Lecture 35: Differential Pairs

- **Announcements**:
- HW#11 will be online soon and due Tuesday, Dec. 10 (more than two weeks from now)
- Lab 6 online and due 5 p.m., Friday, Dec. 13
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- Lecture Topics:
  - Differential Pair
  - MOS Op Amp Design
  - Review of Digital Electronics
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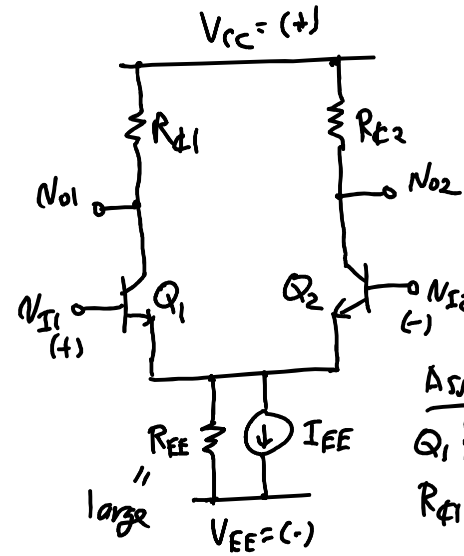
- **Last Time**:
- Started on differential pairs
- Now, continue with this ...

$\boxed{\text{Differential Pair} \rightarrow \text{A Simple Op Amp}}$



Assumption:
Q_1 & Q_2 identical
R_{C1} & R_{C2} identical

Purpose: Amplify the difference between two signals regardless of their common-mode values.

Definitions.

$v_{ID} = v_{I1} - v_{I2}$ (differential input)

$v_{ICM} = \frac{v_{I1} + v_{I2}}{2}$ (common-mode input)

$v_{I1} = v_{ICM} + \frac{v_{ID}}{2}$

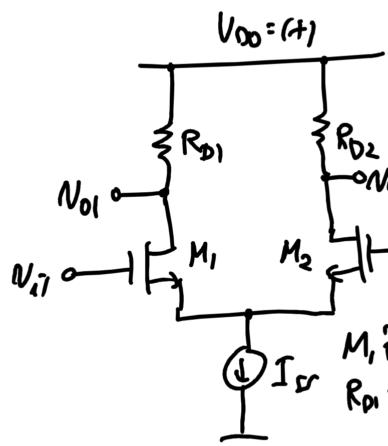
$v_{I2} = v_{ICM} - \frac{v_{ID}}{2}$

Differential Gain: $A_d = \frac{v_{o1}-v_{o2}}{v_{id}} = \frac{v_{od}}{v_{id}}$ (want BIG)

Common-Mode Gain: $A_{cm} = \frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}}$ (want SMALL)

$$\boxed{\text{Common-Mode Rejection Ratio} = CMRR = \frac{A_{dm}}{A_{cm}}}$$

(want BIG)

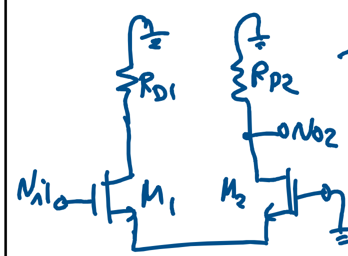


M_1 & M_2 identical
$R_{D1} = R_{D2}$

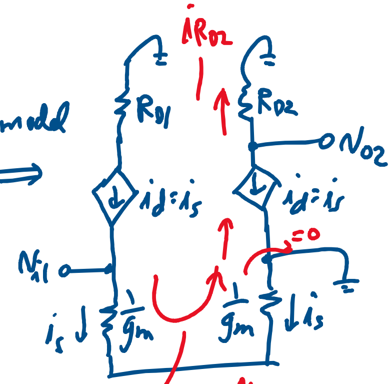
Want $A_d = \frac{v_{o1}-v_{o2}}{v_{i1}-v_{i2}}$:

⇒ use superposition:
- get $v_{o2}' = f(v_{i1})$ & $v_{o2}'' = f(v_{i2})$
- then $v_{o2} = v_{o2}' + v_{o2}''$
- then repeat for v_{o1}

S.S.Ckt.



T-model ⇒



$i_{R_{D2}} = \frac{v_{i1}}{\frac{2}{g_m}} = \frac{1}{2} g_m v_{i1}$

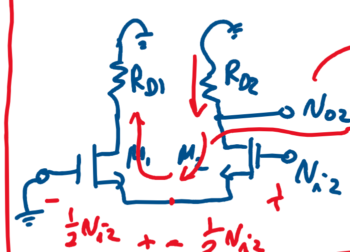
$v_{o2}' = i_{R_{D2}} R_{D2}$

$= +\frac{1}{2} g_m v_{i1} R_{D2}$ → ∴ $\frac{v_{o2}'}{v_{i1}} = \frac{1}{2} g_m R_D$

$[R_D = R_{D1} = R_{D2}]$

$[g_m = g_{m1} = g_{m2}]$

Get $\frac{v_{o2}''}{v_{i2}}$:

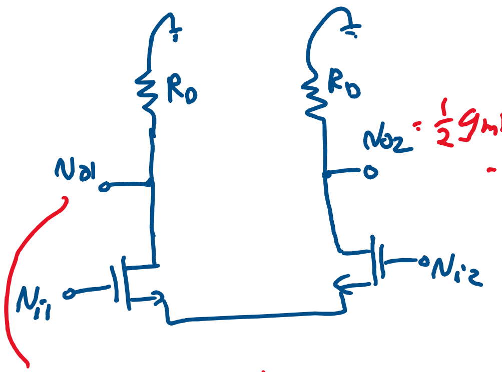


$v_{o2}'' = -\frac{1}{2} g_{m2} v_{i2} R_{D2}$

$i_{d2} = g_{m2}\left(\frac{1}{2} v_{i2}\right) = \frac{1}{2} g_{m2} v_{i2}$

$v_{o2} = v_{o2}' + v_{o2}'' = \frac{1}{2} g_m R_D v_{i1} - \frac{1}{2} g_m R_D v_{i2}$

↳ want v_{o1}



$$v_{o2} = \frac{1}{2} g_m R_D v_{i1} - \frac{1}{2} g_m R_D v_{i2}$$

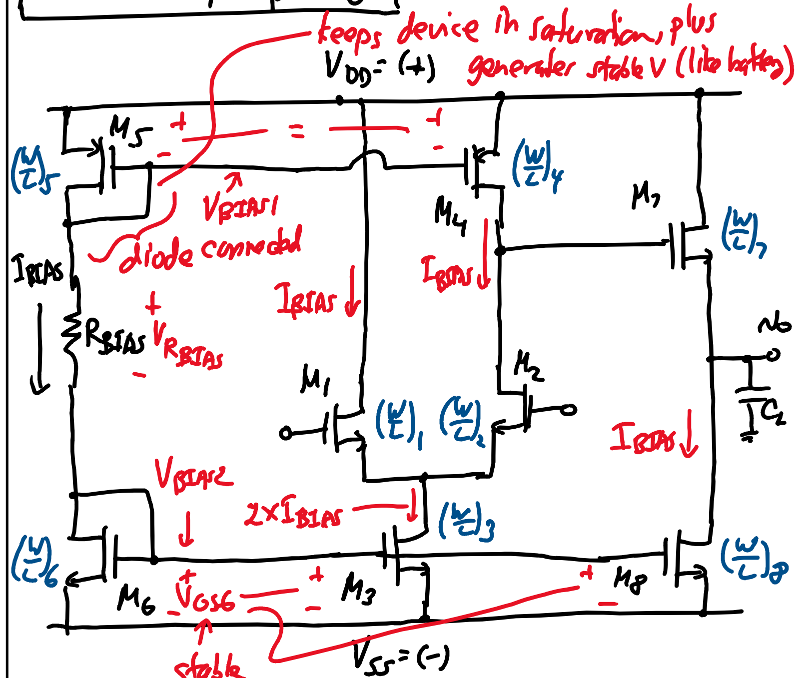
w/ a similar analysis:

$$v_{o1} = -\frac{1}{2} g_m R_D v_{i1} + \frac{1}{2} g_m R_D v_{i2}$$

$$\left.\begin{aligned} v_{o1} &= -\frac{1}{2} g_m R_D (v_{i1} - v_{i2}) \\ v_{o2} &= +\frac{1}{2} g_m R_D (v_{i1} - v_{i2}) \end{aligned}\right\} \quad \begin{aligned} v_{od} &= v_{o1} - v_{o2} \\ &= -g_m R_D (v_{i1} - v_{i2}) \end{aligned}$$

$$\boxed{\frac{v_{od}}{v_{id}} = \frac{v_{o1} - v_{o2}}{v_{i1} - v_{i2}} = A_{dm} = -g_m R_D}$$

An Actual Op Amp Design (in CMOS)



Biasing:

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 \rightarrow$ to eliminate V_{os}

$\left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 \rightarrow$ for $I_{D4} = I_{D5} = I_{BIAS}$

$\left(\frac{W}{L}\right)_8 = \left(\frac{W}{L}\right)_6 \rightarrow$ for $I_{D8} = I_{D6} = I_{BIAS}$

$\left(\frac{W}{L}\right)_3 = 2 \times \left(\frac{W}{L}\right)_6 \rightarrow$ for $I_{D3} = 2 I_{D6} = 2 I_{BIAS}$

To Specify I_{BIAS} → specify R_{BIAS}:

$$R_{BIAS} = \frac{V_{R_{BIAS}}}{I_{BIAS}} = \frac{V_{DD} - |V_{GS5}| - V_{GS6} - V_{SS}}{I_{BIAS}}$$

$$I_{DS} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_5 (|V_{GS5}| - |V_{tp}|)^2$$

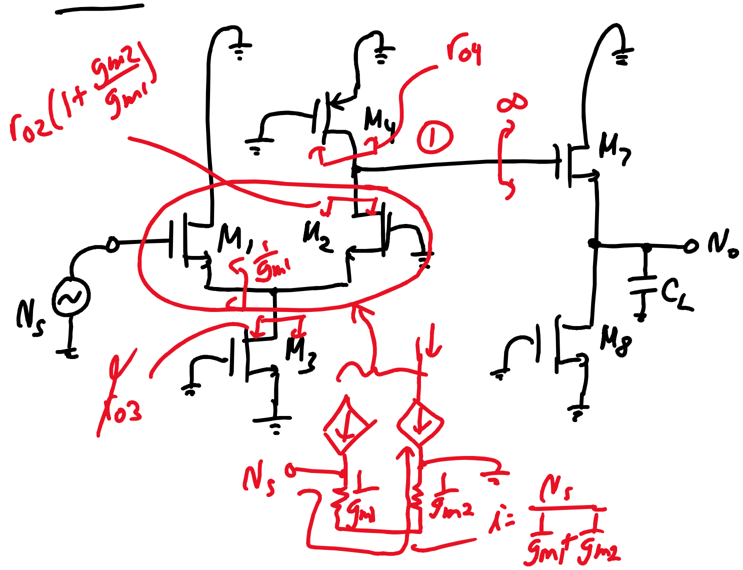
$$\hookrightarrow |V_{GS5}| = |V_{tp}| + \sqrt{\frac{2 I_{BIAS}}{\mu_p C_{ox}(W/L)_5}}$$

Similarly: $V_{GS6} = V_{tn} + \sqrt{\dfrac{2 I_{BIAS}}{\mu_n C_{ox}(W/L)_6}}$

→ Pick an I_{BIAS} → Get the needed R_{BIAS}

Gain: ⇒ focus on the signal path only!

S.S. Ckt:



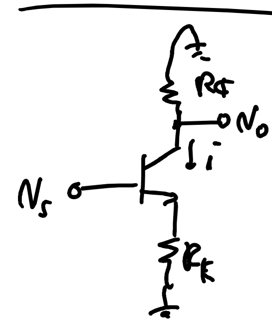
$$\frac{v_o}{v_s} = \frac{v_1}{v_s} \cdot \frac{v_o}{v_1}$$

$$\frac{v_1}{v_s} = G_{mdiff} R_1 = \underbrace{\frac{1}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}}_{G_{mdiff}} \left[r_{o4} \| r_{o2}\underbrace{\left(1 + \frac{g_{m1}}{g_{m2}}\right)}_{2}\right]$$

$$= \frac{1}{2} g_{m1} (r_{o4} \| 2 r_{o2})$$

$$\frac{v_o}{v_1} = \frac{g_{m7}}{g_{m7} + g_{mb7}} \simeq 1$$

$$\boxed{\therefore \frac{v_o}{v_s} = +\frac{1}{2} g_{m1} (r_{o4} \| 2 r_{o2})}$$



$$\frac{v_o}{v_s} = -G_m R_C$$

$$G_m = \frac{i}{v_s} = \frac{g_m}{1 + g_m R_E}$$