Lecture 3: Frequency Response

- **Announcements**:
- HW#1 online and due this Friday
- Discussions this week
- Lab#1 online
- Labs start next week
  - You will need to do your prelabs for Lab 1 before your lab period
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- Lecture Topics:
  - Finish Digital Communications Example
  - Review Impedance
  - Frequency Response
  - Bode Plots
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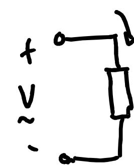
- **Last Time**:
- Going through an example digital communication transmitter as motivation
- Now, continue with this …

- Review of Analog Circuit Concepts:
- We assume you understand the following concepts from previous courses:
  - Transfer functions
  - Gain (voltage, current, power)
  - Input resistance
  - Output resistance
  - Two-port models for amplifiers
  - Bode plots
  - Ideal op amp ckt design and analysis
- We'll review some of these now to jog your memory

$\boxed{\text{Review Impedance}}$



where $\underset{\sim}{V}$ and $\underset{\sim}{I}$ are phasor variables:

Time | Phasor

$$v(t) = V'\cos(\omega t + \phi) \Leftrightarrow \underset{\sim}{V} = V' e^{j\phi}$$

$$i(t) = I'\cos(\omega t + \phi) \Leftrightarrow \underset{\sim}{I} = I' e^{j\phi}$$

and impedance: $\underset{\sim}{Z} = \dfrac{\underset{\sim}{V}}{\underset{\sim}{I}}$

Resistor - $R = \dfrac{v(t)}{i(t)} \underset{\text{phasor}}{\Longleftrightarrow} R = \dfrac{\underset{\sim}{V}}{\underset{\sim}{I}}$

Capacitor - $i(t) = C\dfrac{dv}{dt} \Longleftrightarrow \underset{\sim}{I} = Cj\omega \underset{\sim}{V}$

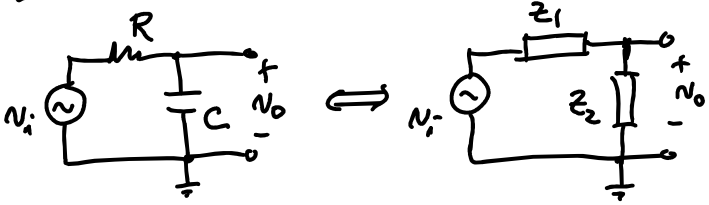
$$\frac{\underset{\sim}{V}}{\underset{\sim}{I}} = Z = \frac{1}{j\omega C} \longleftrightarrow Z = \frac{1}{sC}$$

switch $s = j\omega$

Inductor - $v = L\dfrac{di}{dt} \Longleftrightarrow \underset{\sim}{V} = Lj\omega \underset{\sim}{I}$

$$\frac{\underset{\sim}{V}}{\underset{\sim}{I}} = Z = j\omega L = sL$$

$\boxed{\text{Example. Determine a transfer function}}$



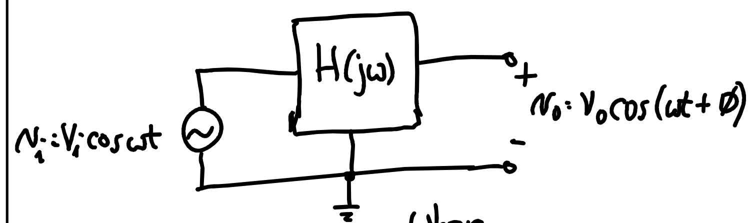
$Z_1 = R$, $Z_2 = \dfrac{1}{sC}$ } Recognize a voltage divider:

$$\therefore v_o = \frac{Z_2 v_i}{Z_1 + Z_2} \Rightarrow \frac{v_o}{v_i}(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$\frac{v_o}{v_i}(s) = \frac{1}{1+sRC} = \frac{1}{1+s\tau_p} = \frac{1}{1+\frac{s}{\omega_p}}$$

$[\tau_p = RC]$ $\quad$ $\left[\omega_p = \dfrac{1}{RC}\right]$

- Frequency Response:
- To measure the frequency response of a given network:
  - Excite the network with a variable-frequency, sinusoidal, constant amplitude source
  - Measure the magnitude and phase of the output signal for different values of input frequency



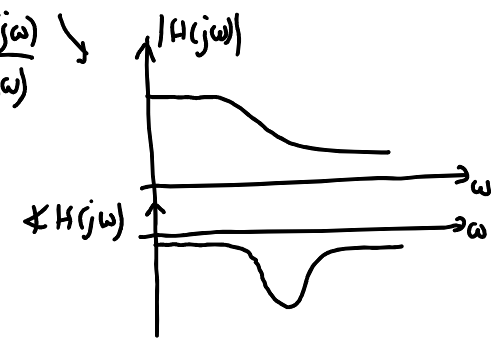
where

$|v_o(j\omega)| = V_o =$ magnitude

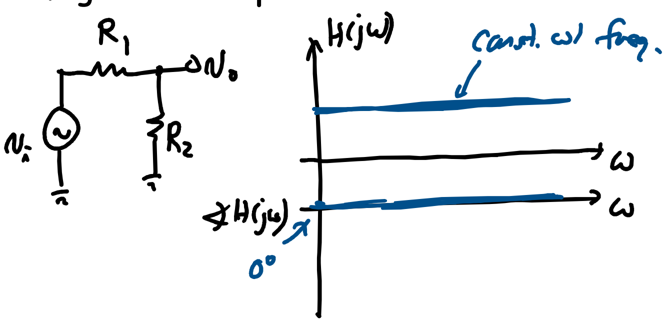
$\measuredangle v_o(j\omega) = \phi =$ phase

- The frequency response of a given network is commonly described by a plot of magnitude and phase versus frequency. Usually, such a plot is of the transfer function of a given network:

$$H(j\omega) = \frac{v_o(j\omega)}{v_i(j\omega)}$$



- For a purely resistive network, the frequency response is constant (i.e., a straight line), both magnitude and phase



- The addition of reactive (energy storage) components, e.g., capacitors, inductors
  - Shapes the frequency response
  - Adds singularities, i.e., poles and zeros
  - Yields the general transfer function:

$$H(s) = \frac{v_o(s)}{v_i(s)} = \underbrace{H_0}_{\text{resistive term}} \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} = H_0 \frac{\prod_{j=1}^{m}(s-z_j)}{\prod_{i=1}^{n}(s-p_i)}$$

where

$z_j = \sigma_{z_j} + j\omega_{z_j}$; $\sigma_{z_j} = Re(z_j)$, $\omega_{z_j} = Im(z_j)$

$p_i = \sigma_{p_i} + j\omega_{p_i}$; $\sigma_{p_i} = Re(p_i)$, $\omega_{p_i} = Im(p_i)$

$v_i(s) \triangleq$ input variable (not necessarily a voltage)

$v_o(s) \triangleq$ output variable (" " ")

$\Rightarrow$ Given this, you should be able to generate a Bode plot!