Lecture 3: Frequency Response

- Announcements:
- HW#1 online and due this Friday
- Discussions this week
- Labs start next week
  - Monday, Sept. 7 is a holiday, so the Monday lab will start one week later
  - The Tuesday lab starts Sept. 8
- Lab#1 Prelab online soon
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- Lecture Topics:
  - Finish Digital Communications Example
  - Review Impedance
  - Frequency Response
  - Bode Plots
- ---
- Last Time:
- Going through an example digital communication transmitter as motivation
- Now, continue with this …

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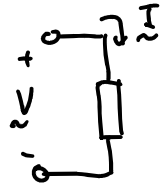
- Review of Analog Circuit Concepts:
- We assume you understand the following concepts from previous courses:
  - Transfer functions
  - Gain (voltage, current, power)
  - Input resistance
  - Output resistance
  - Two-port models for amplifiers
  - Bode plots
  - Ideal op amp ckt design and analysis
- We'll review some of these now to jog your memory

Review of Impedance



where $\underset{\sim}{V}$ and $\underset{\sim}{I}$ are phasor variables

Time		Phasor
$v(t) = V'\cos(\omega t + \phi)$	$\Longleftrightarrow$	$\underset{\sim}{V} = V'e^{j\phi}$
		$= V'(\cos\phi + j\sin\phi)$
$i(t) = I'\cos(\omega t + \phi')$	$\Longleftrightarrow$	$\underset{\sim}{I} = I e^{j\phi'}$

and impedance: $Z = \dfrac{\underset{\sim}{V}}{\underset{\sim}{I}}$

Resistor - $R = \dfrac{v(t)}{i(t)} \underset{\text{phasor}}{\Longleftrightarrow} R = \dfrac{\underset{\sim}{V}}{\underset{\sim}{I}}$

Capacitor - $i(t) = C\dfrac{dv}{dt} \Longleftrightarrow \underset{\sim}{I} = Cj\omega\underset{\sim}{V}$

$\dfrac{\underset{\sim}{V}}{\underset{\sim}{I}} = Z = \dfrac{1}{j\omega C} \longrightarrow Z = \dfrac{1}{sC}$

substitute $s = j\omega$

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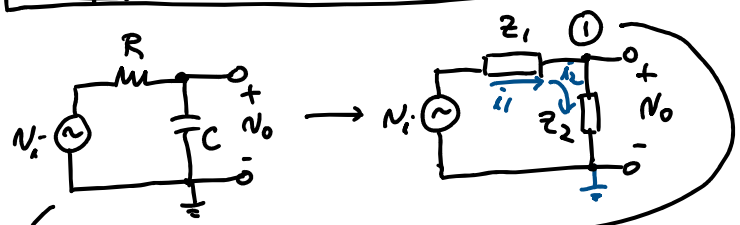
Inductor - $v = L\frac{di}{dt} \Longleftrightarrow \underset{\sim}{V} = Lj\omega \underset{\sim}{I}$

$$\frac{\underset{\sim}{V}}{\underset{\sim}{I}} = j\omega L = sL$$

Example. Determine a transfer function



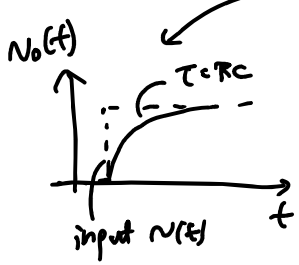
KCL ①: $i_1 = \frac{v_i - v_o}{Z_1}$, $i_2 = \frac{v_o}{Z_2}$

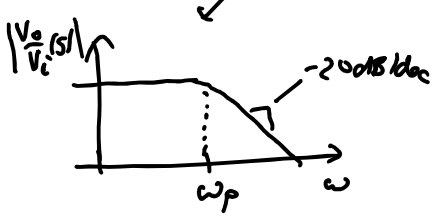
$[i_1 = i_2] \Rightarrow \frac{v_i - v_o}{Z_1} = \frac{v_o}{Z_2} \rightarrow \boxed{\frac{v_o}{v_i}(s) = \frac{Z_2}{Z_1 + Z_2}}$

Recognize Voltage Divider

$$\frac{v_o}{v_i}(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1+sRC} = \frac{1}{1+s\tau_p} = \frac{1}{1+\frac{s}{\omega_p}}$$

$[\tau_p = RC]$ $[\omega_p = \frac{1}{RC}]$





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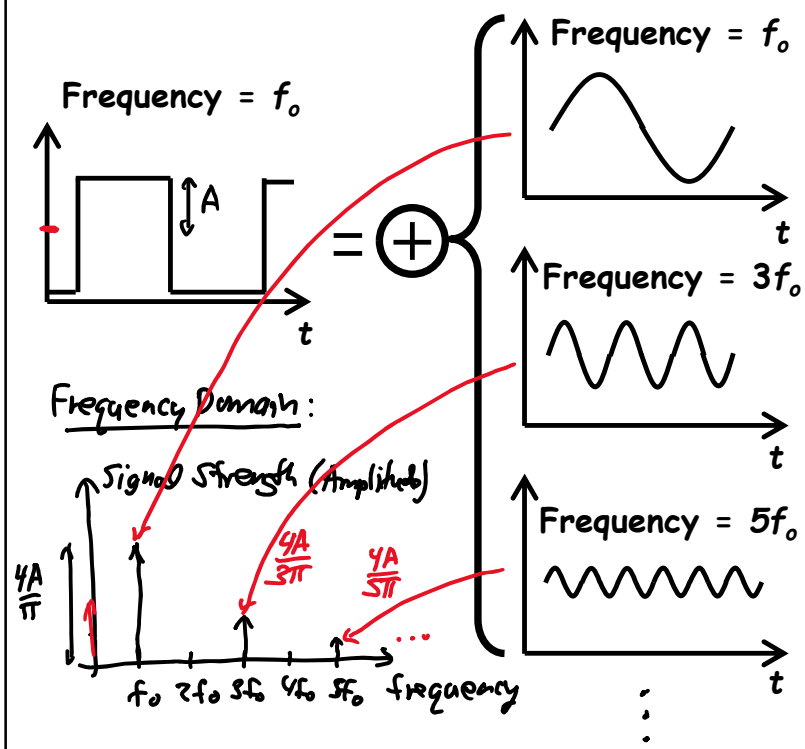
Time and Frequency Domains:

- Most signals of interest to us can be broken down into sums of sinusoids at various amplitudes and frequencies
- Knowledge of the frequencies and amplitudes of these sinusoids conveys as much about the information content as knowledge of time-varying amplitude
- Time Domain description: often conveyed by a plot of amplitude versus time
- Frequency Domain description: often conveyed by a plot of amplitude (& phase) versus frequency



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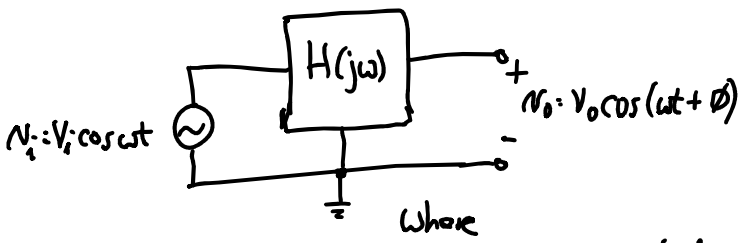
Frequency Response:

- To measure the frequency response of a given network:
  - Excite the network with a variable-frequency, sinusoidal, constant amplitude source
  - Measure the magnitude and phase of the output signal for different values of input frequency



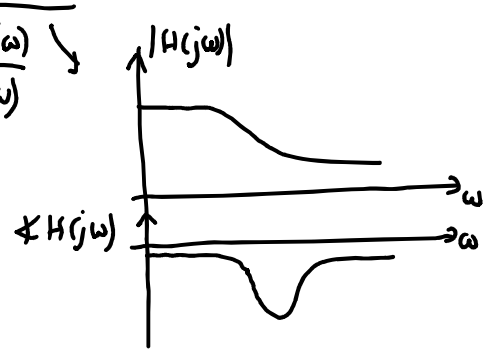
where

$|v_o(j\omega)| = V_o$ = magnitude

$\measuredangle v_o(j\omega) = \phi$ = phase

- The frequency response of a given network is commonly described by a plot of magnitude and phase versus frequency. Usually, such a plot is of the transfer function of a given network:

$$H(j\omega) = \frac{v_o(j\omega)}{v_i(j\omega)}$$



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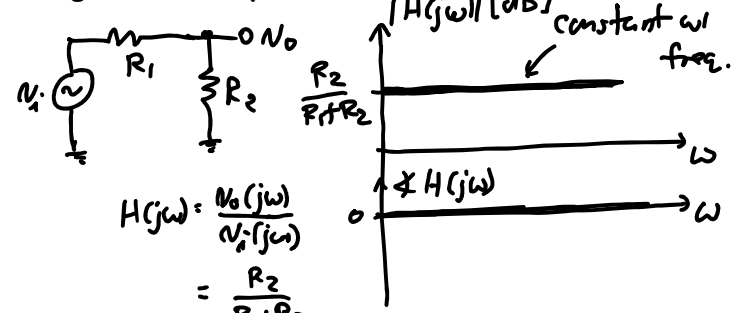
- For a purely resistive network, the frequency response is constant (i.e., a straight line), both magnitude and phase



$$H(j\omega) = \frac{v_o(j\omega)}{v_i(j\omega)} = \frac{R_2}{R_1+R_2}$$

- The addition of reactive (energy storage) components, e.g., capacitors, inductors
  - Shapes the frequency response
  - Adds singularities, i.e., poles and zeros
  - Yields the general transfer function:

$$H(s) = \frac{V_o(s)}{V_i(s)} = H_0 \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} = H_0 \frac{\prod_{j=1}^{m}(s-z_j)}{\prod_{i=1}^{n}(s-p_i)}$$

resistive term (no freq. dependence)

where

$z_j = \sigma_{zj} + j\omega_{zj}$; $\sigma_{zj} = \mathrm{Re}(z_j)$, $\omega_{zj} = \mathrm{Im}(z_j)$

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