Spring 2001 EE105 Homework #14 Solution by: Xuesong Jiang

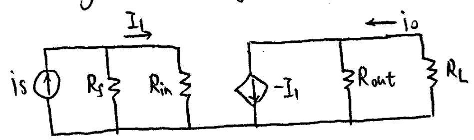
14.1. $I_D = |I_{BIAS}| = I_{SUP} = 200\mu A$

Neglect λ_n when calculating g_m

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \sqrt{2\cdot 50\mu A/V^2 \cdot \frac{100}{2}\cdot 200\mu A} = 1mS$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.05V^{-1}\cdot 200\mu A} = 100k\Omega$$

Small signal model for low frequency current gain

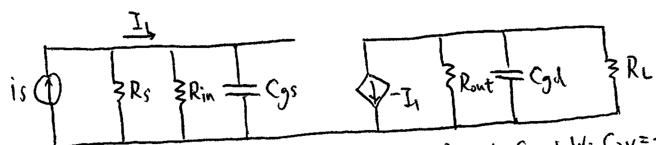


$$R_{in} = \frac{1}{g_m} = 1k\Omega$$

$$R_{out} = r_o + g_m\cdot R_S\cdot r_o = 100k\Omega + 1mS\cdot 10k\Omega\cdot 100k\Omega = 1.1M\Omega$$

$$\boxed{A_i = \frac{i_o}{i_s} = -\frac{R_S}{R_S+R_{in}}\cdot\frac{R_{out}}{R_{out}+R_L}}$$

Small signal model for ω_{3dB}



$$C_{gs} = \frac{2}{3}W\cdot L\cdot C_{ox} + W\cdot C_{ov} = \frac{2}{3}\cdot 100\mu m\cdot 2\mu m\cdot 2.3fF/\mu m^2 + 100\mu m\cdot 0.5fF/\mu m$$
$$= 307fF + 50fF = 357fF$$

$$T_{cgs} = (R_S \| R_{in})\cdot C_{gs}$$

$$C_{gd} = W\cdot C_{ov} = 50fF$$

$$T_{cgd} = (R_{out}\|R_L)C_{gd}$$

$$\boxed{\omega_{3dB} = \frac{1}{(R_S\|R_{in})\cdot C_{gs} + (R_{out}\|R_L)\cdot C_{gd}}}$$

* If you consider source and drain junction capacitance, that will be a more complete solution to this problem and that is more close to the real case, I will encourage you to do it, but junction capacitances are not considered in the textbook, so I neglect junction capacitances in the solution.

(1) $R_S = 100\Omega$ and $R_L = 10k\Omega$.

$$A_i = -\frac{100\Omega}{100\Omega + 1k\Omega} \cdot \frac{1.1M\Omega}{10k\Omega + 1.1M\Omega} = \boxed{-0.09}$$

$$\omega_{3dB} = \frac{1}{(100\Omega \| 1k\Omega) \cdot 357fF + (1.1M\Omega \| 10k\Omega) \cdot 50fF} = \frac{1}{0.032nS + 0.495nS}$$

$$= \boxed{1.9\ G\text{rad/s}}$$

10^9

(2) $R_S = 1k\Omega$ and $R_L = 100k\Omega$

$$A_i = -\frac{1k\Omega}{1k\Omega + 1k\Omega} \cdot \frac{1.1M\Omega}{100k\Omega + 1.1M\Omega} = \boxed{-0.458}$$

$$\omega_{3dB} = \frac{1}{(1k\Omega \| 1k\Omega) \cdot 357fF + (1.1M\Omega \| 100k\Omega) \cdot 50fF} = \frac{1}{0.1785nS + 4.58nS}$$

$$= \boxed{210\ M\text{rad/s}}$$

(3) $R_S = 500\Omega$ and $R_L = 5k\Omega$

$$A_i = -\frac{500\Omega}{500\Omega + 1k\Omega} \cdot \frac{1.1M\Omega}{5k\Omega + 1.1M\Omega} = \boxed{-0.33}$$

$$\omega_{3dB} = \frac{1}{(500\Omega \| 1k\Omega) \cdot 357fF + (1.1M\Omega \| 5k\Omega) \cdot 50fF} = \frac{1}{0.119nS + 0.249nS}$$

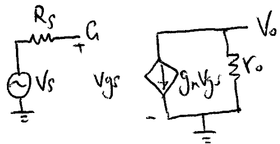
$$= \boxed{2.72\ G\text{rad/s}}$$

14.2 (1) $I_D = I_{sup} = 100\mu A$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}\cdot I_D} = \sqrt{2\cdot 50\mu A/V^2\cdot\frac{40}{2}\cdot 100\mu A} = 0.447\,mS$$

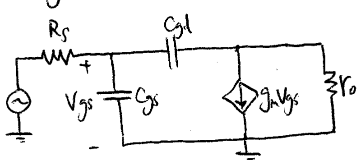
$$r_o = \frac{1}{\lambda\cdot I_D} = \frac{1}{0.05V^{-1}\cdot 100\mu A} = 200k\Omega$$

Small signal model

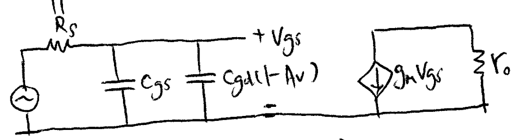


$$A_v = \frac{V_o}{V_s} = \frac{-g_m\cdot r_o\cdot V_{gs}}{V_s} = -\frac{g_m\cdot r_o\cdot V_s}{V_s} = -g_m\cdot r_o = \boxed{-89.4}$$

(2) Small signal model



Miller approximation



$$C_{gs} = \frac{2}{3}\cdot W\cdot L\cdot C_{ox} + W\cdot C_{ov} = \frac{2}{3}\cdot 40\mu m\cdot 2\mu m\cdot 2.3fF/\mu m^2 + 40\mu m\cdot 0.5fF/\mu m = 122.7fF + 20fF$$
$$= 142.7fF$$

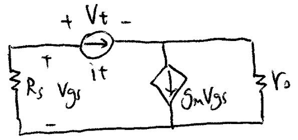
$$C_{gd} = W\cdot C_{ov} = 20fF$$

$$\omega_{3dB} = \frac{1}{R_S\cdot[C_{gs} + C_{gd}\cdot(1-A_v)]} = \frac{1}{10k\Omega\cdot[142.7fF + 20fF\cdot(1+89.4)]}$$

$$= \boxed{51.3\,Mrad/s}$$

(3) Open-circuit time-constant method

$T_1 = T_{C_{gs}} = R_s \cdot C_{gs} = 10k\Omega \cdot 142.7fF = 1.427nS$



$V_t = -i_t \cdot R_s + (g_m V_{gs} - i_t) \cdot r_o$

$= -i_t \cdot R_s - g_m \cdot r_o \cdot R_s \cdot i_t - i_t \cdot r_o$

$= -i_t \cdot (R_s + r_o + g_m \cdot r_o \cdot R_s)$

$\therefore R_{C_{gd}} = R_s + r_o + g_m \cdot r_o \cdot R_s$

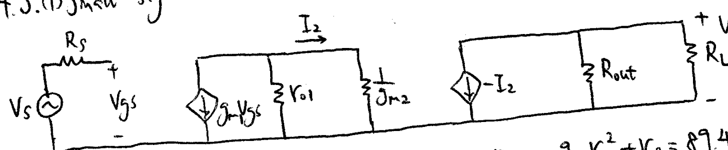
$= 10k\Omega + 200k\Omega + 89.4 \cdot 10k\Omega = 1.104M\Omega$

$T_2 = T_{C_{gd}} = R_{C_{gd}} \cdot C_{gd} = 1.104M\Omega \cdot 20fF = 22.08nS$

$\sum_{i=1}^{2} T_i = 1.427nS + 22.08nS = 23.507nS$

$\omega_{3dB} = \dfrac{1}{\sum_{i=1}^{2} T_i} = \boxed{42.5 Mrad/s}$

14.3.(1) Small signal model



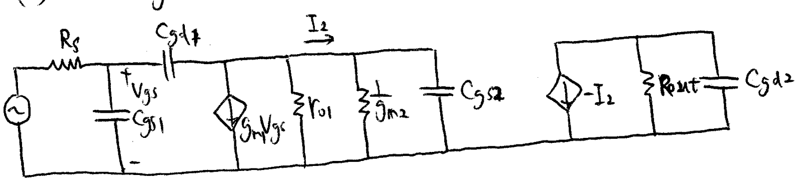
$R_{out} = g_m r_{o2} \cdot r_{o1} + r_{o2}$, $V_s = V_{gs}$

$R_{out} = g_m r_o^2 + r_o = 89.4 \cdot 200k\Omega + 200k\Omega$
$= 18.08M\Omega$

$A_v = \dfrac{V_o}{V_s} = -g_m \cdot \dfrac{r_{o1}}{r_{o1} + \frac{1}{g_{m2}}} (R_{out} \| R_L)$

$= -0.447mS \cdot \dfrac{200k\Omega}{200k\Omega + \frac{1}{0.447mS}} \cdot 18.08M\Omega = \boxed{-7992}$

(2) Small signal model



$$T_{cgs1} = R_S \cdot C_{gs1} = 10k\Omega \cdot 142.7fF = 1.427ns$$

$$R_{cgd1} = R_S + (r_{o1} \| \frac{1}{g_{m2}}) + g_m \cdot R_S \cdot (r_{o1} \| \frac{1}{g_{m2}}) = 10k\Omega + 2.212k\Omega + 9.89k\Omega = 22.1k\Omega$$

$$T_{cgd1} = R_{cgd1} \cdot C_{gd} = 22.1k\Omega \cdot 20fF = 0.442ns$$

$$T_{cgs2} = (r_{o1} \| \frac{1}{g_{m2}}) \cdot C_{gs2} = 2.212k\Omega \cdot 142.7fF = 0.316ns$$

$$T_{cgd2} = R_{out} \cdot C_{gd2} = 18.08M\Omega \cdot 20fF = 361.6ns$$

$$\therefore \omega_{3dB} = \frac{1}{\Sigma T} = \frac{1}{1.427ns + 0.442ns + 0.316ns + 361.6ns} = \frac{1}{363.8ns}$$

$$= \boxed{2.75\ Mrad/s}$$