Lecture 31: Multi-Transistor High Frequency

- **Announcements**:
- HW#10 online, but not due till Friday next week
- Lab#6 coming soon
- Midterm 2 this Friday, Nov. 15, @ 7 p.m., in 160 Kroeber Hall
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- Lecture Topics:
  - Multi-Transistor Example (Inspection Analysis)
    - —Input/Output Resistances
    - —Gain
    - —High Frequency
  - MOS Inspection Analysis
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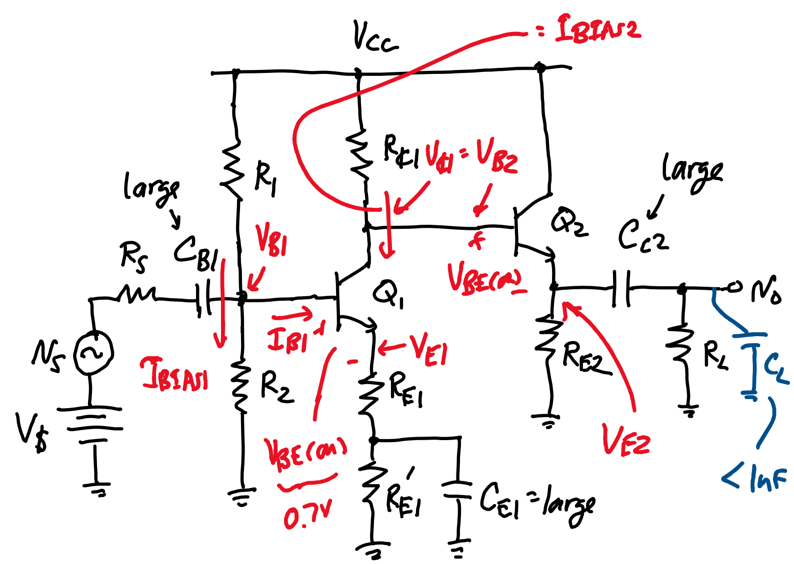
- **Last Time**:
- Got the gain of the amplifier
- Now, get the high frequency cut-off

Example. Multi-Transistor Amplifier Inspection Analysis

(C.E. w/ Degeneration, C.C. Cascade)



Find $R_i, R_o, a_v = \frac{v_o}{v_s}$, and f_H.

First, find the DC operating pt:

Good Design: (stable bias pt.) $\Rightarrow$ $I_{BIAS1} > 10 I_{B1}$

$$V_{B1} = \frac{R_2}{R_1+R_2} V_{CC} \rightarrow V_{E1} = V_{B1} - \underbrace{V_{BE(on)}}_{0.7V}$$

$$I_{C1} \simeq I_{E1} = \frac{V_{E1}}{R_{E1}+R_{E1}'} = \frac{V_{B1}-V_{BE(on)}}{R_{E1}+R_{E1}'}$$

$$V_{C1} = V_{CC} - I_{C1}R_{C1} = V_{B2} \rightarrow V_{E2} = V_{B2} - V_{BE(on)}$$

$I_{C2} \cong I_{E2} = \frac{V_{E2}}{R_{E2}}$

Remarks.

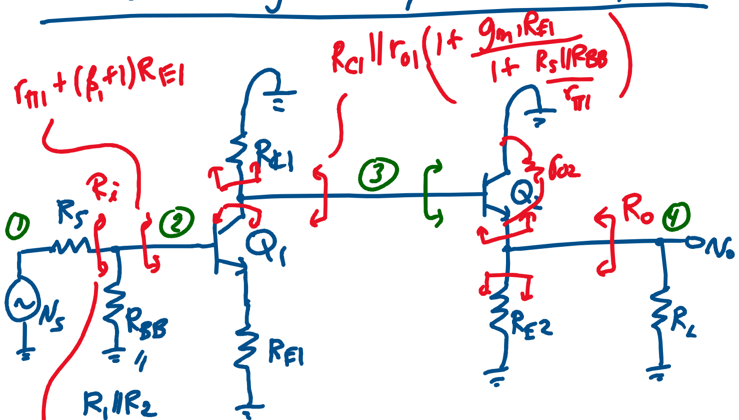
① Look for the $V_{BE(on)}$'s → well-defined voltages

② Current usually determined by $\frac{V_E}{R_E}$'s.

For bias stability:

$I_{BIAS1} > 10 I_{B1}$, also $I_{C1} = I_{BIAS2} > 10 I_{B2}$

Midband Small-Signal Analysis for Gain, R_i, and R_o:



$$\boxed{R_i = R_{BB}\,\|\,(r_{\pi1}+(\beta_1+1)R_{E1})}$$

$$R_o = \underbrace{R_{E2}\,\|\,r_{o2}}_{\text{big!}}\,\|\left\{\frac{r_{\pi2}+R_{C1}\,\|\,\overbrace{r_{o1}\left(1+\frac{g_{m1}R_{E1}}{1+\frac{R_S\|R_{BB}}{r_\pi}}\right)}^{\text{big!}}}{\beta_2+1}\right\}$$

$$\boxed{R_o = R_{E2}\,\|\left(\frac{r_{\pi2}+R_{C1}}{\beta_2+1}\right)}$$

Get gain $\frac{v_o}{v_s}$: (gain from ① to ④)

$$\frac{v_o}{v_s} = \frac{v_2}{v_s}\cdot\frac{v_3}{v_2}\cdot\frac{v_o}{v_3}$$

$$\frac{v_2}{v_s} = \frac{R_i}{R_S+R_i} = \frac{R_{BB}\|(r_{\pi1}+(\beta_1+1)R_{E1})}{R_S+R_{BB}\|(r_{\pi1}+(\beta_1+1)R_{E1})}$$

(voltage divider)

$$\frac{v_3}{v_2} = -G_{m1}R_{C1}$$

$$= -\frac{g_{m1}}{1+g_{m1}R_{E1}}\left\{R_{C1}\,\|\,\overbrace{r_{o1}\left(1+\frac{g_{m1}R_{E1}}{1+\frac{R_S\|R_{BB}}{r_{\pi1}}}\right)}^{\text{large}}\,\|\,\overbrace{\underbrace{\bigcirc}_{r_{\pi2}+(\beta_2+1)(R_{E2}\|R_L\|r_{o2})}}^{\text{large}}\right\}$$

$$\frac{v_3}{v_2} = -\frac{g_{m1}R_{C1}}{1+g_{m1}R_{E1}} \xrightarrow[{[g_{m1}R_{E1}\gg 1]}]{} -\frac{g_{m1}R_{C1}}{g_{m1}R_{E1}} \approx -\frac{R_{C1}}{R_{E1}}$$

$$\frac{v_4}{v_3} = \frac{R_{E2}\|R_L\|r_{o2}}{\underbrace{r_{e2}}_{\frac{1}{g_{m2}}}+R_{E2}\|R_L\|r_{o2}} \approx 1$$

$$\therefore \frac{v_o}{v_s} = \frac{v_2}{v_s}\cdot\frac{v_3}{v_2}\cdot\frac{v_o}{v_3}$$

$$\boxed{= -\frac{R_{BB}\,||\,(r_{\pi 1}+(\beta_1+1)R_{E1})}{R_S + R_{BB}\,||\,(r_{\pi 1}+(\beta_1+1)R_{E1})}\;\frac{g_{m1}R_{E1}}{1+g_{m1}R_{E1}} = \frac{v_o}{v_s}}$$

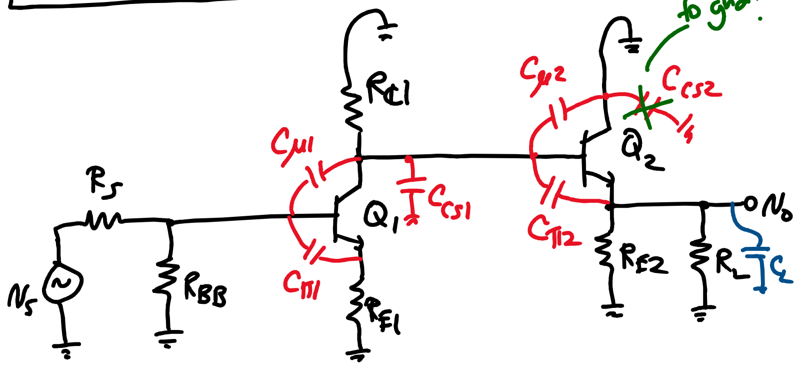
<u>Procedure for Midband Gain Inspection Analysis</u>:

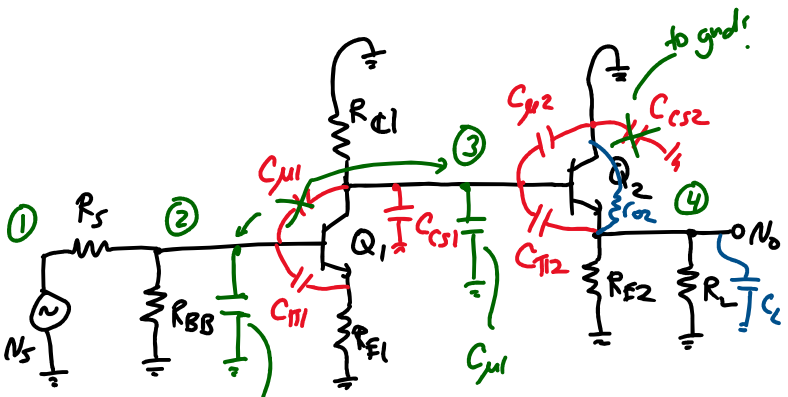
- Identify and label all signal path nodes
- Get stage gain from node to node
  - For each stage, be sure to account for loading by the next stage, specifically load resistance to ground
  - For transistor terminal-to-terminal gains, will likely need to determine output node resistance to ground
    - including loading by the next stage, and
    - even the influence of loading by the previous stage, e.g., when determining R_c
- Take the product of all node-to-node gains to get the total gain
- Can do all of this by inspection if
  - There is no feedback
  - You know all the terminal-to-terminal gain equations or can "see" or "derive" them quickly
  - You know all the equations for resistances looking into the transistor terminals (to ground) or can "see" or "derive" them quickly
  - "see" or "derive" quickly can often be done by following the currents

<u>Procedure for High Frequency Inspection Analysis</u>:

- Identify and label all signal path nodes
- Draw in the small transistor capacitors
- Use the Miller transform to turn the base-to-collector or gate-to-drain capacitor into shunt capacitors to ground
- For the base-to-emitter or gate-to-source capacitor you will need to know the equation for driving point resistance, i.e., resistance in parallel
- Get the time constant for each node by
  - Determining the total capacitance C_{node} from that node to ground
  - Determining the total resistance R_{node} from that node to ground
  - Time constant = $R_{node}*C_{node}$
- Handle each feedback capacitor separately using knowledge of its driving point R equation (or derive the equation from scratch using the hybrid-π model
- Add up all the time constants and take the reciprocal to get the ω_H

High Freq. Analysis





$$C_{M1} = C_{\mu1}(1 - a_{v1}) = C_{\mu1}(1 + G_{m1} R_{3})$$

Using OCTC Analysis: "τ_m"

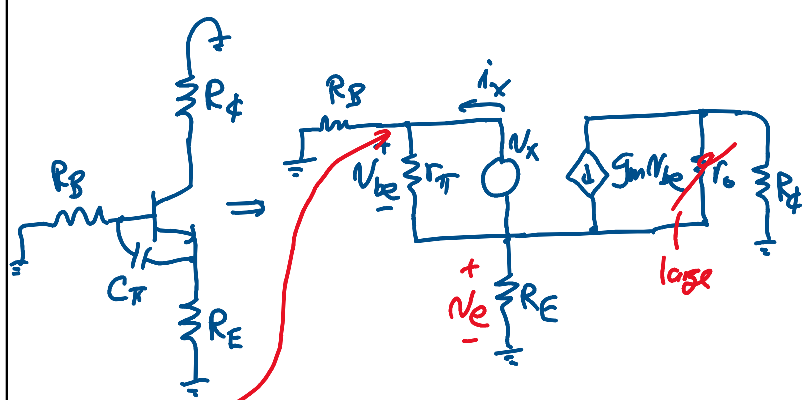
$$\omega_H = \frac{1}{C_2 R_2 + C_3 R_3 + C_4 R_4 + C_{\pi1} R_{\pi1} + C_{\pi2} R_{\pi2}}$$

total shunt C @ node 2

total shunt R @ node 2

haven't seen these yet
↳ need to determine R_π's → *

*↳ Do it for the general case:



KCL: $i_x = \frac{v_x}{r_\pi} + \frac{v_e + v_x}{R_B} = \frac{v_x}{r_\pi} + \frac{v_e}{R_B} + \frac{v_x}{R_B}$

$$v_e = R_E\left(\frac{v_x}{r_\pi} - i_x + g_m v_x\right)$$

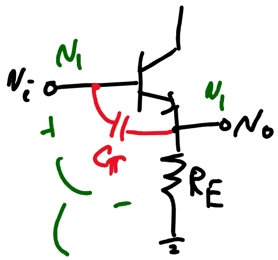
$$i_x = \frac{v_x}{r_\pi} + \frac{v_x}{R_B} + \frac{R_E}{R_B}\left(\frac{v_x}{r_\pi} - i_x + g_m v_x\right)$$

$$i_x\left(1 + \frac{R_E}{R_B}\right) = v_x\left(\frac{1}{r_\pi} + \frac{1}{R_B} + \frac{R_E}{r_\pi R_B} + \frac{g_m R_E}{R_B}\right)$$

$$R_{\pi0} = \frac{v_x}{i_x} = \frac{1}{\frac{\cancel{(R_E+R_B)}}{r_\pi \cancel{(R_E+R_B)}} + \frac{1+g_m R_E}{R_B+R_E}} = \boxed{r_\pi \| \frac{R_B+R_E}{1+g_m R_E} = R_{\pi0}}$$

R_E = large → $R_{\pi0} \approx \underbrace{r_\pi}_{25\Omega \text{ for } 1mA = I_C} \| \frac{1}{g_m}$ = small → oftentimes can be neglected!

$\gg R_B$



$$\frac{v_o}{v_i} = \frac{\overset{1k\Omega}{R_E}}{\underset{25\Omega}{\frac{1}{g_m}} + R_E} \approx 1$$

$v_i - v_1 = 0 \rightarrow C_\pi$ is effectively not there!

Now, get the τ's to get ω_H:

$$C_{②} = C_{\mu1}(1 + G_{m1}R_{③})$$

$$= C_{\mu1}\left(1 + \frac{g_{m1}}{1+g_{m1}R_{E1}}\left\{R_{C1} \,\|\, \underbrace{r_{o1}\left(1 + \frac{g_{m1}R_{E1}}{1+\frac{R_S\|R_{BB}}{r_{\pi1}}}\right)}_{huge} \,\|\, \bigcirc\right\}\right)$$

$$\bigcirc = \underbrace{r_{\pi2} + (\beta_2+1)(R_{E2}\|R_L\|r_{o2})}_{large}$$

$$C_{②} = C_{\mu1}\left(1 + \frac{g_{m1}R_{C1}}{1+g_{m1}R_{E1}}\right)$$

$$R_{②} = R_S \| R_{BB} \| (\underbrace{r_{\pi1} + (\beta_1+1)R_{E1}}_{large}) \approx R_S \| \underset{\uparrow\ large}{R_{BB}} \approx R_S$$

$$\boxed{\tau_{②} = C_{\mu1}\left(1 + \frac{g_{m1}R_{C1}}{1+g_{m1}R_{E1}}\right)R_S}$$

$$C_{③} = C_{cs1} + C_{\mu1} + C_{\mu2}$$

$$R_{③} = R_{C1} \,\|\, \underbrace{r_{o1}\left(1 + \frac{g_{m1}R_{E1}}{1+\frac{R_S\|R_{BB}}{r_{\pi1}}}\right)}_{large} \,\|\, \underbrace{\left(r_{\pi2} + (\beta_2+1)(R_{E2}\|R_L\|r_{o2})\right)}_{large}$$

$$\cong R_{C1}$$

$$\boxed{\tau_{③} = (C_{cs1} + C_{\mu1} + C_{\mu2})R_{C1}}$$

$$C_{④} = C_L$$

$$R_{④} = R_{E2} \| R_L \| \frac{r_{\pi2} + R_{C1} \| \cancel{r_{o1}\left(1 + \frac{g_{m1}R_{E1}}{1+\frac{R_S\|R_{BB}}{r_{\pi}}}\right)}^{\,large}}{\beta_2+1}$$

$$\cong R_{E2} \| R_L \| \left(\frac{r_{\pi2} + R_{C1}}{\beta_2+1}\right)$$

$$\boxed{\tau_{④} = C_L\left(R_{E2} \| R_L \| \left(\frac{r_{\pi2} + R_{C1}}{\beta_2+1}\right)\right)}$$