Lecture 5: Ideal Op Amps

- Announcements:
- HW#2 online and due next Friday via Gradescope
- Labs have started
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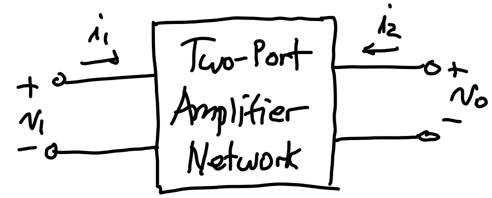
- Lecture Topics:
  - Amplifier Models (2-port networks)
  - Input R_i
  - Output R_o
  - Ideal Voltage Amplifier
  - Ideal Op Amps
  - Negative Feedback
  - Op Amp Circuits
- ------------------------------------
- Last Time:
- Going through amplifier models
- Now, continue with this …

- All of these are equivalent representations, each comprising a gain factor along with an input and output resistance that model the resistance seen looking into the amplifier terminals
- Take for example a voltage amplifier:

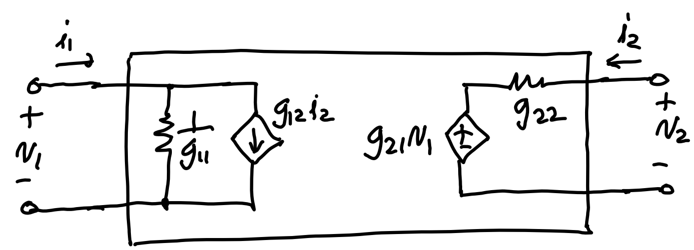
$\boxed{\text{Voltage Amplifier}}$

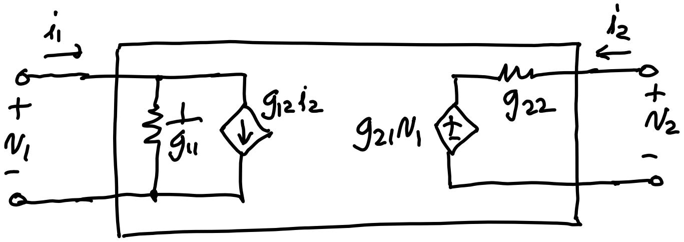
⇒ most appropriate general model is the g-parameter model → Defining Equations:

Note the matrix form →
$$\begin{cases} i_1 = g_{11}v_1 + g_{12}i_2 \\ v_2 = g_{21}v_1 + g_{22}i_2 \end{cases}$$



$$\equiv$$





where:

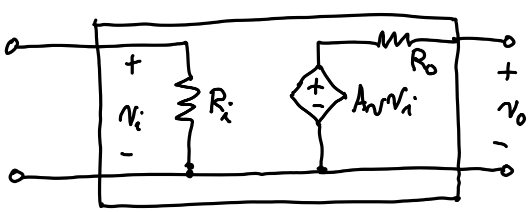
$g_{11} = \left.\frac{i_1}{v_1}\right|_{i_2=0}$ = open-circuit input conductance (measure i_1; input v_1; $i_2 = 0$ ← open-ckt the output)

$g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0}$ = reverse short-circuit current gain (meas i_1; input i_2; $v_1 = 0$ ← short-ckt the input)

$g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0}$ = forward open-circuit voltage gain

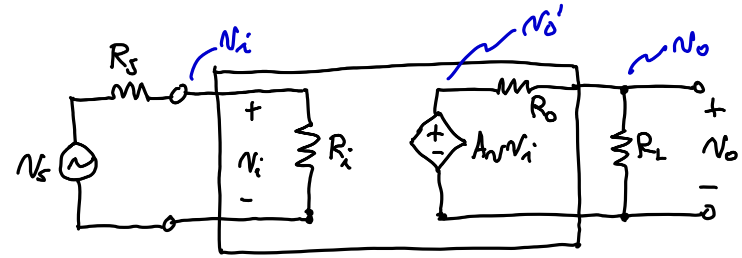
$g_{22} = \left.\frac{v_2}{i_2}\right|_{v_1=0}$ = short-circuit output resistance

Assuming a design to amplify in the forward direction → neglect the current source $g_{12}i_2$ in the g-parameter model → the result:



Voltage Amplifier Model

⇒ R_i & R_o directly influence the total voltage gain via the input & output voltage dividers:



$\frac{v_i}{v_s} = \frac{R_i}{R_i + R_s}$ $\qquad \frac{v_o'}{v_i} = A_v$ $\qquad \frac{v_o}{v_o'} = \frac{R_L}{R_L + R_o}$

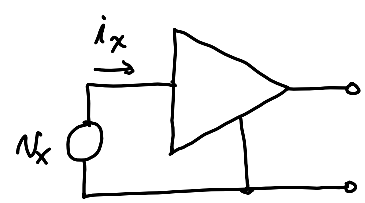
$\therefore \frac{v_o}{v_s} = \underbrace{\frac{R_i}{R_i + R_s}} A_v \underbrace{\frac{R_L}{R_L + R_o}}$ = total voltage gain $\neq A_v$ ← ideal

these terms attenuate the intended gain!

$\frac{v_o}{v_s} = \frac{v_i}{v_s} \cdot \frac{v_o'}{v_i} \cdot \frac{v_o}{v_o'}$

get when $R_i = \infty$ & $R_o = 0$ ← These values define an ideal voltage amplifier!

To Determine R_i:

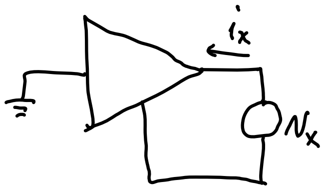


① Apply test voltage source, v_x.

② Measure resulting current, i_x.

③ $R_i = \frac{v_x}{i_x}$ ← same as $\frac{1}{g_{11}} = \left.\frac{v_1}{i_1}\right|_{i_2=0}$ from the g-parameter model

To Determine R_o (for a voltage amplifier)



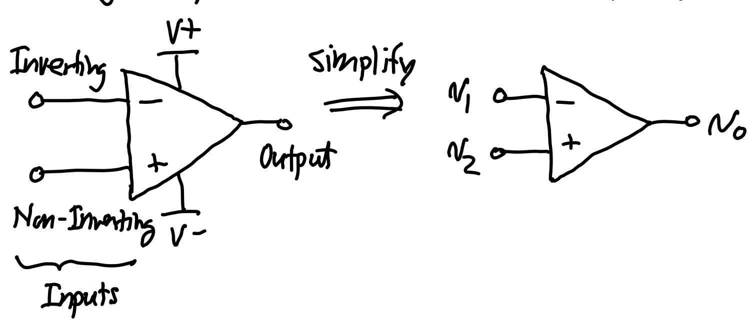
1. Ground input. (Zero out all independent sources.)
2. Apply test voltage source, v_x.
3. Measure resulting current, i_x.
4. $R_o = \frac{v_x}{i_x}$ ← Same as $g_{22} = \frac{v_2}{i_2}\Big|_{v_1=0}$ from the g-parameter model

Ideal Operational Amplifier (op amps)

⇒ the work horse of analog electronics

⇒ Combinations of op amps w/ feedback components allow realization of analog computers, sampled-data systems, A/D converters, DAC's, instrumentation amplifiers, ...

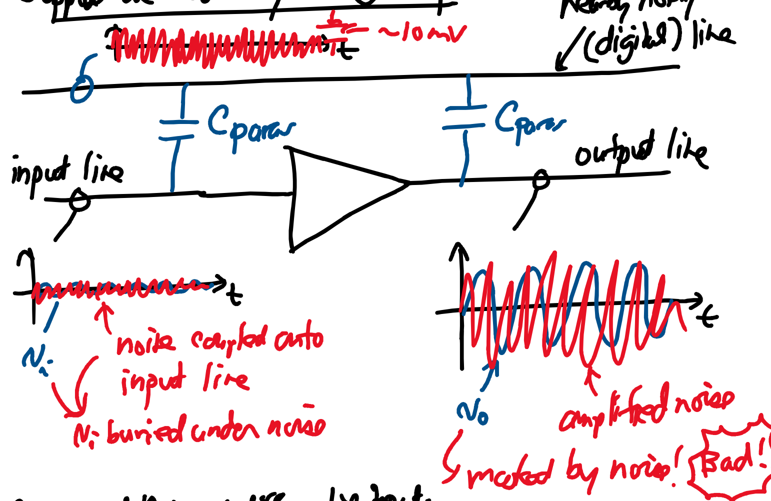
⇒ in general, have a minimum of 5 terminals:



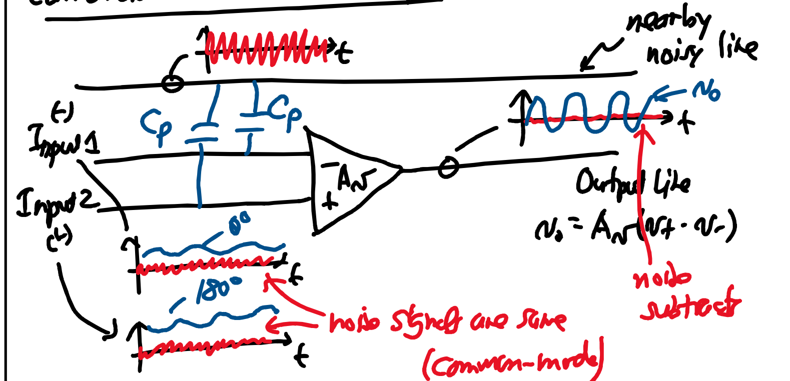
Why Have 2 Inputs?

1. To get a virtual ground for op amp ckts.
2. To suppress common-mode noise.

Suppose we had only a single input:

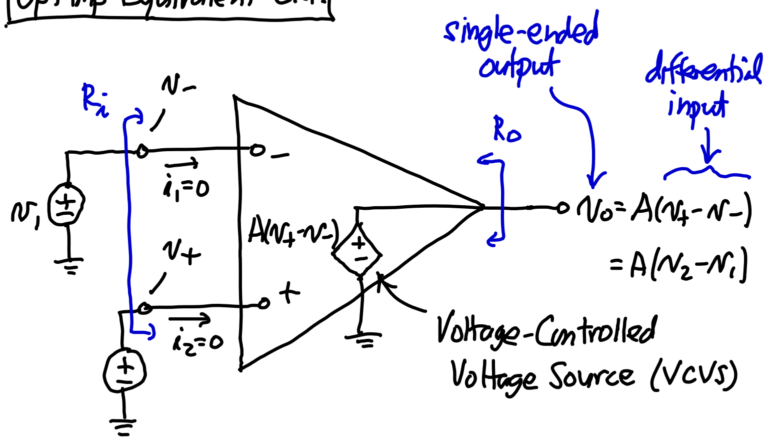


Can avoid this w/ differential input:



Perhaps the best way to define an op amp is via its equivalent ckt:

Op Amp Equivalent Ckt.



Properties of Ideal Op Amps:

① $R_i=\infty \longrightarrow$ ④ $i_+=i_-=0$

② $R_o=0$

③ $A_v=\infty \longrightarrow$ ⑤ $v_+=v_-$, assuming v_o = finite

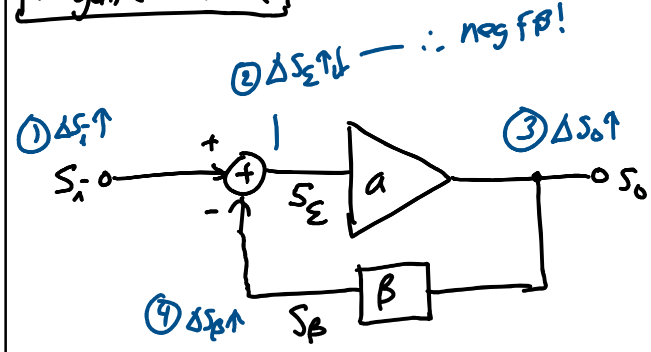
Big Assumption! only holds when we have negative feedback!

Why? $\infty(v_+-v_-)=v_o$ = finite

$\therefore v_+-v_-=0 \rightarrow v_+=v_-$

source of "virtual ground"

Negative Feedback



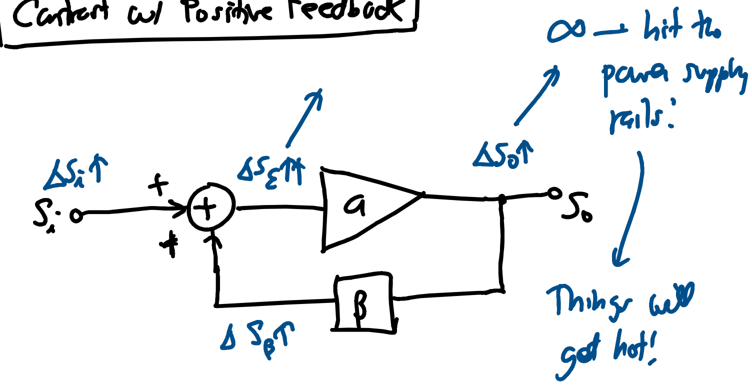
$$\left.\begin{array}{l} S_o = aS_\epsilon \\ S_\epsilon = S_i - \beta S_o \end{array}\right\} \quad S_o = a(S_i-\beta S_o)$$

$$S_o(1+a\beta) = aS_i \rightarrow \boxed{\frac{S_o}{S_i} = \frac{a}{1+a\beta}}$$

$$[a\to\infty] \Rightarrow \frac{S_o}{S_i} \underset{\uparrow}{\approx} \frac{a}{a\beta} = \frac{1}{\beta} = \text{finite!}$$

$[a\beta \gg 1]$

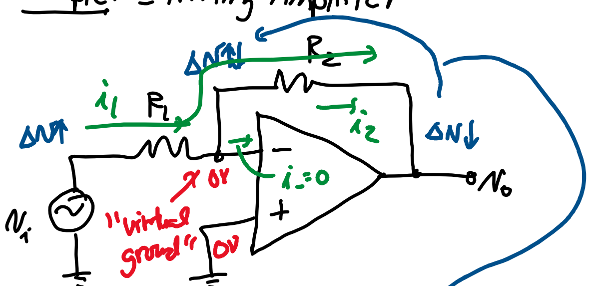
Contrast w/ Positive Feedback



- Remarks: (on neg. FB)
- Neg. FB can insure v_o = finite even with a=infinity
- Overall closed-loop gain (or transfer function) is dependent only on external components (e.g., β)
- Overall closed-loop gain S_o/S_i is independent of amplifier gain a
- This is very important, since it's easy to get large amplifier gain, but it's hard to get an exact value
  - If you're shooting for a=50,000, you might get 47,000 or 60,000 instead
  - But it won't matter much in the feedback ckt.

Op Amp Ckts.

Example. Inverting Amplifier



① Verify that we have neg. FB ✓ ∴ ideal op amp rules apply!

② ∵ v_o = finite → $v_+ = v_- =$ virtual ground

③ $i_- = 0$

$$\left.\begin{array}{l} i_1 = \dfrac{v_i - 0}{R_1} = \dfrac{v_i}{R_1} = i_2 \\ v_o = -i_2 R_2 \end{array}\right\} \quad v_o = -\left(\frac{v_i}{R_1}\right)R_2 = -\frac{R_2}{R_1}v_i$$

$$\therefore \boxed{\frac{v_o}{v_i} = -\frac{R_2}{R_1}}$$