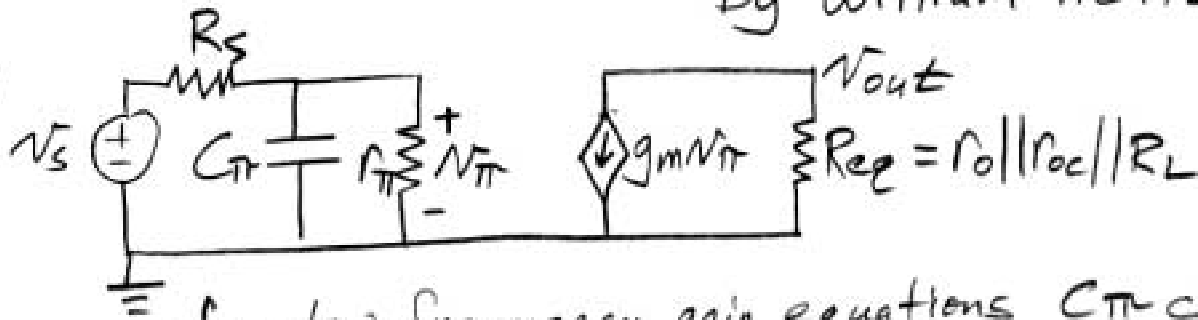
EE130 – Spring 2001 – HW#13 Solution

oops, wrong class - EE105

by William Holtz

1)



for low frequency gain equations C_π can be ignored

$$v_\pi = v_s\left(\frac{r_\pi}{R_s + r_\pi}\right) \; ; \; v_{out} = g_m v_\pi R_{eq}$$

$$\frac{v_{out}}{v_s} = g_m\left(\frac{r_\pi}{R_s + r_\pi}\right)R_{eq} \equiv A_o \; ; \quad r_o = \frac{V_A}{I_c} ; \; g_m = \frac{I_c}{V_{th}} ; \; r_\pi = \frac{B}{g_m}$$

$$A_o = \frac{I_c}{V_{th}}\left(\frac{\frac{B}{g_m}}{R_s + \frac{B}{g_m}}\right)\left(\frac{1}{\frac{2I_c}{V_A} + \frac{1}{R_L}}\right) = \frac{I_c}{V_{th}}\left(\frac{\frac{BV_{th}}{I_c}}{R_s + \frac{BV_{th}}{I_c}}\right)\left(\frac{V_A R_L}{2I_c R_L + V_A}\right)$$

$$A_o = \frac{I_c}{V_{th}}\left(\frac{BV_{th}}{I_c R_s + BV_{th}}\right)\left(\frac{V_A R_L}{2I_c R_L + V_A}\right) = \frac{I_c B V_A R_L}{(I_c R_s + BV_{th})(2I_c R_L + V_A)}$$

$$A_o(2I_c^2 R_s R_L + I_c R_s V_A + BV_{th} 2I_c R_L + BV_{th} V_A) - I_c B V_A R_L = 0$$

$$I_c^2(2A_o R_s R_L) + I_c(A_o R_s V_A + 2A_o B V_{th} R_L - BV_A R_L) + (BV_{th} V_A) = 0$$

$\boxed{I_c = 260.6\,\mu A}$ or $16.6\,mA$ – use min current for min power

$$Z_\pi = \frac{1}{j\omega C_\pi} \Big\| r_\pi = \frac{\frac{r_\pi}{j\omega C_\pi}}{\frac{1}{j\omega C_\pi} + r_\pi} = \frac{r_\pi}{1 + j\omega r_\pi C_\pi}$$

for frequency analysis we only need to consider the first half of the circuit as second half of the circuit just causes a frequency independent gain term.

$$\frac{Z_\pi}{Z_\pi + R_S} = \left(\frac{r_\pi}{1+j\omega r_\pi C_\pi}\right)\left(\frac{1}{\frac{r_\pi}{1+j\omega r_\pi C_\pi} + R_S}\right)$$

$$= \left(\frac{r_\pi}{1+j\omega r_\pi C_\pi}\right)\left(\frac{1+j\omega r_\pi C_\pi}{r_\pi + R_S + j\omega R_S r_\pi C_\pi}\right) = \frac{\left(\frac{r_\pi}{r_\pi + R_S}\right)}{1+j\omega\left(\frac{R_S r_\pi C_\pi}{r_\pi + R_S}\right)}$$

$$\frac{r_\pi + R_S}{R_S r_\pi C_\pi} = 2\pi f_p$$ f_p is frequency of first pole

$$\frac{r_\pi + R_S}{2\pi f_p R_S r_\pi} = C_\pi = C_b + C_{jE} = \tau_F g_m + \sqrt{2}\, C_{jE0}$$

$$= \frac{W_B^2 I_C}{2 D_{nB} V_{th}} + \sqrt{2}\, A \sqrt{\frac{q \varepsilon_s N_a N_{dE}}{2 \phi_B (N_a + N_{dE})}}$$

Solve for area, A

$$\left(\frac{r_\pi + R_S}{2\pi f_p R_S r_\pi} - \frac{W_B^2 I_C}{2 D_{nB} V_{th}}\right)\left(\frac{q \varepsilon_s N_a N_{dE}}{2 \phi_B (N_a + N_{dE})}\right)^{-1/2} = A$$

$D_{nB} = 450 \frac{cm^2}{Vs}\, 0.026V = 11.7 cm^2/s$; $\phi_B = 0.06\left(\log\left(\frac{N_a}{n_i}\right) + \log\left(\frac{N_{dE}}{n_i}\right)\right)$

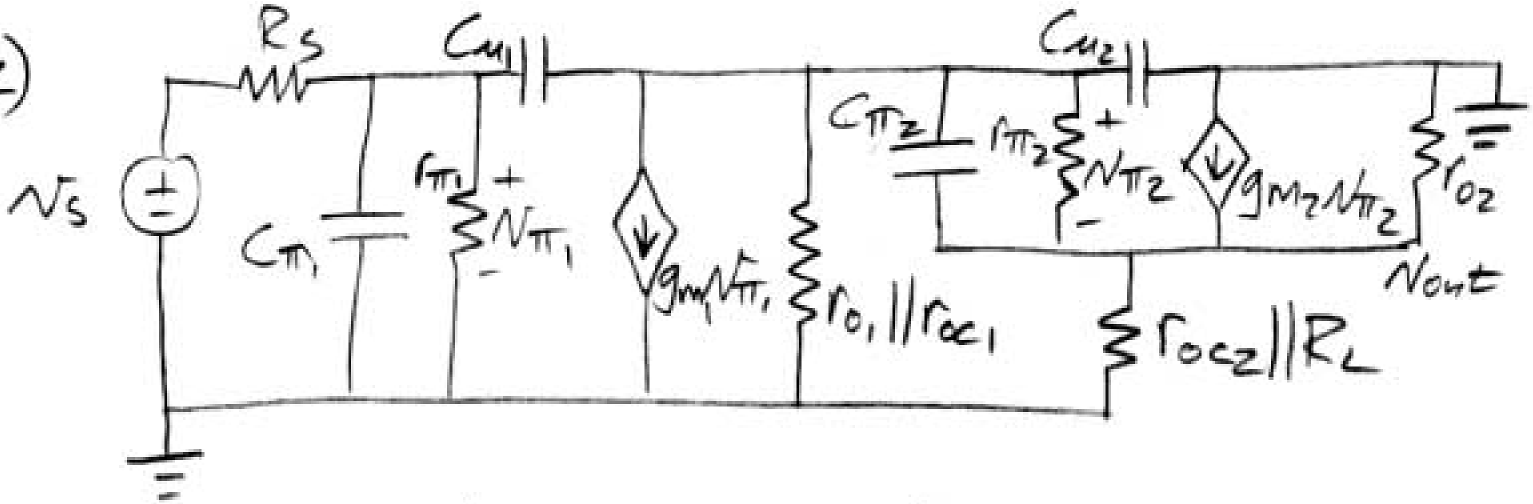
$\phi_B = 1.00V$

$$\boxed{A = 4.78 \times 10^{-6} cm^2}$$

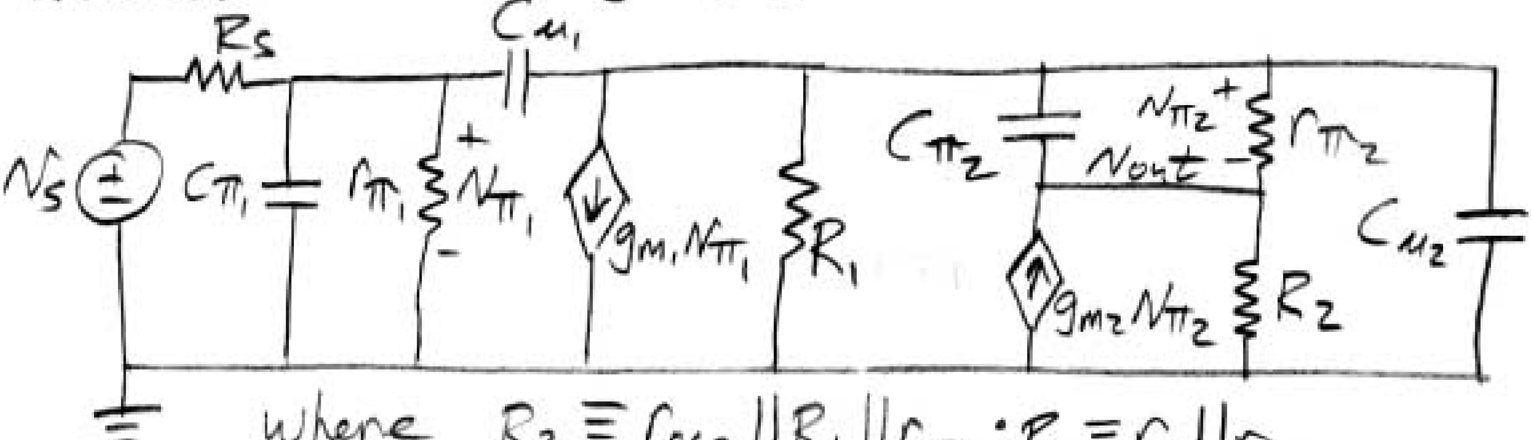
$V_{BE} = V_{th} \ln\left(\frac{I_C}{I_S}\right)$; $V_{BIAS} - \frac{R_S I_C}{B} - V_{BE} + 2.5 = 0$

$$V_{BIAS} = \frac{R_S I_C}{B} + V_{th} \ln\left(\frac{I_C}{I_S}\right) - 2.5 = \boxed{-1.755\,V = V_{BIAS}}$$

2)



Redraw to better group ground



where $R_2 \equiv r_{oc2}||R_L||r_{o2}$; $R_1 \equiv r_{o1}||r_{oc1}$

$I_{E2} = 1mA$; $I_{C2} = I_{E2}\left(\frac{B}{B+1}\right) = 0.99mA$; $I_{B2} = \frac{I_{C2}}{B} = 9.9\mu A$

$I_{C1} = I_{sup1} - I_{B2} = 0.99mA$; $r_o = \frac{V_A}{I_C}$; $g_m = \frac{I_C}{V_{th}}$; $r_\pi = \frac{B}{g_m}$

$$r_{o1} = r_{o2} = \frac{20}{990\times10^{-6}} = 20.2k\Omega$$

$$g_{m1} = g_{m2} = \frac{990\times10^{-6}}{0.026} = 38.1mS$$

$$r_{\pi 1} = r_{\pi 2} = \frac{100}{0.0381} = 2626\Omega$$

$$C_{\pi 1} = C_{\pi 2} = \frac{W_B^2 I_C}{2D_{nB}V_{th}} + A\sqrt{\frac{q\varepsilon_s N_a N_{dE}}{\phi_B(N_a+N_{dE})}}$$

$$= \frac{(10^{-4})^2\, 990\times10^{-6}}{11.7\times2\times0.026} + (10^{-3})^2\sqrt{\frac{1.6\times10^{-19}\times1.035\times10^{-12}\times10^{19}\times5\times10^{17}}{1(10^{19}+5\times10^{17})}}$$

$C_{\pi_1} = C_{\pi_2} = 16.55\,pF$

$$C_{\mu_1} = \frac{C_{\mu_{10}}}{\sqrt{1+V_{CB}/\phi_{BC}}} = A\sqrt{\frac{q\varepsilon_s N_a N_{dc}}{2\phi_{BC}(N_a+N_{dc})}}\left(1+\frac{V_{CB}}{\phi_{BC}}\right)^{-1/2}$$

$$\phi_{BC} = 0.06\left(\log\left(\frac{N_a}{n_i}\right)+\log\left(\frac{N_{dc}}{n_i}\right)\right) = 0.822V$$

$$V_{BE_1} = V_{BE_2} = V_{th}\ln\left(\frac{I_{C1}}{I_S}\right) = 0.778V$$

$$V_{C_1} = V_{OUT} + V_{BE_2} = 0.778$$

$$V_{CB_1} = V_{C_1} - V_{B_1} = V_{C_1} - (-2.5 + V_{BE_1}) = 2.5V$$

$$C_{\mu_{10}} = (10^{-3})^2\sqrt{\frac{1.6\times10^{-19}\times1.035\times10^{-12}\times5\times10^{17}\times10^{16}}{2\times0.822\times(5\times10^{17}+10^{16})}} = 35.3fF$$

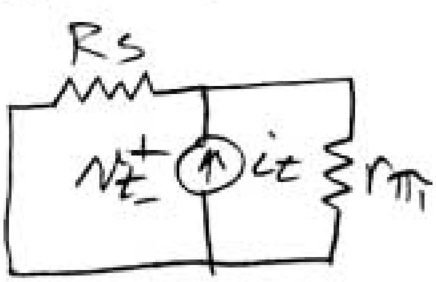
$$C_{\mu_1} = 35.3\times10^{-15}\left(1+\frac{2.5}{0.822}\right)^{-1/2} = 17.6fF$$

$$V_{CB_2} = 2.5 - V_{B_2} = 2.5 - V_{BE_2} = 2.5 - 0.778 = 1.722V$$

$$C_{\mu_2} = 35.3\times10^{-15}\left(1+\frac{1.722}{0.822}\right)^{-1/2} = 20.1fF$$

Use open circuit time constants

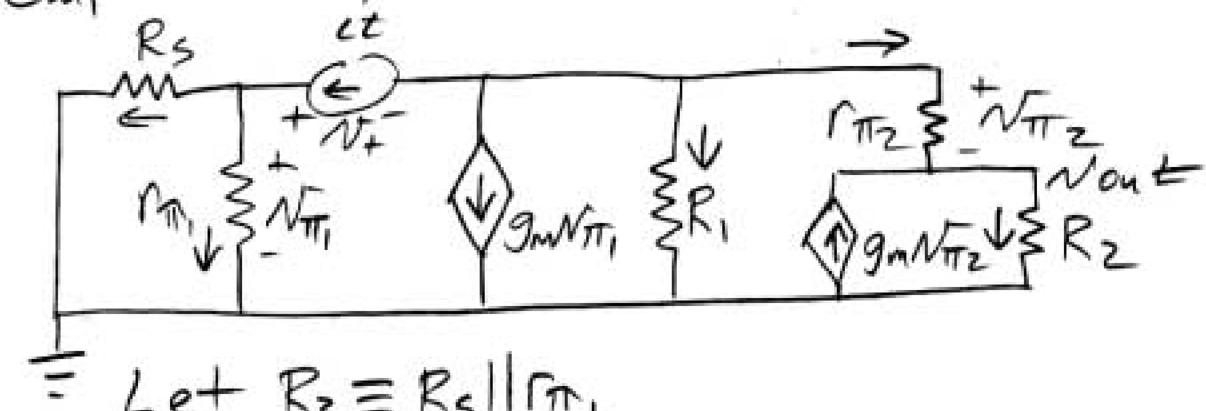
First C_{π_1}



$$R_{C\pi_1} = R_s \| r_{\pi_1} = \frac{R_s r_{\pi_1}}{R_s + r_{\pi_1}} = 488\Omega$$

$$\frac{1}{\tau_{C\pi_1}} = \frac{1}{C_{\pi_1}R_{C\pi_1}} = 123.7\,Mrad/s$$

C_{μ_1}



Let $R_3 \equiv R_S \| r_{\pi_1}$

$$i_t = \frac{v_{\pi_1}}{R_3} \; ; \; g_m v_{\pi_2} + \frac{v_{\pi_2}}{r_{\pi_2}} = \frac{v_{out}}{R_2} \; ; v_{out} + v_{\pi_2} + v_t = v_{\pi_1}$$

$$i_t + g_m v_{\pi_1} + \frac{v_{\pi_2} + v_{out}}{R_1} + \frac{v_{\pi_2}}{r_{\pi_2}} = 0$$

substitute for vout

$$i_t + g_m v_{\pi_1} + \frac{v_{\pi_2} + v_{\pi_1} - v_{\pi_2} - v_t}{R_1} + \frac{v_{\pi_2}}{r_{\pi_2}} = 0$$

solve for v_{π_2}

$$v_{\pi_2} = -r_{\pi_2}\left(i_t + g_m v_{\pi_1} + \frac{v_{\pi_1} - v_t}{R_1}\right)$$

substitute for vout in remaining equation

$$g_m v_{\pi_2} + \frac{v_{\pi_2}}{r_{\pi_2}} = \frac{v_{\pi_1} - v_{\pi_2} - v_t}{R_2}$$

factor out v_{π_2}

$$v_{\pi_2}\left(g_m + \frac{1}{r_{\pi_2}} + \frac{1}{R_2}\right) = \frac{v_{\pi_1} - v_t}{R_2}$$

substitute for v_{π_2}

$$-r_{\pi_2}\left(i_t + g_m v_{\pi_1} + \frac{v_{\pi_1} - v_t}{R_1}\right)\left(g_m + \frac{1}{r_{\pi_2}} + \frac{1}{R_2}\right) = \frac{v_{\pi_1} - v_t}{R_2}$$

substitute for v_{π_1}

$$-r_{\pi_2}\left(i_t+g_m i_t R_3+\frac{i_t R_3-v_t}{R_1}\right)\left(g_m+\frac{1}{r_{\pi_2}}+\frac{1}{R_2}\right)=\frac{i_t R_3-v_t}{R_2}$$

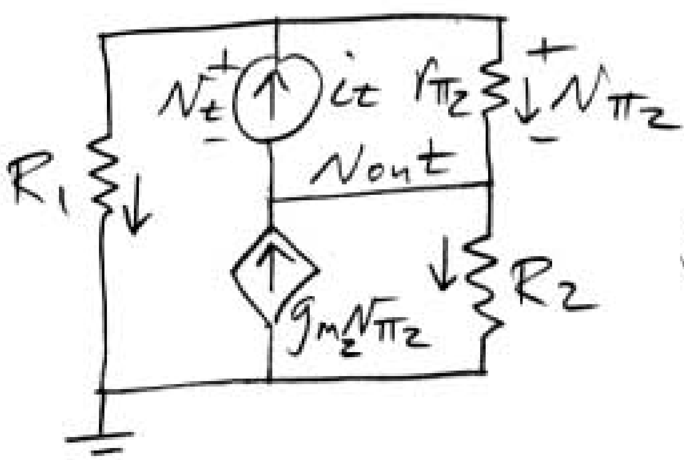
Let $X \equiv -r_{\pi_2} g_m - 1 - \frac{r_{\pi_2}}{R_2}$

$$i_t\left(1+g_m R_3+\frac{R_3}{R_1}-\frac{R_3}{R_2 X}\right)=v_t\left(\frac{1}{R_1}-\frac{1}{R_2 X}\right)$$

$$R_{C_{\mu_1}}=\frac{1+g_m R_3+\frac{R_3}{R_1}-\frac{R_3}{R_2 X}}{\frac{1}{R_1}-\frac{1}{R_2 X}}$$; $R_1=10.1K$; $R_2=5.0K$; $R_3=488$; $X=-102$

$R_{C_{\mu_1}}=195 k\Omega$; $\frac{1}{\tau_{C_{\mu_1}}}=\frac{1}{R_{C_{\mu_1}}C_{\mu_1}}=292$ Mrad/s

C_{π_2}



$$i_t=\frac{v_{out}+v_{\pi_2}}{R_1}+\frac{v_{\pi_2}}{r_{\pi_2}}$$

$$g_{m_2}v_{\pi_2}+\frac{v_{\pi_2}}{r_{\pi_2}}=i_t+\frac{v_{out}}{R_2}$$

$$v_t=v_{\pi_2}$$

solve for v_{out}

$$R_2\left(g_{m_2}v_t+\frac{v_t}{r_{\pi_2}}-i_t\right)=v_{out}$$

substitute to eliminate v_{out}

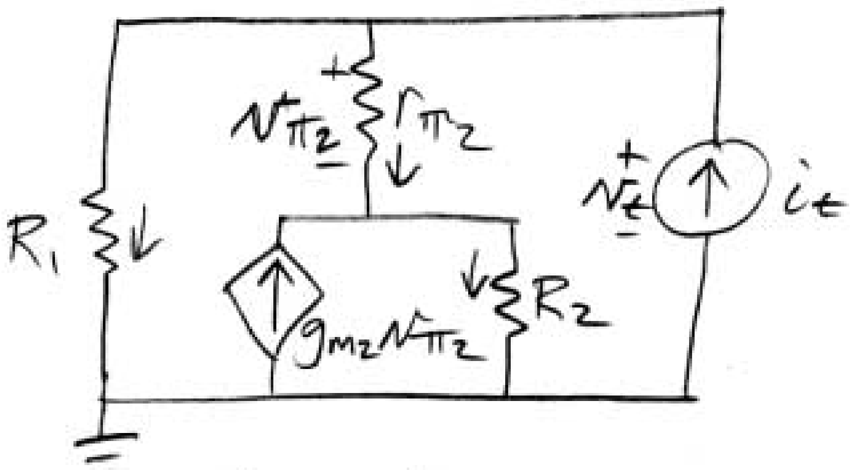
$$i_t=\frac{R_2\left(g_{m_2}v_t+\frac{v_t}{r_{\pi_2}}-i_t\right)+v_t}{R_1}+\frac{v_t}{r_{\pi_2}}$$

$$i_t\left(1+\frac{R_2}{R_1}\right)=v_t\left(\frac{R_2 g_{m_2}}{R_1}+\frac{R_2}{r_{\pi_2}R_1}+\frac{1}{R_1}+\frac{1}{r_{\pi_2}}\right)$$

$$R_{C\pi_2} = \frac{1 + \frac{R_2}{R_1}}{\left(\frac{R_2 g_{m2}}{R_1} + \frac{R_2}{r_{\pi 2} R_1} + \frac{1}{R_1} + \frac{1}{r_{\pi 2}}\right)} = 76\,\Omega$$

$$\frac{1}{\tau_{C\pi_2}} = \frac{1}{R_{C\pi_2} C_{\pi_2}} = 4.6 \times 10^{12}\ \text{rad/s}$$

C_{μ_2}



$$i_t = \frac{v_t}{R_1} + \frac{v_{\pi 2}}{r_{\pi 2}}\ ; \quad g_{m2} v_{\pi 2} + \frac{v_{\pi 2}}{r_{\pi 2}} = \frac{v_t - v_{\pi 2}}{R_2}$$

$$r_{\pi 2}\left(i_t - \frac{v_t}{R_1}\right) = v_{\pi 2}\ ; \quad v_{\pi 2}\left(g_{m2} + \frac{1}{r_{\pi 2}} + \frac{1}{R_2}\right) = \frac{v_t}{R_2}$$

$$r_{\pi 2}\left(i_t - \frac{v_t}{R_1}\right)\left(g_{m2} + \frac{1}{r_{\pi 2}} + \frac{1}{R_2}\right) = \frac{v_t}{R_2}$$

$$i_t = v_t\left[\frac{1}{\left(g_{m2} r_{\pi 2} + 1 + \frac{r_{\pi 2}}{R_2}\right) R_2} + \frac{1}{R_1}\right]$$

$$R_{C\mu_2} = \left[\frac{1}{\left(g_{m2} r_{\pi 2} + 1 + \frac{r_{\pi 2}}{R_2}\right) R_2} + \frac{1}{R_1}\right]^{-1}$$

$R_{C\mu_2} = 9.9K\Omega$; $\frac{1}{\tau_{C\mu_2}} = \frac{1}{R_{C\mu_2} C_{\mu_2}} = 5.02\ Grad/sec$

Smallest time constant is $\tau_{C\pi_1}$

$f_p = \frac{1}{2\pi \tau_{C\mu_2}} = \boxed{19.7MHz \text{ smallest pole}}$

should be C_pi_1 not C_mu_2, but correct value was used...

$\tau_{-3dB} = \tau_{C\pi_1} + \tau_{C\pi_2} + \tau_{C\mu_1} + \tau_{C\mu_2} = 1.17 \times 10^{-8} sec$

$F_{-3dB} = \frac{1}{2\pi \tau_{-3dB}} = \boxed{13.6MHz \text{ -3dB Freq.}}$

3) first find low freq gain across 100pF cap.

Voltage across dependent source is

$-0.001 v_{in} (20K + 8K)$

Voltage on other side of cap is just v_{in}, so

gain is $-0.001 \times 28 \times 10^3 = -28$

$C_M = 100pF(1 - -28) = 2.9nF$

Let $R_1 = 600$
$R_2 = 3K$
$C = 2.905pF$

$$R_2 \| C = \frac{\frac{R_2}{jwC}}{R_2 + \frac{1}{jwC}} = \frac{R_2}{1 + jwR_2C}$$

$$v_{in} = v_s \left(\frac{\left(\frac{R_2}{1+jwR_2C}\right)}{R_1 + \frac{R_2}{1+jwR_2C}} \right) = \frac{v_s R_2}{R_1 + R_2 + jwR_1R_2C}$$

$$\frac{v_{in}}{v_s} = \frac{\left(\frac{R_2}{R_1+R_2}\right)}{1+jw\left(\frac{R_1R_2C}{R_1+R_2}\right)}$$

pole at Freq $\dfrac{1}{2\pi\left(\frac{R_1R_2C}{R_1+R_2}\right)}$

$$\boxed{F_p = 110\,KHz}$$ (a)

for part b there is no voltage drop across the 100pF cap (v_{in} on both sides), so gain across cap is 1

$C_M = 100pF(1-1) = 0$

can just remove the 100pF cap from circuit

let $R_1 = 50k$

$R_2 = 400k$

$C = 5\times10^{-12}$

using above formula

$$\boxed{F_p = 716\,KHz}$$ (b)

9/9