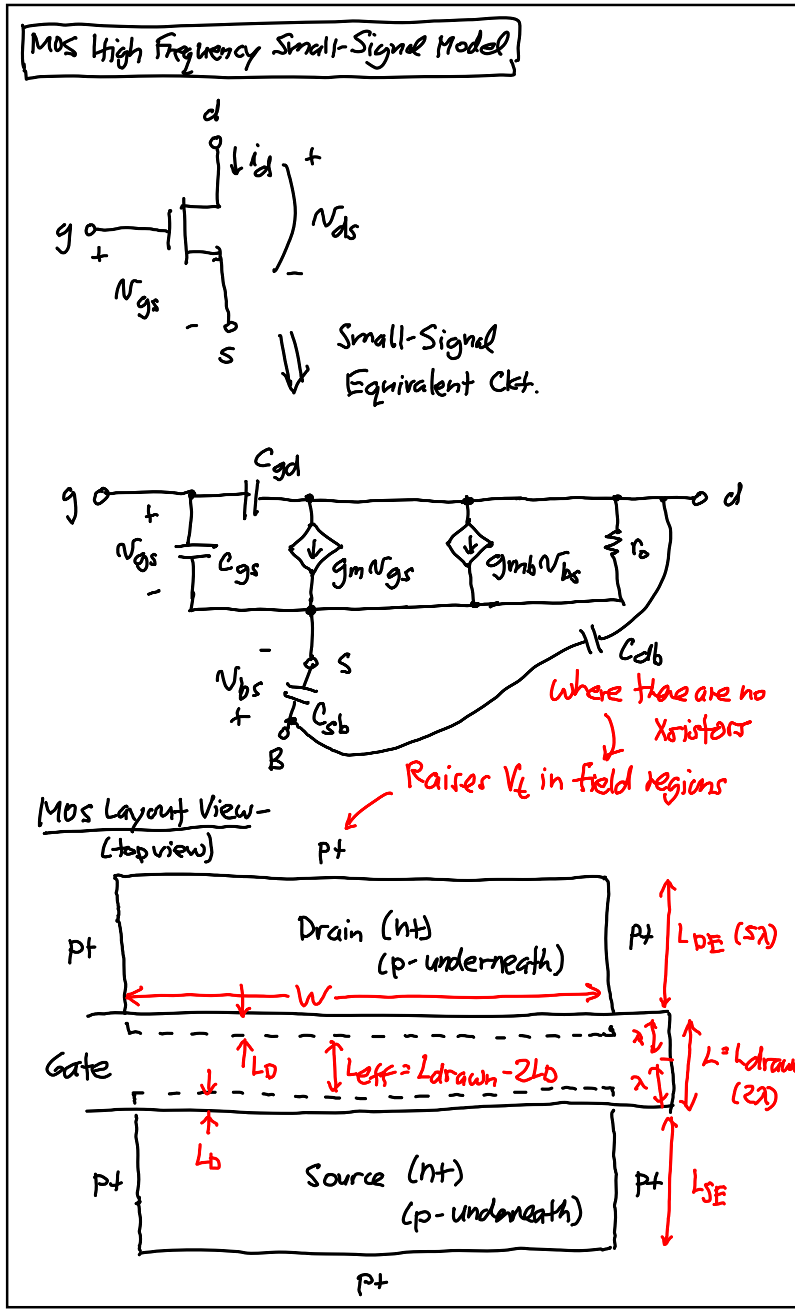
Lecture 25: High Frequency Circuit Analysis

- Announcements:
- HW#8 online and due Friday via Gradescope
- Lab#5 due Tuesday, Nov. 12, 5 p.m.
- This is a recorded video, since the PG&E shutdown forced class cancellation on Monday, 10/28
- -----------------------------------
- Lecture Topics:
  - MOS High Frequency Model
  - Brute Force CE HF Analysis
  - Open Circuit Time Constant (OCTC) Analysis
- -----------------------------------
- Last Time:
- Covered BJT high frequency model
- Now, continue with MOS high frequency model …



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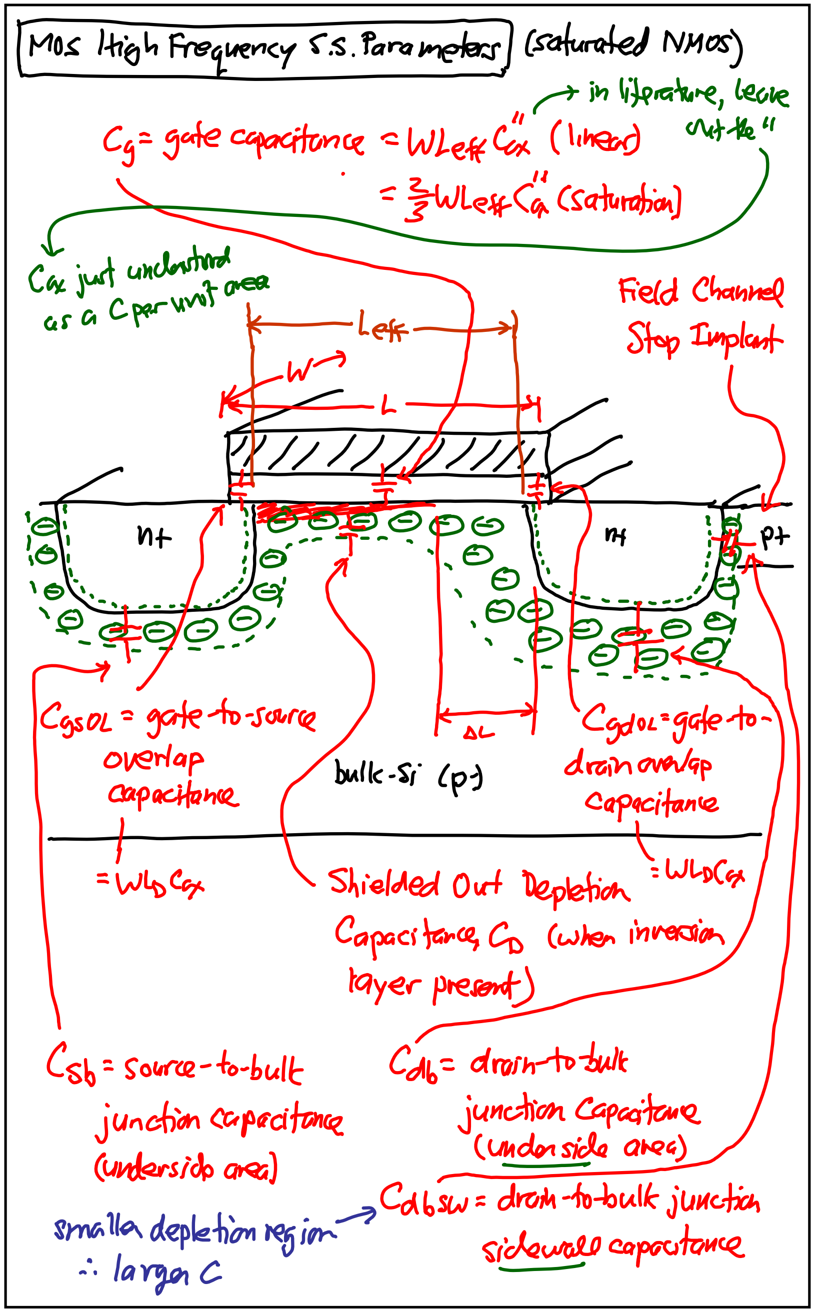


Gate-to-Source Capacitor, C_{gs}: (saturated MOS)

$$C_{gs} = C_{OL} + \frac{2}{3} W L_{eff} C_{ox}$$

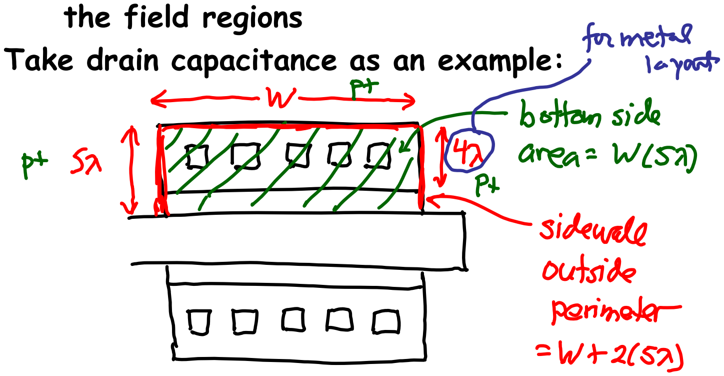
$C_{OL} = W L_D C_{ox}$; the $\frac{2}{3} W L_{eff} C_{ox}$ term accounts for the fact that the inversion charge is only under a portion of the gate

Gate-to-Drain Capacitance, C_{gd}:

$$C_{gd} = C_{OL} \quad (C_{OL} = W L_D C_{ox})$$

(no inversion charge near the drain in the saturation region)

- Source/Drain Junction Capacitance, C_{sb} & C_{db}:
- These are depletion capacitors associated with bulk-to-drain and bulk-to-source pn junctions
- Bottom capacitance per unit area differs from sidewall capacitance due to higher p+ bulk doping at the sidewalls
- The higher doping is near the silicon surface and designed to raise the threshold voltage in field areas, i.e., areas between transistors
  - ↳ This way unwanted inversion does not occur in the field regions
- Take drain capacitance as an example:



$$C_{db} = \frac{C_{dbo}}{\sqrt{1 + \frac{V_{DB}}{\phi_j}}}, \text{ where:}$$

$C_{dbo} \triangleq$ depletion capacitance w/ $V_{SB} = 0V$

$= (\text{junction bottom-side area}) \times C_{jo}$

$+ (\text{junction outside perimeter}) \times C_{jswo}$

$= W(5\lambda) \times C_{jo} + (W + 2(5\lambda)) \times C_{jswo}$

and where:

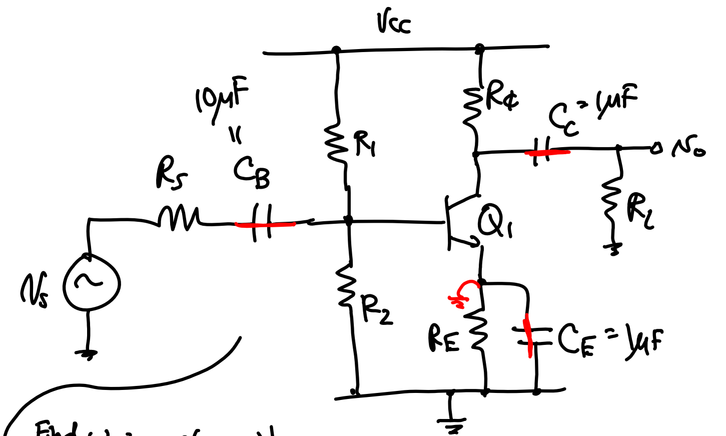
$$C_{jo} = \sqrt{\frac{q\epsilon_s N_B}{2\phi_j}} \quad \text{and} \quad C_{jswo} = \sqrt{\frac{q\epsilon_s N_{CS}}{2\phi_j}} \times x_j$$

N_B: bulk doping concentration

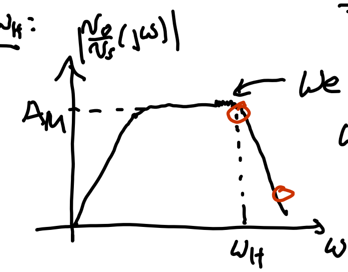
N_{CS}: channel-stop implant doping concentration

x_j: S/D junction depth

C.E. High Freq Analysis (Brute Force)



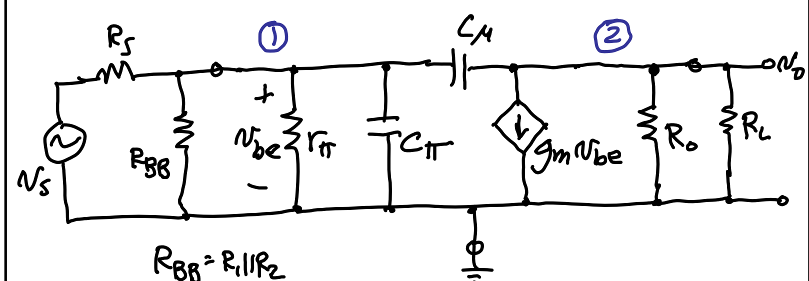
Find ω_H:



We want this frequency, which then gives us

$$\frac{v_o}{v_s}(s) = A_M \frac{1}{1 + \frac{s}{\omega_H}}$$

Small-Signal Ckt.



$R_{BB} = R_1 \| R_2$

$R_o = r_o \| R_c$

KCL①: $\frac{v_s - v_{be}}{R_s} = \frac{v_{be}}{r_\pi || R_{BB}} + v_{be}(sC_\pi) + (v_{be} - v_o)(sC_\mu)$

KCL②: $(v_{be} - v_o)(sC_\mu) = g_m v_{be} + \frac{v_o}{R_o || R_L} = g_m v_{be} + R''$

$[R'' = R_o || R_L]$

Rearrange:

$\frac{v_s}{R_s} = v_{be}\left[\frac{1}{R_s} + \frac{1}{r_\pi || R_{BB}} + s(C_\pi + C_\mu)\right] - v_o(sC_\mu)$

$\frac{1}{r_\pi || R_s || R_{BB}} = \frac{1}{R'}$ $[R' = r_\pi || R_s || R_{BB}]$

... math ...

$$\frac{v_o}{v_s}(s) = -\frac{g_m R' R''}{R_s}\left\{\frac{\left(1 - s\frac{C_\mu}{g_m}\right)}{\left(1 + s\left[R'(C_\pi + C_\mu) + R''C_\mu + g_m C_\mu R' R''\right] + s^2 R' R'' C_\pi C_\mu\right)}\right\}$$

$A_M = -g_m \frac{(r_\pi || R_{BB})(r_o || R_C || R_L)}{R_s + r_\pi || R_{BB}}$ — constant ≜ midband gain

$F_H(s)$ ← models the freq. response

*

$$F_H(s) = \frac{1 - s\frac{C_\mu}{g_m}}{1 + s\left[R'(C_\pi + C_\mu) + R''C_\mu + g_m C_\mu R' R''\right] + s^2 R' R'' C_\pi C_\mu}$$

$$= \frac{1 - \frac{s}{z_1}}{\left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)} = \frac{1 + \frac{s}{\omega_z}}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)} = \frac{N(s)}{D(s)}$$

Zeros:

⇒ identify a RHP zero: $z_1 = +\frac{g_m}{C_\mu} \rightarrow \omega_z = \frac{g_m}{C_\mu}$

Note that $\frac{g_m}{C_\mu} \gg \omega_T$ (since $\frac{g_m}{C_\mu} \gg \frac{g_m}{C_\pi + C_\mu}$ and $C_\pi \gg C_\mu$)

∴ ω_z is a very high freq.!

↳ can ignore relative to the lower freq. poles

Poles:

⇒ often, $\omega_{p1} \ll \omega_{p2}$ (i.e., one pole frequency is much lower than other)

(ω_{p1} dominates over ω_{p2})

$$D(s) = \left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)$$

$$= 1 + s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

$$= 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$ } Dominant pole approximation

Comparing w/ the denominator of $F_H(s)$:

$$\omega_{p1} = -p_1 \cong \frac{1}{R'(C_\pi + C_\mu) + R''C_\mu + g_m C_\mu R'R''}$$

$$\boxed{\omega_{p1} \cong \frac{1}{\left(C_\pi + C_\mu\left(1 + g_m R'' + \frac{R''}{R'}\right)\right)R'}}$$

$$\omega_{p1}\omega_{p2} \cong \frac{1}{R'R''C_\pi C_\mu} \Rightarrow \omega_{p2} \cong \frac{\left[C_\pi + C_\mu\left(1 + g_m R'' + \frac{R''}{R'}\right)\right]R'}{R'R''C_\pi C_\mu}$$

$$\therefore \omega_{p2} = \frac{g_m}{C_\pi} + \frac{1}{R''C_\mu} + \frac{1}{C_\pi}\left(\frac{1}{R'} + \frac{1}{R''}\right)$$

$$\frac{g_m}{C_\pi} > \omega_T \left(= \frac{g_m}{C_\pi + C_\mu}\right)$$

∴ ω_{p2} is indeed at a very high freq. $\gg \omega_{p1}$

∴ satisfies our dominant pole approx. ✓

Take a closer look @ the form of ω_{p1}:

$$\omega_{p1} \cong \frac{1}{R'C_\pi + (R' + R'' + g_m R'R'')C_\mu}$$ *

$\tau_{\pi o} = R_{\pi o} C_\pi$; $\tau_{\mu o} = R_{\mu o} C_\mu$

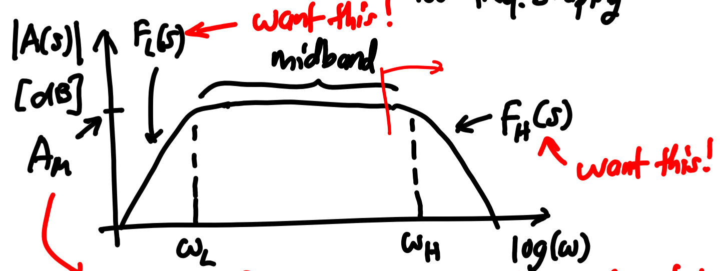
Can think of the denominator as the sum of time constants associated w/ each of the capacitor in the ckt!

Freq. Response

Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

$$A(s) = A_M F_L(s) F_H(s) \qquad s = j\omega$$

- A_M: midband gain
- $F_L(s)$: low freq. shaping
- $F_H(s)$: high frequency shaping



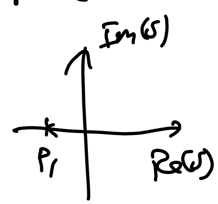
we've already found this using small-signal analysis

High Freq Response Determination Using Open Ckt. Time Constant (OCTC) Analysis

In general:

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_{n_z} s^{n_z}}{1 + b_1 s + b_2 s^2 + \cdots + b_{n_p} s^{n_p}}, \quad n_p > n_z$$

$$= \frac{\prod_{j=1}^{n_z}\left(1 - \frac{s}{z_j}\right)}{\prod_{i=1}^{n_p}\left(1 - \frac{s}{p_i}\right)} = \frac{\prod_{j=1}^{n_z}\left(1 + \frac{s}{\omega_{zj}}\right)}{\prod_{i=1}^{n_p}\left(1 + \frac{s}{\omega_{pi}}\right)}$$



from which:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn_p}} = \sum_{i=1}^{n_p} \frac{1}{\omega_{pi}} = \sum_{k=1}^{n_p} \tau_{pk}$$

b_1 ← coeff. of the 1st order term

Through network theory, one can prove that: (see Gray & Meyer, Chpt. 7)

$$\sum_{i=1}^{n_p} \tau_{pi} = \sum_j C_j R_{jo} = \sum_j \tau_{jo} = b_1$$

where C_j are capacitors in the H.F. ckt., i.e., small ones

$R_{jo} \triangleq$ driving pt. resistance seen between the terminals of C_j determined with

1. all small (<1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage sources, open current sources)
3. short all large (coupling/bypass) capacitors (i.e., >1μF or >1nF)

In calculating the H.F. response, we use the dominant pole approximation:

(i) $\omega_{p1} \ll \omega_{p2}, \ldots, \omega_{pn_p}$

(ii) $F_H(s) \cong \frac{1}{1 + b_1 s} = \frac{1}{1 + \frac{s}{\omega_H}}$

(iii) $\boxed{\omega_H \cong \frac{1}{b_1} = \frac{1}{\sum_j C_j R_{jo}} = \frac{1}{\sum_j \tau_{jo}}}$

When there is no dominant pole, an approximate expression for ω_H is:

$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \cdots - \frac{1}{\omega_{z1}^2} - \frac{1}{\omega_{z2}^2} - \cdots}}$$

(just FYI)

- Now, use OCTC analysis on the CE amplifier to find the upper cut-off frequency, ω_H

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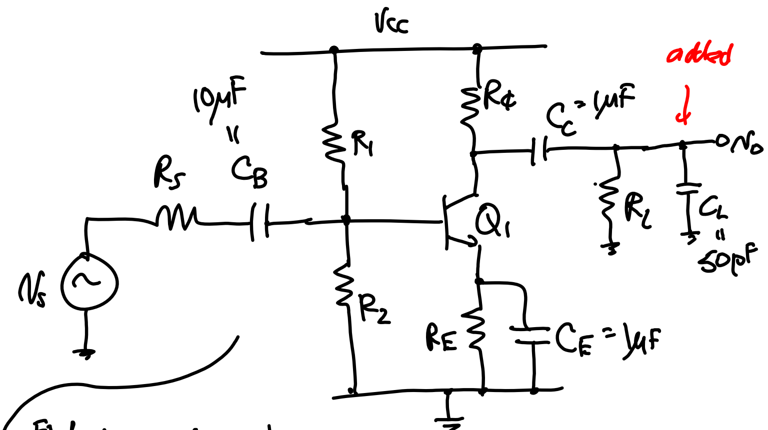
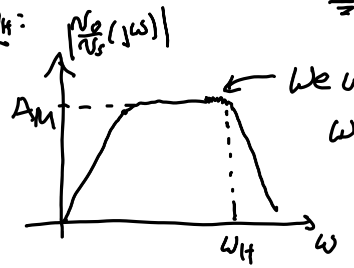
C.E. High Freq. Analysis Using OCTC



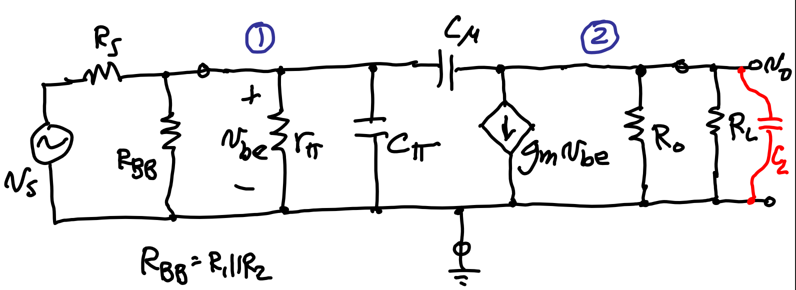
Find ω_H:



We want this frequency, which then gives us

$$\frac{v_o}{v_s}(s) = A_M \frac{1}{1 + \frac{s}{\omega_H}}$$

Small-Signal Ckt.

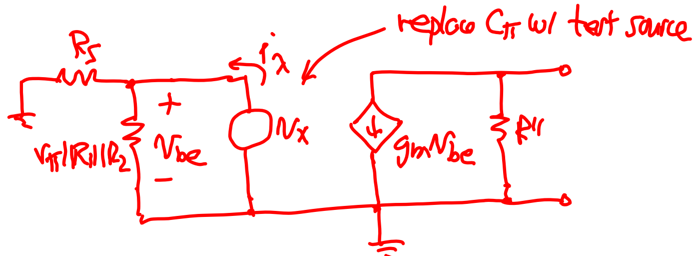


$R_{BB} = R_1 || R_2$

$R_o = r_o || R_C$

(a) Determine $\tau_{\pi o} = C_\pi R_{\pi o}$

⇒ open ckt. all C's, zero out all sources, and determine the driving point impedance, $R_{\pi o}$:

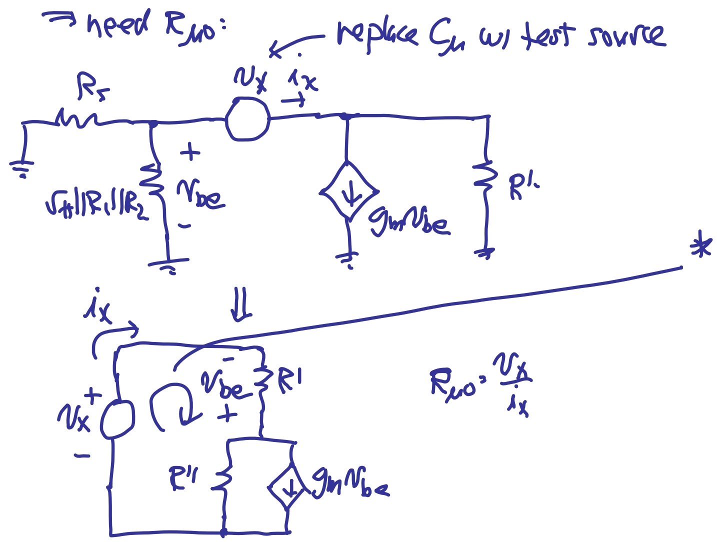


$R_{\pi o} = \frac{v_x}{i_x} = r_\pi || R_1 || R_2 || R_S = R'$ (by inspection)

$\therefore$ $\boxed{\tau_{\pi o} = C_\pi R'}$

(b) Determine $\tau_{\mu o} = C_\mu R_{\mu o}$:

⇒ need $R_{\mu o}$:



$R_{\mu o} = \frac{v_x}{i_x}$

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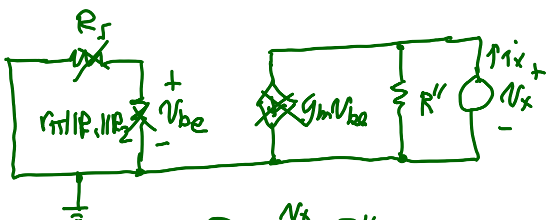
* KVL: $v_x = i_x R' + R''(i_x - g_m v_{be})$

$[v_{be} = -i_x R'] \rightarrow v_x = i_x R' + R''(i_x + i_x R' g_m)$

$\therefore R_{\mu o} = \frac{v_x}{i_x} = R' + R'' + g_m R' R''$

$$\therefore \tau_{\mu o} = C_\mu (R' + R'' + g_m R' R'')$$

(c) Determine $\tau_{Lo} = C_L R_{Lo}$:



$R_{Lo} = \frac{v_x}{i_x} = R''$

$$\therefore \tau_{Lo} = C_L R''$$

Thus:

$$\omega_H = \frac{1}{\sum_j \tau_{jo}} = \frac{1}{C_\pi R' + C_\mu (R' + R'' + g_m R' R'') + C_L R''}$$

= *