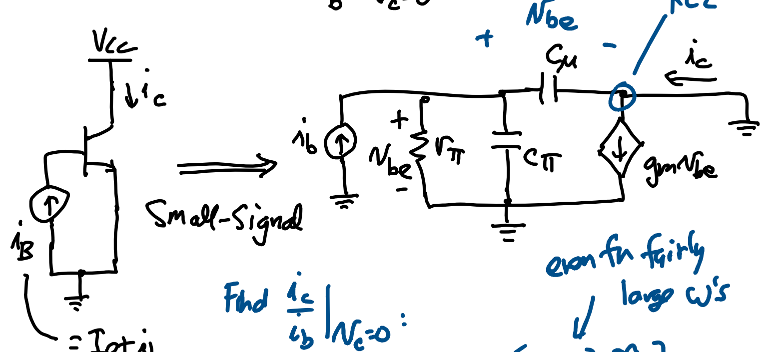


⇒ can measure C_μ vs. V_{CB}



- To find C_π, find an expression for C_π in terms of C_μ and known measurable parameters
- One parameter we can conveniently measure is the short-circuit current gain:

$$h_{fe} = \left.\frac{i_c}{i_b}\right|_{v_c=0}$$



Find $\left.\frac{i_c}{i_b}\right|_{v_c=0}$:

even for fairly large ω's

$[g_m \gg sC_\mu]$

$v_{be} = i_b \left(r_\pi \| \frac{1}{sC_\pi} \| \frac{1}{sC_\mu}\right)$

$i_c = g_m v_{be} - sC_\mu v_{be} = (g_m - sC_\mu) v_{be} \approx g_m v_{be}$

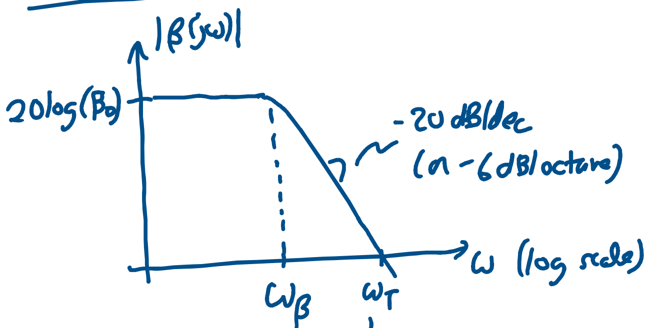
$i_c = g_m \left(r_\pi \| \frac{1}{sC_\pi} \| \frac{1}{sC_\mu}\right) i_b$

$$\frac{i_c}{i_b} = \frac{g_m}{\frac{1}{r_\pi} + s(C_\pi + C_\mu)} = \frac{g_m r_\pi}{1 + s r_\pi (C_\pi + C_\mu)} = \frac{\beta_0}{1 + s r_\pi (C_\pi + C_\mu)}$$

[$\beta_0 = g_m r_\pi \to$ low freq β]

$$\boxed{\beta(j\omega) = \frac{\beta_0}{1 + \frac{j\omega}{\omega_\beta}}, \text{ where } \omega_\beta = \frac{1}{r_\pi (C_\pi + C_\mu)}}$$

Plot $|\beta(j\omega)|$: (Bode plot)



$\triangleq$ defined as the radian frequency where $|\beta(j\omega)| = 1$

$$|\beta(j\omega_T)| \cong \frac{\beta_0}{\omega_T r_\pi (C_\pi + C_\mu)} = 1$$

$$\boxed{\omega_T = \frac{g_m}{C_\pi + C_\mu}} \Rightarrow \underbrace{f_T = \frac{\omega_T}{2\pi}}$$

Figure of Merit for the freq. performance of a Xsistor

for bipolar: $f_T \sim 100$ MHz $\to$ 50 GHz

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$$\boxed{C_\pi = \frac{g_m}{\omega_T} - C_\mu}$$ (to determine C_π from measurable parameters)