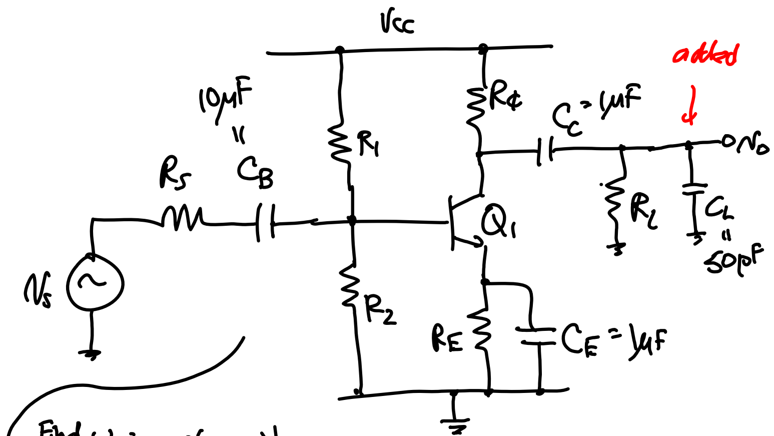
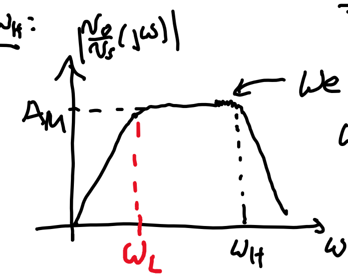
Lecture 26: Low Frequency Circuit Analysis

- **Announcements**:
- HW#8 online and due Friday via Gradescope
- Lab#5 due Tuesday, Nov. 12, 5 p.m.
- Hopefully, you watched Monday's video lecture
- --
- Lecture Topics:
  - Short Circuit Time Constant (SCTC) Analysis
  - Intro. to Inspection Analysis
  - C.E. Design Project Hints
- --
- **Last Time**:
- Finished OCTC analysis for high frequency

C.E. High Freq. Analysis Using OCTC



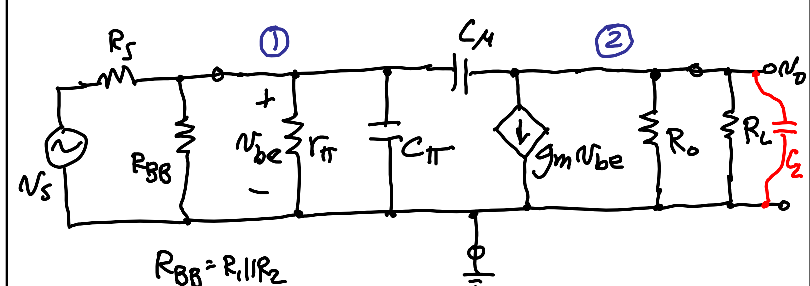
Find ω_H:



We want this frequency, which then gives us

$$\frac{v_o}{v_s}(s) = A_M \frac{1}{1+\frac{s}{\omega_H}}$$

Small-Signal Ckt.



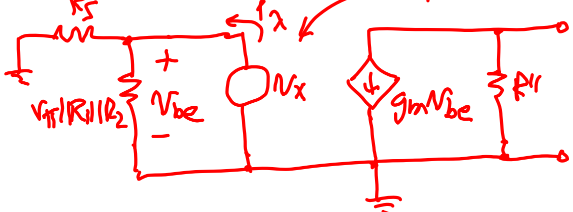
$R_{BB} = R_1 \| R_2$

$R_o = r_o \| R_C$

(a) Determine $\tau_{\pi o} = C_\pi R_{\pi o}$

⇒ open ckt. all C's, zero out all sources, and determine the driving point impedance, $R_{\pi o}$:

replace C_π w/ test source

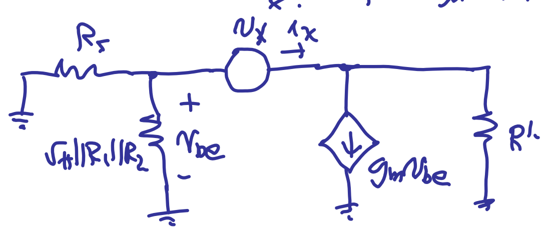


$$R_{\pi o} = \frac{v_x}{i_x} = r_\pi \| R_1 \| R_2 \| R_S = R' \quad \text{(by inspection)}$$

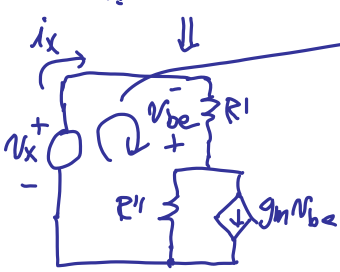
$$\therefore \boxed{\tau_{\pi o} = C_\pi R'}$$

(b) Determine $\tau_{\mu o} = C_\mu R_{\mu o}$:

⇒ need $R_{\mu o}$: replace C_μ w/ test source



⇓



$$R_{\mu o} = \frac{v_x}{i_x}$$

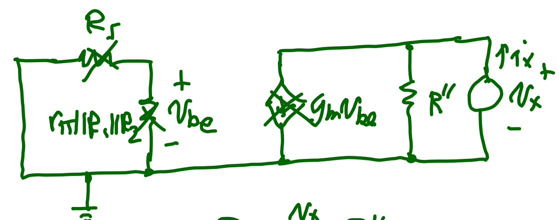
* KVL: $v_x = i_x R' + R''(i_x - g_m v_{be})$

$[v_{be} = -i_x R'] \rightarrow v_x = i_x R' + R''(i_x + i_x R' g_m)$

$$\therefore R_{\mu o} = \frac{v_x}{i_x} = R' + R'' + g_m R' R''$$

$$\therefore \boxed{\tau_{\mu o} = C_\mu (R' + R'' + g_m R' R'')}$$

(c) Determine $\tau_{Lo} = C_L R_{Lo}$:



$$R_{Lo} = \frac{v_x}{i_x} = R''$$

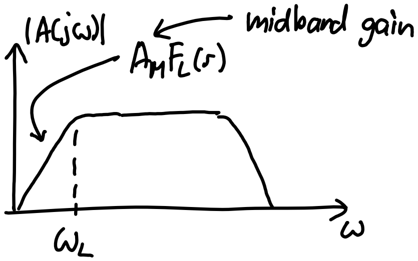
$$\therefore \boxed{\tau_{Lo} = C_L R''}$$

Thus:

$$\boxed{\omega_H = \frac{1}{\sum_j \tau_{jo}} = \frac{1}{C_\pi R' + C_\mu (R' + R'' + g_m R' R'') + C_L R''}}$$

Low Freq. Amplifier Response Using Short Circuit Time Constant Analysis (SCTC)

Recall:



In general, for the low freq. response:

$$F_L(s) = \frac{s^{n_L} + d_1 s^{(n_L-1)} + \cdots}{s^{n_L} + e_1 s^{(n_L-1)} + \cdots}, \quad n_L = \#\text{poles} = \#\text{zeros}$$

We can express the coefficient e_1 by:

$$e_1 = \omega_{p1} + \omega_{p2} + \cdots + \omega_{pn_L}$$

Similar analysis to that used for OCTC...

For the case of a dominant pole:
↳ i.e., the highest freq. pole

$$F_L(s) \cong \frac{s}{s+\omega_L} = \frac{s}{s+e_1} \rightarrow e_1 \cong \omega_{p1} = \omega_L$$

$$\therefore \boxed{\omega_L \cong e_1 = \sum_j \omega_{pj} = \sum_j \frac{1}{C_j R_{js}} = \sum_j \frac{1}{\tau_{js}}}$$

where $C_j \triangleq$ various large (>10nF) capacitors in the ckt. (e.g., the bypass caps.)

$R_{js} \triangleq$ driving point resistance seen between the terminals of C_j determined with:

For reading, can go to Sedra & Smith

① all large capacitors short-circuited, except C_j, which is replaced by the test voltage source for R determination

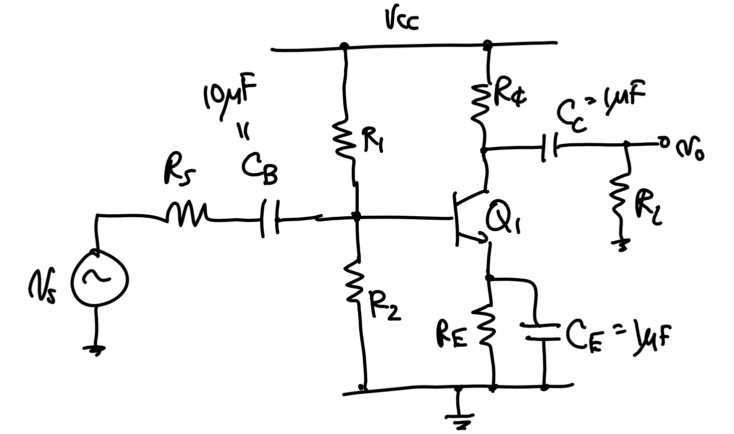
② all independent sources eliminated (i.e., short voltage sources, open current sources)

③ open all H.F. capacitors (i.e., small caps in the pF range, or < 1nF)

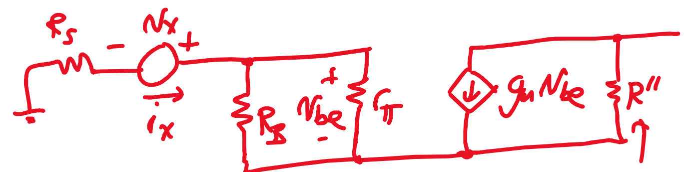
Again, for the case where there are no dominant poles, a reasonable approximation is:

$$\omega_L \cong \sqrt{\omega_{p1}^2 + \omega_{p2}^2 - 2\omega_{z1}^2 - 2\omega_{z2}^2}$$

Ex: Determine the L.F. response of the C.E. Amplifier

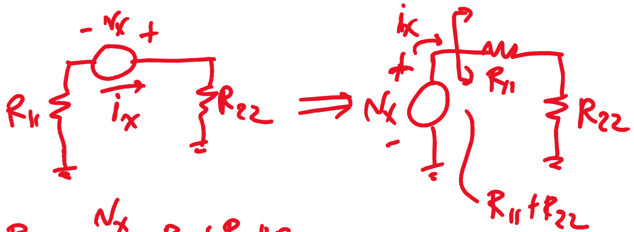


(a) C_B: short ckt. C_E & C_C, then determine R_{Bs}



$R'' = r_o \| R_C \| R_L \approx R_C \| R_L$

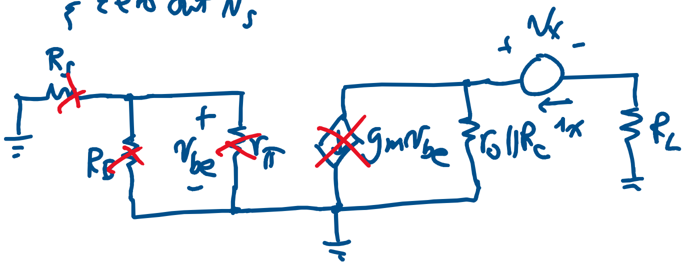
$R_B = R_1 \| R_2$



$$R_{BS} = \frac{v_x}{i_x} = R_S + R_B \| r_\pi$$

$$\therefore \boxed{\tau_{BS} = C_B(R_S + r_\pi \| R_B)} \rightarrow \boxed{\omega_{BS} = \frac{1}{\tau_{BS}} = \frac{1}{C_B(R_S + r_\pi \| R_B)}} \;\; \text{"}\omega_{p1}$$

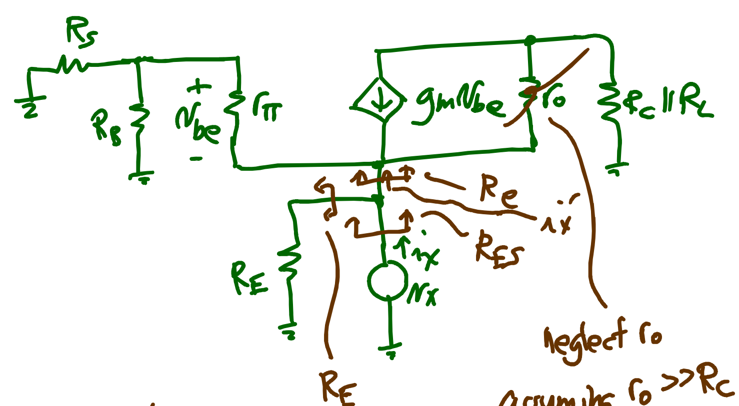
(b) $\underline{C_C}$: short ckt. C_B & C_E, then determine R_{CS} & zero out v_s



$$R_{CS} = R_L + r_o \| R_C \underset{\uparrow}{\approx} R_L + R_C$$
$[r_o \gg R_C]$

$$\boxed{\tau_{CS} = C_C(R_L + R_C)} \rightarrow \boxed{\omega_{CS} = \frac{1}{\tau_{CS}} = \frac{1}{C_C(R_L + R_C)}} \;\; \text{"}\omega_{p2}$$

(c) $\underline{C_E}$: short C_B & C_C; zero out v_s; determine R_{ES}



$$R_{ES} = R_E \| R_e$$

$\underline{R_e}$: $$v_{be} = \frac{-r_\pi v_x}{r_\pi + (R_S \| R_B)}$$

$$\therefore i_x = \frac{v_x}{r_\pi + (R_S \| R_B)} - g_m v_{be} = v_x\left(\frac{1}{r_\pi + (R_S \| R_B)} + \frac{\overbrace{g_m r_\pi}^{\beta}}{r_\pi + (R_S \| R_B)}\right)$$

$$R_e = \frac{v_x}{i_x} = \frac{r_\pi + (R_S \| R_B)}{\beta + 1}$$

$$R_{ES} = R_E \| R_e = R_E \| \frac{r_\pi + (R_S \| R_B)}{\beta + 1}$$

$$\boxed{\tau_{ES} = C_E\left(R_E \| \frac{r_\pi + (R_S \| R_B)}{\beta + 1}\right)} \rightarrow \boxed{\omega_{ES} = \frac{1}{C_E\left(R_E \| \frac{r_\pi + (R_S \| R_B)}{\beta + 1}\right)}} \;\; \text{"}\omega_{p3}$$

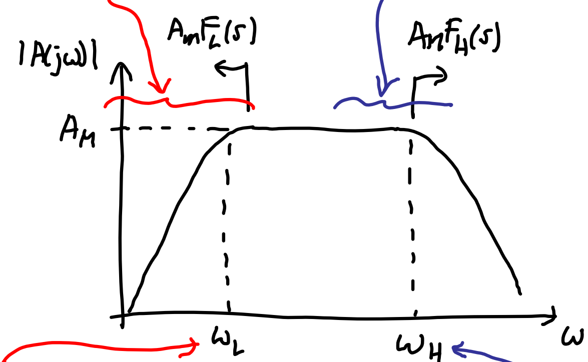
$$\boxed{\omega_L \cong \sum_j \frac{1}{\tau_j} = \sum_j \omega_{pj}}$$

Summarize:

- Which capacitors to use for OCTC? Which for SCTC?
- Separate caps into two categories:
  - ↳ Large caps → C_{Lj}'s
  - ↳ Small caps → C_{Sj}'s

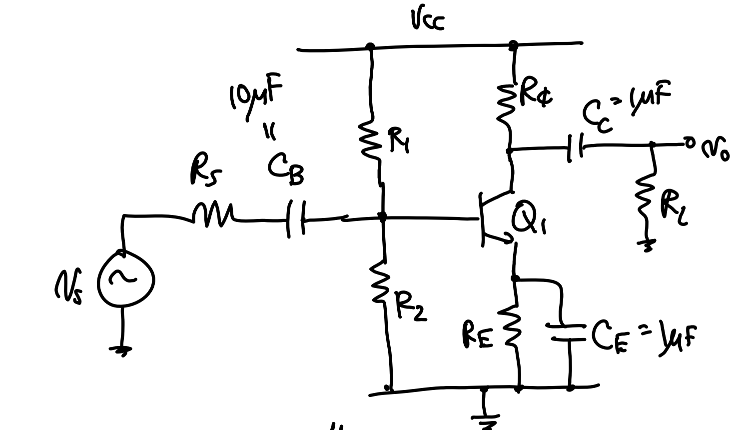
Determined by large caps C_{Lj}'s

Shape determined by small caps C_{Sj}'s

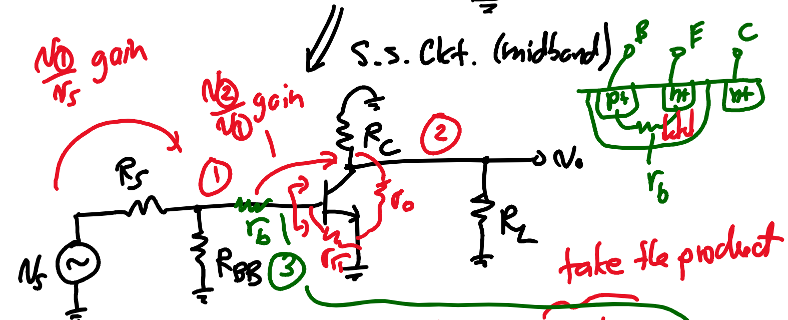


- Find using SCTC
  - ↳ Get time constants using capacitors that contribute to low frequency poles and zeros, which generally means bypass or coupling caps
  - ↳ Open smaller capacitors (e.g., hybrid-π ones)
  - ↳ Use C_{Lj}'s; open C_{Sj}'s
- Find using OCTC
  - ↳ Get time constants using capacitors that contribute high freq. poles & zeros, which generally means hybrid-π or any small caps
  - ↳ Short larger caps (e.g., bypass or coupling capacitors)
  - ↳ Use C_{Sj}'s; short C_{Lj}'s

Intro. to Inspection Analysis (w/ Lab#5 hints)



S.S. Ckt. (midband)



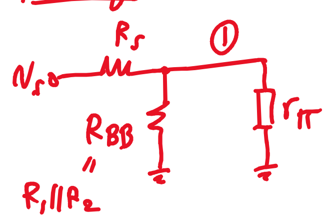
take the product

⇒ get gains from node to node, then combine

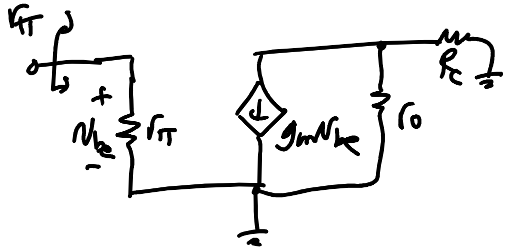
⇒ account for load resistance for each gain calculation for each stage

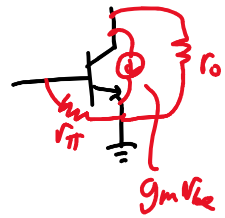
might need to account for r_b in your lab

1st Stage:



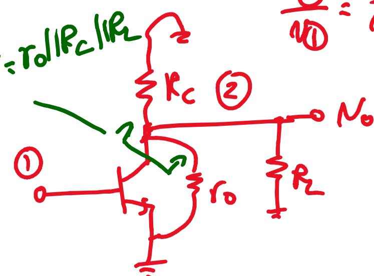
$$\frac{v_{①}}{v_s} = \frac{r_\pi \| R_1 \| R_2}{R_s + r_\pi \| R_1 \| R_2}$$





2nd Stage:

$R'' = r_o \| R_C \| R_L$



$$\frac{v_{(2)}}{v_{(1)}} = \frac{v_o}{v_{(1)}} = -\underbrace{g_m R''}_{\text{from memory}} = -g_m(r_o \| R_C \| R_L)$$

$[r_o = \text{large}]$

$$-g_m(R_C \| R_L)$$

$$\frac{v_o}{v_s} = \frac{v_{(1)}}{v_s} \cdot \frac{v_o}{v_{(1)}} = \boxed{-\frac{r_\pi \| R_1 \| R_2}{R_s + r_\pi \| R_1 \| R_2} g_m (R_C \| R_L)}$$