Lecture 7: Non-Ideal Op Amp Circuits

- Announcements:
- HW#3 online and due Friday via Gradescope
- Due in lab session next week:
  - Lab#2 prelab
  - Lab#1 writeup
- ------------------------------------
- Lecture Topics:
  - Closed Loop Amplifier Freq. Response
    - Non-Inverting Amplifier
    - Inverting Amplifier
- ------------------------------------
- Last Time:
- Non-inverting amplifier using finite gain-BW op amp
- Now, continue with this …

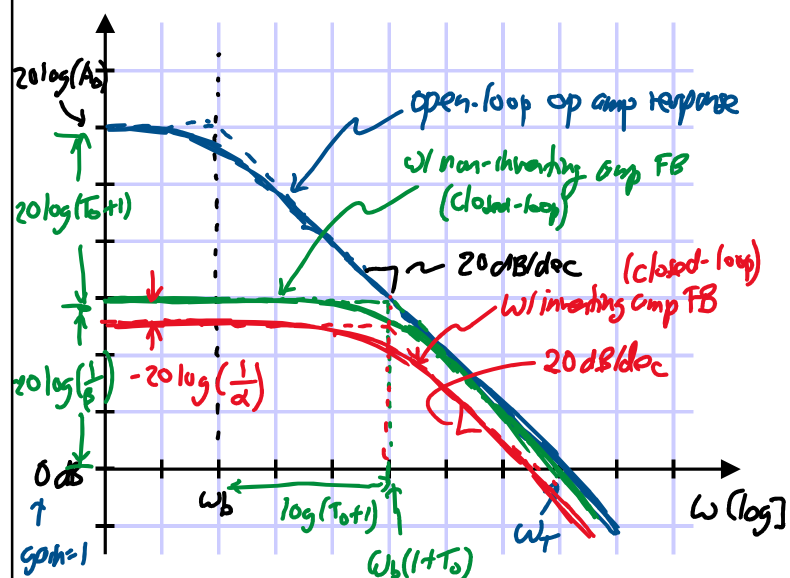
Non-Ideal Op Amps

Finite Op Amp Gain & Bandwidth

For an ideal op amp, $A = \infty$. (gain)

In reality, the gain goes as: $A(s) = \dfrac{A_o}{1 + \frac{s}{\omega_b}}$ ← finite DC gain

$|A(j\omega)|$ [dB]



$\omega_T \triangleq$ unity gain frequency = freq. @ which $|A(j\omega)| = 1$ (= 0dB)

At ω_T: $|A(j\omega_T)| = \dfrac{A_o}{\sqrt{1+\left(\frac{\omega_T}{\omega_b}\right)^2}} \xrightarrow[\;[\omega_T \gg \omega_b]\;]{} \dfrac{A_o}{\frac{\omega_T}{\omega_b}} = 1 \rightarrow \boxed{\omega_T = A_o \omega_b}$

gain bandwidth

$[\text{For } \omega >> \omega_b] \Rightarrow A(s) = \frac{A_o}{\frac{s}{\omega_b}} = \frac{A_o \omega_b}{s} = \frac{\omega_T}{s} \Rightarrow \frac{f_T}{f}$ (f: freq.)

$A(s) = \frac{A_o}{1 + \frac{s}{\omega_b}} \Rightarrow A(j\omega) = \frac{A_o}{1 + \frac{j\omega}{\omega_b}} \Rightarrow |A(j\omega)| = \frac{A_o}{\sqrt{1 + \left(\frac{\omega}{\omega_b}\right)^2}}$

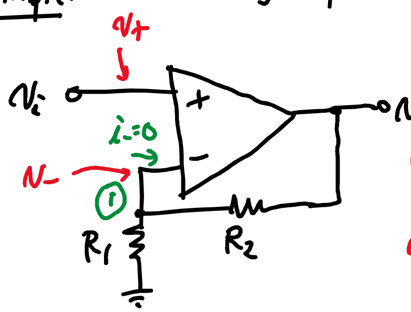
$[\omega >> \omega_b] \Rightarrow \frac{A_o}{\sqrt{\left(\frac{\omega}{\omega_b}\right)}}$

An op amp ultimately is an integrator with time constant $\tau = \frac{1}{\omega_T}$.

ω_T in datasheets

Frequency Response of a Closed Loop Amplifier

Example. Non-Inverting Amplifier



$v_- = v_+ - \frac{v_o}{A(s)}$

$v_- = v_i - \frac{v_o}{A(s)}$

Find an expression for gain as a function of freq.

① Brute Force Determination:

KCL ①: $\frac{v_o - v_-}{R_2} = \frac{v_-}{R_1} + \frac{v_o}{R_2} = v_-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$\frac{v_o}{R_2} = \left(v_i - \frac{v_o}{A(s)}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \Rightarrow \frac{v_o}{v_i}(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A(s)}\left(1 + \frac{R_2}{R_1}\right)}$

$\left[A(s) = \frac{A_o}{1 + \frac{s}{\omega_b}}\right]$

Accounting for finite gain & BW

$\frac{v_o}{v_i}(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \cancel{\frac{1}{A_o}\left(1 + \frac{R_2}{R_1}\right)}} \cdot \frac{1}{1 + \frac{s}{\omega_b} \cdot \frac{\frac{1}{A_o}\left(1 + \frac{R_2}{R_1}\right)}{1 + \cancel{\frac{1}{A_o}\left(1 + \frac{R_2}{R_1}\right)}}}$

$[A_o = \text{large}] \Rightarrow \left(\frac{1}{A_o}\left(1 + \frac{R_2}{R_1}\right) << 1\right)$ → Here, we're essentially saying that the op amp gain is close to ideal & the non-ideality is mostly finite BW

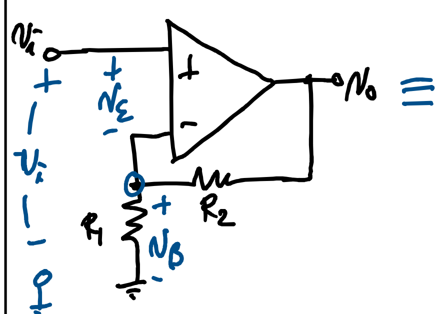
$$\boxed{\frac{v_o}{v_i}(s) = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + \frac{s}{A_o \omega_b \left(\frac{R_1}{R_1 + R_2}\right)}}}$$

Closed-loop T.F.

Near Ideal Gain, but Finite BW

= ✓ ★★

(2) More insightful way to do this:



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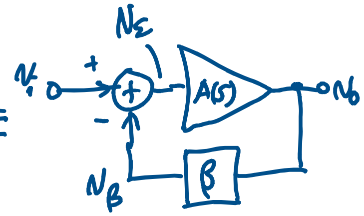
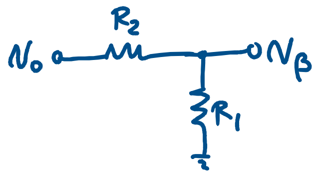


$$\frac{v_o}{v_i}(s) = \frac{A(s)}{1+\beta A(s)}$$

Closed-loop gain

What is β?



$$\boxed{\beta = \frac{v_\beta}{v_o} = \frac{R_1}{R_1+R_2}}$$

Recall f/ previous analysis:

$$\frac{v_o}{v_i}(s) = \frac{A(s)}{1+\beta A(s)}$$

$$\left[A(s) = \frac{A_o}{1+\frac{s}{\omega_b}}\right] \Rightarrow \frac{v_o}{v_i}(s) = \frac{\frac{A_o}{1+\frac{s}{\omega_b}}}{1+\beta\left(\frac{A_o}{1+\frac{s}{\omega_b}}\right)}$$

open-loop gain

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$$\frac{v_o}{v_i}(s) = \underbrace{\frac{A_o}{1+\beta A_o}}_{\text{DC Gain Term (midband gain)}} \; \underbrace{\frac{1}{1+\frac{s}{\omega_b(1+\beta A_o)}}}_{\text{Frequency Shaping Term}}$$

$$\frac{A_o}{1+T_o}, \quad T_o = \beta A_o$$

T_o: $\beta A_o \triangleq$ "loop gain" @ $\omega = 0$ (i.e., @ DC)

If $A_o \to \infty$ or if $\beta A_o \gg 1$ $\Rightarrow$ DC gain $\cong \frac{1}{\beta}$

Plug in β:

$$[\beta A_o \gg 1] \Rightarrow \frac{v_o}{v_i}(s) \cong \frac{1}{\beta}\,\frac{1}{1+\frac{s}{\omega_b \beta A_o}}$$

$$\left[\beta = \frac{R_1}{R_1+R_2}\right] \Rightarrow \boxed{= \left(1+\frac{R_2}{R_1}\right)\frac{1}{1+\frac{s}{\omega_b A_o\left(\frac{R_1}{R_1+R_2}\right)}} = \frac{v_o}{v_i}(s)}$$

Closed-loop T.F.

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Observations:

① Closed loop DC gain $= \dfrac{A_o}{1+\beta A_o} = \dfrac{A_o}{1+T_o} \approx \dfrac{A_o}{T_o}$ $[T_o \gg 1]$

i.e., the closed loop gain is reduced from the open loop gain by $1+T_o$ → show this on graph

② Alternatively, closed loop DC gain $\approx \dfrac{A_o}{\beta A_o} = \dfrac{1}{\beta}$ $[T_o \gg 1]$

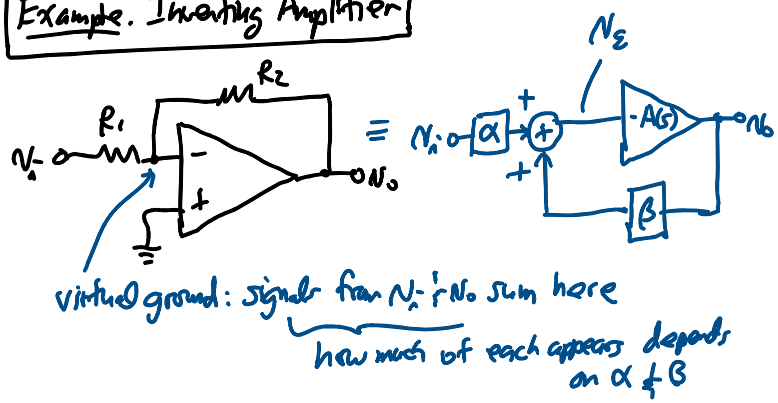
③ ω_{-3dB} has increased from $\omega_b \rightarrow \omega_b(1+A_o\beta) = \omega_b(1+T_o)$

↳ To draw the Bode plot, just find the dc gain, draw a horizontal line across, then follow the open loop response after running into it!

④ Gain-BW Product $= \dfrac{A_o}{1+\beta A_o}\omega_b(1+\beta A_o) = A_o\omega_b = \omega_T$

∴ the Gain-BW product remains the same for the closed & open loop FB cases!

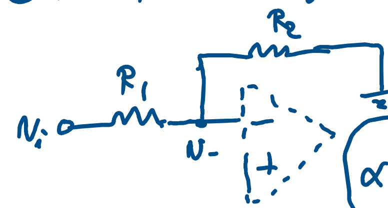
$\boxed{\text{Example. Inverting Amplifier}}$



Determine α:

① Ground output & open loop

② Feed v_i forward to get T.F. to the summing pt.

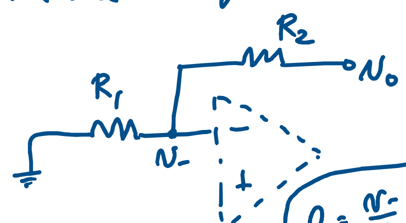


$$\alpha = \frac{v_-}{v_i}(s) = \frac{R_2}{R_1+R_2}$$

Determine β:

① Ground input, open loop

② Feedback v_o to get T.F. to the summing pt.



$$\beta = \frac{v_-}{v_o}(s) = \frac{R_1}{R_1+R_2}$$

Now, get the T.F. for the system block diagram:

$$\left.\begin{aligned} v_o &= -A(s)v_\varepsilon \\ v_\varepsilon &= \alpha v_i + \beta v_o \end{aligned}\right\} \quad \begin{aligned} v_o &= -\alpha A(s)v_i - \beta A(s)v_o \\ v_o(1+\beta A(s)) &= -\alpha A(s)v_i \end{aligned}$$

$$\therefore \frac{v_o}{v_i}(s) = -\frac{\alpha A(s)}{1+\beta A(s)}$$

$$\left[A(s) = \frac{A_o}{1+\frac{s}{\omega_b}}\right] \Rightarrow \frac{v_o}{v_i}(s) = \frac{-\alpha \frac{A_o}{1+\frac{s}{\omega_b}}}{1+\frac{\beta A_o}{1+\frac{s}{\omega_b}}}$$

$$\therefore \boxed{\frac{v_o}{v_i}(s) = \frac{-\alpha A_o}{1+\beta A_o} \; \frac{1}{1+\frac{s}{\omega_b(1+\beta A_o)}}}$$

for an inverting amplifier

Remarks.

① Closed loop DC gain now modified by α.

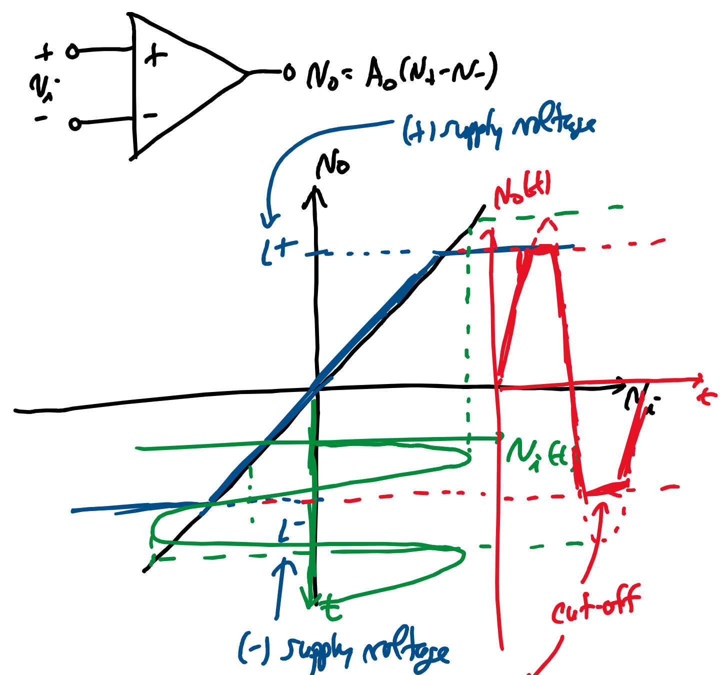
② BW still $\omega_b(1+\beta A_o)$ → same as for non-inverting case

③ ω_T or Gain-BW product now smaller than that of open-loop amplifier:

$$\text{gain-BW} = \alpha \omega_T$$

[remember, $\alpha < 1$]

Output Saturation



output beyond the linear range

⇓

get distortion

input sinusoid → output squared off