Lecture 6: Finite Gain & Bandwidth

- Announcements:
- HW#2 online and due Friday via Gradescope
- Lab#2 online
- Lots of SPICE this week
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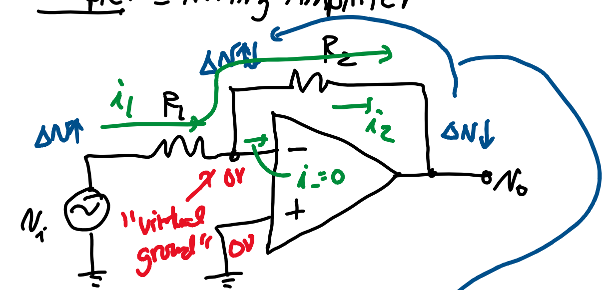
- Lecture Topics:
  - Ideal Op Amp Circuits
  - Non-Ideal Op Amp Bode Plot
  - Finite Gain & Bandwidth
  - Closed Loop Amplifier Freq. Response
    - —Non-Inverting Amplifier
    - —Inverting Amplifier
- -------------------------------------
- Last Time:
- Op amp circuit example
- Now, continue with this …

- Remarks: (on neg. FB)
- Neg. FB can insure v_o = finite even with a=infinity
- Overall closed-loop gain (or transfer function) is dependent only on external components (e.g., β)
- Overall closed-loop gain S_o/S_i is independent of amplifier gain a
- This is very important, since it's easy to get large amplifier gain, but it's hard to get an exact value
  - If you're shooting for a=50,000, you might get 47,000 or 60,000 instead
  - But it won't matter much in the feedback ckt.

Op Amp Ckts.

Example. Inverting Amplifier



① Verify that we have neg. FB ✓ ∴ ideal op amp rules apply!

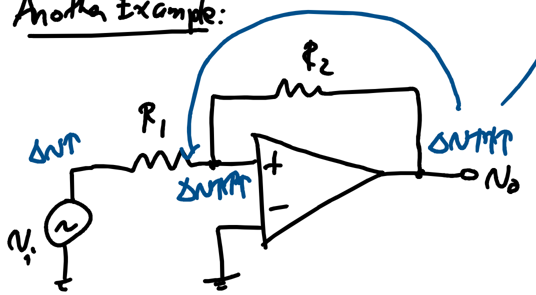
② $\therefore v_o$ = finite → $v_+ = v_-$ = virtual ground

③ $i_- = 0$

$$\left.\begin{aligned} i_1 &= \frac{v_i - 0}{R_1} = \frac{v_i}{R_1} = i_2 \\ v_o &= -i_2 R_2 \end{aligned}\right\} \quad v_o = -\left(\frac{v_i}{R_1}\right) R_2 = -\frac{R_2}{R_1} v_i$$

$$\therefore \boxed{\frac{v_o}{v_i} = -\frac{R_2}{R_1}}$$

Another Example:



Blow up!
↓
Head for the supply rails: either L^+ or L^-

① Verify neg. FB ✗
↓
Can no longer say $v_+ = v_-$

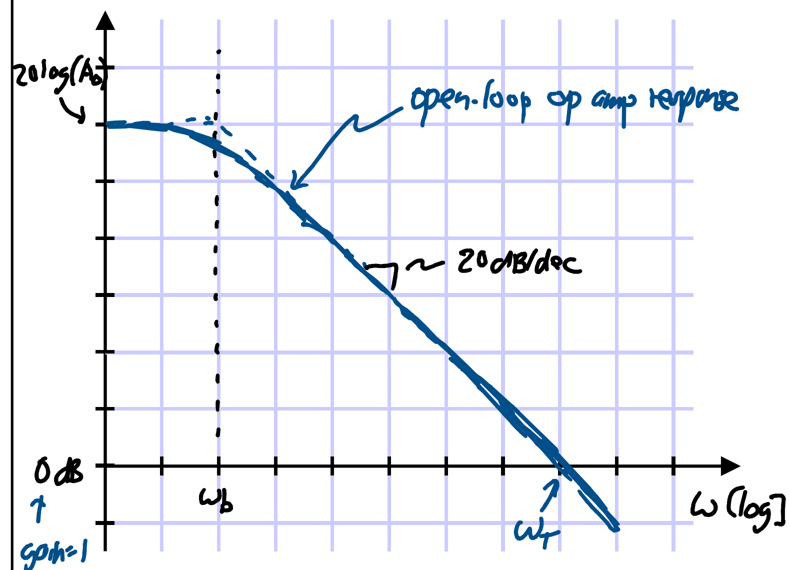
Can still say: $i_- = i_+ = 0$, since $R_i = \infty$

Non-Ideal Op Amps

Finite Op Amp Gain & Bandwidth

For an ideal op amp, $A = \infty$. (gain)

In reality, the gain goes as: $A(s) = \dfrac{A_0}{1 + \frac{s}{\omega_b}}$ ← finite DC gain



$\omega_T \triangleq$ unity gain frequency = freq. @ which $|A(j\omega)| = 1$ (= 0dB)

At ω_T: $|A(j\omega_T)| = \dfrac{A_0}{\sqrt{1 + \left(\frac{\omega_T}{\omega_b}\right)^2}} \underset{[\omega_T \gg \omega_b]}{\Longrightarrow} \dfrac{A_0}{\frac{\omega_T}{\omega_b}} = 1 \rightarrow \boxed{\omega_T = A_0 \omega_b}$

gain bandwidth

$$[\text{For } \omega \gg \omega_b] \Rightarrow A(s) = \frac{A_o}{\frac{s}{\omega_b}} = \frac{A_o \omega_b}{s} = \frac{\omega_T}{s} \Rightarrow \frac{f_T}{f}$$

f ← freq.

$$A(s) = \frac{A_o}{1 + \frac{s}{\omega_b}} \Rightarrow A(j\omega) = \frac{A_o}{1 + \frac{j\omega}{\omega_b}} \Rightarrow |A(j\omega)| = \frac{A_o}{\sqrt{1 + \left(\frac{\omega}{\omega_b}\right)^2}}$$

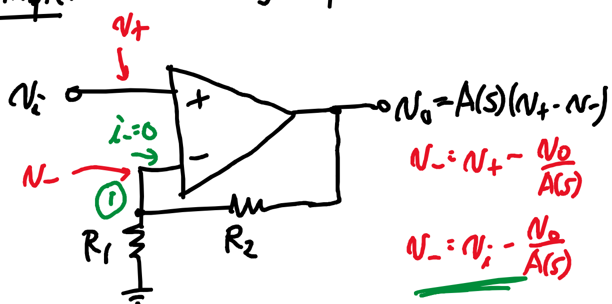
$$[\omega \gg \omega_b] \rightarrow \frac{A_o}{\sqrt{\left(\frac{\omega}{\omega_b}\right)}}$$

An op amp ultimately is an integrator with time constant $\tau = \frac{1}{\omega_T}$.

ω_T in datasheets

$$\boxed{\text{Frequency Response of a Closed Loop Amplifier}}$$

Example. Non-Inverting Amplifier



$$v_- = v_+ - \frac{v_o}{A(s)}$$

$$v_- = v_i - \frac{v_o}{A(s)}$$

Find an expression for gain as a function of freq.

① Brute Force Determination:

$$\text{KCL ①}: \frac{v_o - v_-}{R_2} = \frac{v_-}{R_1} + \frac{v_o}{R_2} = v_-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{v_o}{R_2} = \left(v_i - \frac{v_o}{A(s)}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \Rightarrow \frac{v_o}{v_i}(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A(s)}\left(1 + \frac{R_2}{R_1}\right)}$$

$$\left[A(s) = \frac{A_o}{1 + \frac{s}{\omega_b}}\right]$$

Accounting for finite gain & BW

$$\frac{v_o}{v_i}(s) = \frac{1 + \frac{R_2}{R_1}}{\cancel{1 + \frac{1}{A_o}\left(1 + \frac{R_2}{R_1}\right)}} \cdot \frac{1}{1 + \frac{s}{\omega_b} \frac{\frac{1}{A_o}\left(1 + \frac{R_2}{R_1}\right)}{\cancel{1 + \frac{1}{A_o}\left(1 + \frac{R_2}{R_1}\right)}}}$$

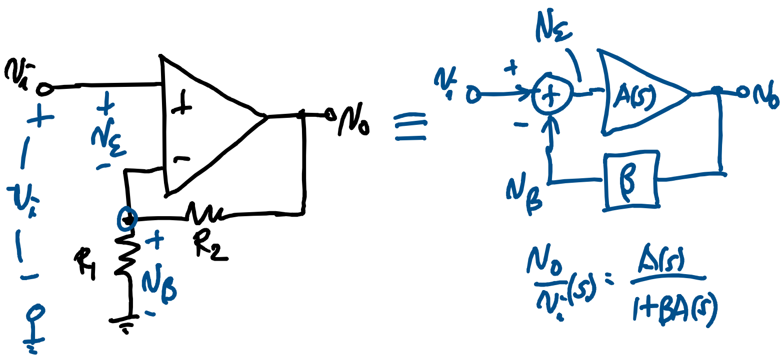
$$[A_o = \text{large}] \Rightarrow \left(\frac{1}{A_o}\left(1 + \frac{R_2}{R_1}\right) \ll 1\right)$$

Here, we're essentially saying that the op amp gain is close to ideal & the non-ideality is mostly finite BW

$$\boxed{\frac{v_o}{v_i}(s) = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + \frac{s}{A_o \omega_b \left(\frac{R_1}{R_1 + R_2}\right)}}}$$

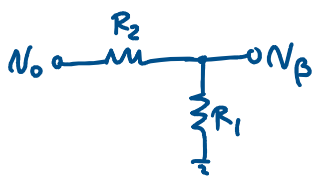
Near Ideal Gain, but Finite BW

② More insightful way to do this:



$$\frac{v_o}{v_i}(s) = \frac{A(s)}{1+\beta A(s)}$$

What is β?



$$\boxed{\beta = \frac{v_\beta}{v_o} = \frac{R_1}{R_1+R_2}}$$

Recall fr previous analysis:

$$\frac{v_o}{v_i}(s) = \frac{A(s)}{1+\beta A(s)}$$

$$\left[A(s) = \frac{A_0}{1+\frac{s}{\omega_b}}\right] \rightarrow \frac{v_o}{v_i}(s) = \frac{\frac{A_0}{1+\frac{s}{\omega_b}}}{1+\beta\left(\frac{A_0}{1+\frac{s}{\omega_b}}\right)}$$

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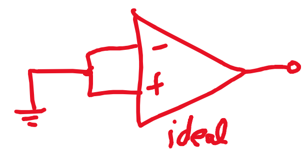
$$\boxed{\frac{v_o}{v_i}(s) = \underbrace{\frac{A_0}{1+\beta A_0}}_{\text{Dc Gain Term (midband gain)}} \underbrace{\frac{1}{1+\frac{s}{\omega_b(1+\beta A_0)}}}_{\text{Frequency Shaping Term}}}$$

For lab #2:



$v_o = A(s)(v_+ - v_-) = 0$ ← expect

actually $\neq 0$