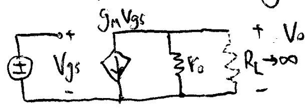
Spring 2001 EE105 Homework #11 Solution

by Xuesong Jiang

11.1 (1) when $R_L \to \infty$, the small signal model of the CE amplifier:



The transistor is in saturation

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{Tn})^2 \cdot (1+\lambda_n V_{DS})$$

$$\therefore g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{Tn})\cdot(1+\lambda_n V_{DS}) = \frac{2 \cdot I_D}{V_{GS}-V_{Tn}}$$

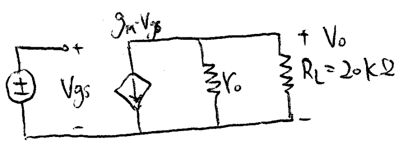
$$1/r_o = \frac{\partial I_D}{\partial V_{DS}} = \lambda_n \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{Tn})^2 = \frac{\lambda_n I_D}{1+\lambda_n V_{DS}}$$

$$A_v = -g_m r_o = -\frac{2 \cdot I_D}{V_{GS}-V_{Tn}} \cdot \frac{1+\lambda_n V_{DS}}{\lambda_n I_D} = -\frac{2(1+\lambda_n V_{DS})}{\lambda_n (V_{GS}-V_{Tn})} = -100$$

$$\therefore V_{GS} = V_{Tn} + \frac{2(1+\lambda_n V_{DS})}{100 \cdot \lambda_n} = 0.7V + \frac{2\cdot(1+0.05V^{-1}\cdot 2.5V)}{100 \cdot 0.05V^{-1}} = 1.15V$$

$$V_{BIAS} = V_{GS} + (-2.5V) = \boxed{-1.35V}$$

when $R_L = 20k\Omega$, follow the similar way when $R_L \to \infty$



$$A_v = -g_m \cdot (r_o \| R_L) = -g_m \cdot \frac{r_o \cdot R_L}{r_o + R_L} = -\underbrace{g_m r_o}_{100} \cdot \frac{R_L}{r_o + R_L} = -20$$

$$\therefore \frac{R_L}{r_o+R_L} = \frac{20}{100} = \frac{1}{5} \Rightarrow r_o = 80k\Omega.$$

$$r_o = \frac{1+\lambda V_{DS}}{\lambda_n \cdot I_D} \Rightarrow I_D = \frac{1+\lambda V_{DS}}{\lambda_n r_o} = \frac{1+0.05V^{-1}\cdot 2.5V}{0.05V^{-1}\cdot 80k\Omega} = \boxed{0.28125\,mA = I_{SUP}}$$ (since $V_o = 0V$)

$$W = \frac{2 \cdot I_{sup} \cdot L}{\mu_n C_{ox} \cdot (V_{GS}-V_{Tn})^2 (1+\lambda V_{DS})} = \frac{2 \cdot 0.28125\,mA \cdot 2\,\mu m}{50\,\mu A/V^2 \cdot (0.45V)^2 (1+0.05V^{-1} \cdot 2.5V)}$$

$$\doteq \boxed{98.765\,\mu m}$$

(2)

	Simulated
V_o	$\sim 100\,mV$
$A_v (R_L \to \infty)$	-100.0004
$A_v (R_L = 20\,k\Omega)$	-19.9999

Based on your assumptions, it is also possible that simulation and hand analysis doesn't match, but the assumptions should be reasonable

The simulation matches the hand analysis.

If in your answers, the simulation and the hand analysis doesn't match, explain what assumptions you have made.

(3) when $V_s \doteq 0.1V$, the output waveform distorts and clips

The peak-to-peak output swing is about 3.9V when the distortion appears.

For 11.1 (2)


```
**** Rl->infinity

.model nmos nmos level=1 vto=0.7 kp=5e-5 lambda=0.05

m1 d g vss vss nmos w=98.765u l=2u
vdd vdd 0 2.5
vss vss 0 -2.5
isup vdd d 0.28125m
vbias g 0 -1.35

.op
.tf v(d) vbias
.end

****     Operating points

+0:d        =  98.0359u 0:g        =  -1.3500   0:vdd      =   2.5000
+0:vss      =  -2.5000

****     small-signal transfer characteristics

     v(d)/vbias                                 = -100.0004
     input resistance at          vbias         =  1.000e+20
     output resistance at v(d)                  =   80.0003k


****     Rl=20kohm

.model nmos nmos level=1 vto=0.7 kp=5e-5 lambda=0.05

m1 d g vss vss nmos w=98.765u l=2u
vdd vdd 0 2.5
vss vss 0 -2.5
isup vdd d 0.28125m
vbias g 0 -1.35
rl d 0 20k

.op
.tf v(d) vbias
.end

****     Operating points

+0:d        =  19.6071u 0:g        =  -1.3500   0:vdd      =   2.5000
+0:vss      =  -2.5000

****     small-signal transfer characteristics

     v(d)/vbias                                 =  -19.9999
     input resistance at          vbias         =  1.000e+20
     output resistance at v(d)                  =   16.0000k
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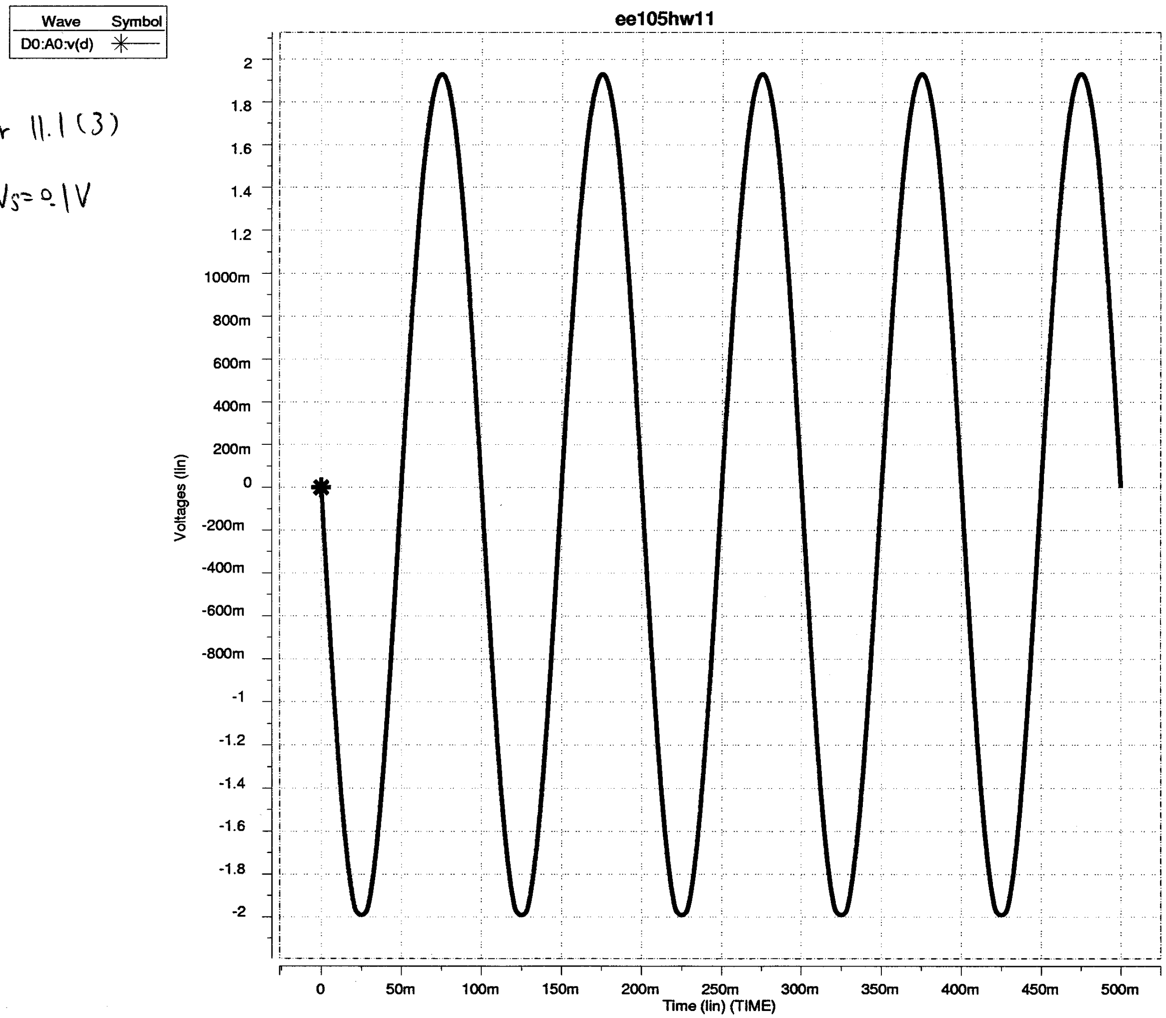
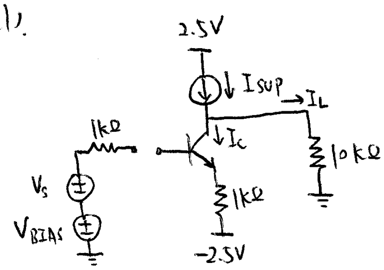
For 11.1 (3)

$V_S = 0.1V$



11.2 (1).



$I_{sup} = I_C + I_L$

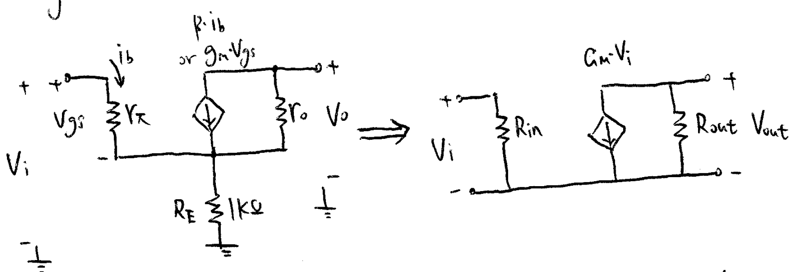
$V_o = 0 \rightarrow I_L = 0$

$\therefore I_C = I_{sup} = 200\mu A \quad , \quad I_B = \frac{I_C}{\beta} = 2\mu A$

$V_{BE} = V_T \ln \frac{I_C}{I_S} = 0.026V \cdot \ln \frac{200\mu A}{10^{-15} A} = 0.676 V$

$\therefore V_{BIAS} = I_B \cdot 1k\Omega + V_{BE} + I_B(\beta+1) \cdot 1k\Omega + (-2.5V) = \boxed{-1.62V}$

(2) small-signal model



$r_\pi = \frac{\beta}{g_m} = 13k\Omega \qquad g_m = \frac{I_C}{V_T} = \frac{200\mu A}{0.026V} = 7.69 mS \qquad r_o = \frac{V_A}{I_C} = 125k\Omega$

$R_{in} = r_\pi(1 + g_m \cdot R_E) = 13k\Omega \cdot (1 + 7.69mS \cdot 1k\Omega) = \boxed{113 k\Omega}$

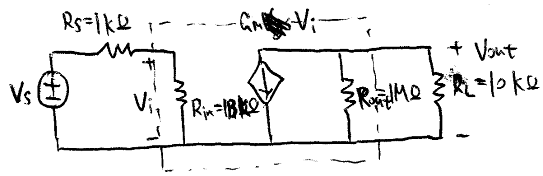
$R_{out} = r_o(1 + g_m \cdot R_E) = 125k\Omega \cdot (1 + 7.69mS \cdot 1k\Omega) = \boxed{1.086 M\Omega}$

(3) $G_m = \frac{g_m}{1 + g_m \cdot R_E} = \frac{7.69mS}{1 + 7.69mS \cdot 1k\Omega} = \boxed{0.885 mS}$ ← I will consider this is right if it is just because of different understanding of overall transconductance

Overall transconductance

$G_m' = \frac{R_{in}}{R_{in} + R_s} G_m \cdot \frac{R_{out}}{R_{out} + R_L} = \boxed{0.869 mS}$

(4).



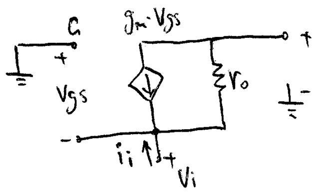
$$A_V = -\frac{R_{in}}{R_s+R_{in}}\cdot G_m \cdot \frac{R_{out}\cdot R_L}{R_{out}+R_L} = -\frac{113k\Omega}{1k\Omega+113k\Omega}\cdot 0.885mS \cdot \frac{1M\Omega\cdot 10k\Omega}{1M\Omega+10k\Omega}$$

$$\fallingdotseq \boxed{-8.7}$$

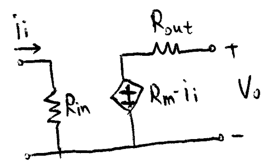
11.3 (1) $V_o = 0V$, $I_o = 0A$

$$I_{BIAS} = -I_{SUP} = \boxed{-100\mu A}$$

(2)



$\Rightarrow$



Here, I neglect λ_n for g_m calculation

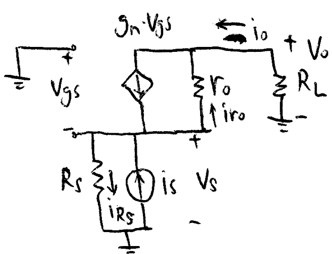
$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}\cdot I_D} = \sqrt{2\cdot 50\mu A/V^2\cdot 4/2\cdot 100\mu A} = 141.4\mu S$$

$$r_o = \frac{1}{\lambda_n I_D} = \frac{1}{0.05V^{-1}\cdot 100\mu A} = 200k\Omega$$

$$R_{in} = \frac{1}{g_m} = \boxed{7.07k\Omega}$$

$$R_{out} = r_o(1+g_m\cdot R_s) = 200k\Omega\cdot(1+141.4\mu S\cdot 10k\Omega) = \boxed{483k\Omega}$$

(3)



$i_o = -(g_m V_{gs} + i_{ro})$

$i_s = -i_o + i_{Rs}$

Since $r_o \gg R_s$ ($200\,k\Omega$ vs. $10\,k\Omega$), neglect r_o

$$\therefore\ i_s = i_{R_s} - g_m V_{gs} = \frac{V_s}{R_s} + g_m \cdot V_s \qquad (V_s = -V_{gs})$$

$$i_o = +g_m V_{gs} = -g_m \cdot V_s = -g_m \cdot \frac{i_s}{g_m + \frac{1}{R_s}} = -\frac{g_m \cdot R_s}{1 + g_m \cdot R_s}\, i_s$$

$$\therefore\ A_i = \frac{i_o}{i_s} = -\frac{g_m \cdot R_s}{1 + g_m \cdot R_s} = -\frac{141.4\,\mu S \cdot 10\,k\Omega}{1 + 141.4\,\mu S \cdot 10\,k\Omega} = \boxed{-0.586}$$

(4). $V_o = -i_o \cdot R_L = -A_i \cdot i_s \cdot R_L$

$$R_m = \frac{V_o}{i_s} = -A_i \cdot R_L = +0.586 \cdot 10\,k\Omega = \boxed{5.86\,k\Omega}$$