EE105 - Spring 2001 - Homework 3 solutions
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3.1) $J^{diff} = J_n^{diff} = q D_n \frac{dn}{dx}$

$$\frac{D_n}{\mu_n} = \frac{kT}{q} \Rightarrow D_n = \frac{\mu_n kT}{q} \Rightarrow J^{diff} = q\left(\frac{\mu_n kT}{q}\right)\frac{dn}{dx}$$

$$J^{diff} = q \mu_n V_{th} \left(\frac{N_d(x=\ell) - N_d(x=0)}{\ell - 0}\right)$$

$$J^{diff} = 1.6\times10^{-19}C \times 1400\frac{cm^2}{Vs} \times 0.026V \left(\frac{10^{15}/cm^3 - 10^{10}/cm^3}{10^{-4}cm}\right)$$

$$\boxed{J^{diff} = 58.24\frac{A}{cm^2}}$$ (a)

$$J^{dr}(x) = J_n^{dr}(x) = q\,n(x)\mu_n E$$

$$J_n^{dr}(x=0.5\mu m) = q\,n(x=0.5\mu m)\mu_n E = -J^{diff}$$

$$E = \frac{-J^{diff}}{q\,n(x=0.5\mu m)\mu_n} = \frac{-58.24\frac{A}{cm^2}}{1.6\times10^{-19}C \times 5\times10^{12}/cm^3 \times 1400\frac{cm^2}{Vs}}$$

$$\boxed{E = 52kV/cm}$$ (b)

linearly doped implies n(0.5um)=[n(1um)-n(0)]/2 ~ 5e14
n(0.5um) != 5e12
Therefore the answer to part b should be -520V/cm

$$J(x) = J^{diff} + J^{dr}(x) = J^{diff} + q\,n(x)\mu_n E$$

$$J(0) = 58.24\frac{A}{cm^2} + 1.6\times10^{-19}C\times10^{10}/cm^3 \times 1400\frac{cm^2}{Vs} \times 52\times10^3\frac{V}{cm}$$

$$\boxed{J(0) = 58.36\frac{A}{cm^2}}$$ (c)

carrying down the value from part b...
J(0)=58.239A/cm^2

$$J(x=1\mu m) = 58.24\frac{A}{cm^2} + 1.6\times10^{-19}C \times 10^{15}/cm^3 \times 1400\frac{cm^2}{Vs} \times 52\times10^3\frac{V}{cm}$$

$$\boxed{J(x=1\mu m) = 11.71\times10^3 A/cm^2}$$ (d)

carrying down the value from part b...
J(1um)= -58.24A/cm^2

3.2) $E_{max} = \frac{-q N_d x_n}{\varepsilon_s} = \frac{-q N_a x_p}{\varepsilon_s}$

$$x_n = \frac{-E_{max}\varepsilon_s}{q N_d} \; ; \; x_p = \frac{-E_{max}\varepsilon_s}{q N_a}$$

$$X_d = x_n + x_p = \frac{-E_{max}\varepsilon_s}{q}\left(\frac{1}{N_d} + \frac{1}{N_a}\right)$$

$$X_d = \frac{5\times10^{5}\,V/cm \times 1.035\times10^{-12}\,\frac{F}{cm}}{1.6\times10^{-19}\,C}\left(\frac{1}{5\times10^{19}/cm^3} + \frac{1}{10^{16}/cm^3}\right)$$

$$\boxed{X_d = 3.24\,\mu m} \quad (a)$$

$$X_d(V_D) = \sqrt{\left(\frac{2\varepsilon_s(\phi_B - V_D)}{q}\right)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

$$\phi_B = \phi_n - \phi_p = 540\,mV - (-360\,mV) = 900\,mV$$

$$\frac{-X_d^2 q}{2\varepsilon_s\left(\frac{1}{N_a} + \frac{1}{N_d}\right)} + \phi_B = V_D = \frac{-(3.24\times10^{-4}\,cm)^2\,1.6\times10^{-19}\,C}{2\times1.035\times10^{-12}\,\frac{F}{cm}\left(\frac{1\,cm^3}{5\times10^{19}} + \frac{1\,cm^3}{10^{16}}\right)} + 900\,mV$$

$$\boxed{V_D = -80.22\,V} \quad (b)$$

$$X_d(V_D) = X_{d0}\sqrt{1 - V_D/\phi_B} \Rightarrow \frac{X_d(V_D)}{X_{d0}} = \sqrt{1 - \frac{V_D}{\phi_B}}$$

$$\frac{X_d(V_D)}{X_{d0}} = \sqrt{1 - \frac{-80.22\,V}{0.9\,V}} = \boxed{9.49 \text{ times larger}} \quad (c)$$

3.3) $V_{Tn} = V_{FB} - 2\phi_p + \left(\frac{t_{ox}}{\varepsilon_{ox}}\right)\sqrt{2q\varepsilon_s N_a(-2\phi_p)}$

$V_{FB} = -(\phi_n - \phi_p) = -(540mV - -360mV) = -0.9V$

$2\phi_p = (2)(-360mV) = -0.72V$

$$V_{Tn} = -0.9V + 0.72V + \left(\frac{50\times10^{-7}cm}{3.45\times10^{-13}F/cm}\right)\sqrt{2\times1.6\times10^{-19}C\times1.035\times10^{-12}F/cm}$$
$$\times\sqrt{10^{16}/cm^3\,(0.72V)}$$

$V_{Tn} = 0.528V$

$V_{GB} > V_{Tn} \Rightarrow$ inversion

$$X_{d,max} = \sqrt{2\frac{\varepsilon_s}{qN_a}(-2\phi_p)}$$

$$X_{d,max} = \sqrt{\frac{2\times1.035\times10^{-12}F/cm\times0.72V}{1.6\times10^{-19}C\times10^{16}/cm^3}} = 0.305\mu m$$

$$Q_N = -\left(\frac{\varepsilon_{ox}}{t_{ox}}\right)(V_{GB} - V_{Tn}) = -\left(\frac{3.45\times10^{-13}F/cm}{50\times10^{-7}cm}\right)(1-0.528) = -32.57nC$$

$Q_G = -(Q_N - qN_aX_{d,max})$

$Q_G = -(-32.57\times10^{-9}C - 1.6\times10^{-19}C\times10^{16}/cm^3\times0.305\times10^{-4}cm)$

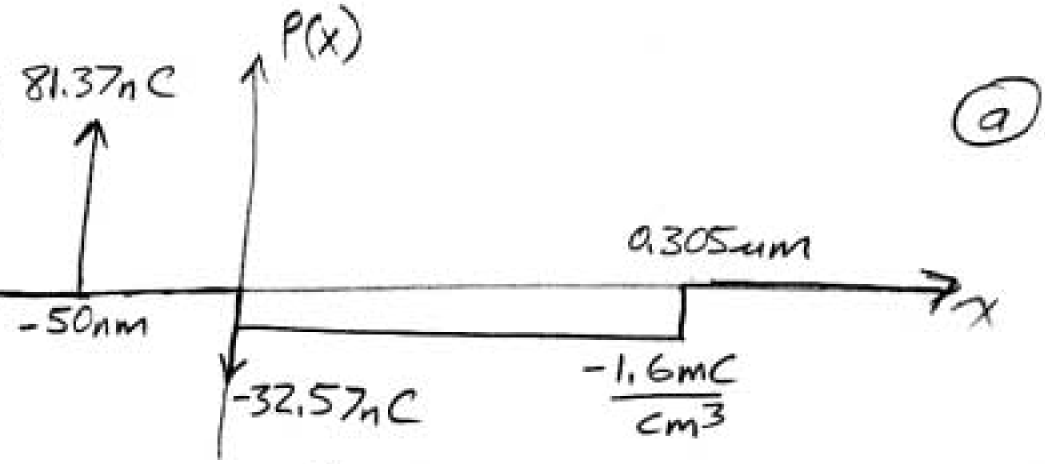
$Q_G = 81.37nC$

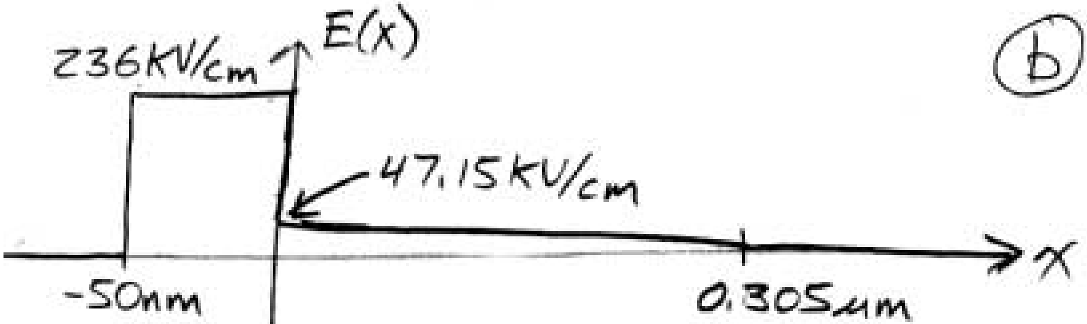
$$E_{ox} = \frac{Q_G}{\varepsilon_{ox}} = \frac{81.37\times10^{-9}C}{3.45\times10^{-13}F/cm} = 236kV/cm$$

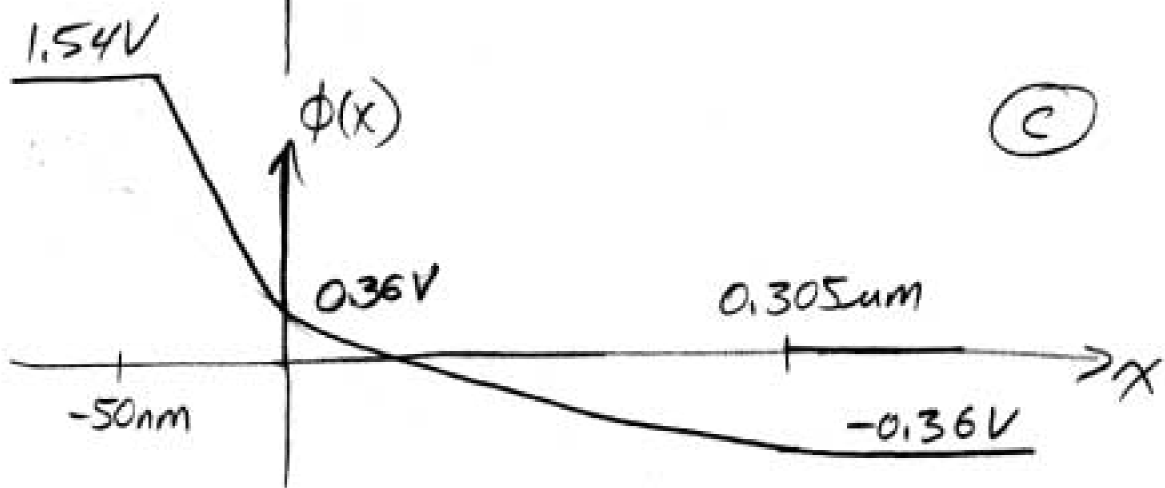
$$E(0^+) = \frac{(Q_G + Q_N)}{\varepsilon_s} = \frac{(81.37nC - 32.57nC)}{1.035\times10^{-12}F/cm} = 47.15kV/cm$$

$V_{ox} = E_{ox}t_{ox} = 236\times10^3V/cm\times50\times10^{-7}cm = 1.18V$

3.3) cont







Change t_{ox} by 2x

$$V_{Tn} = -0.9V + 0.72V + \left(\frac{100\times10^{-7}\,cm}{3.45\times10^{-13}\,F/cm}\right)\sqrt{2\times1.6\times10^{-19}\,C\times1.035\times10^{-12}\,F/cm\times10^{16}\times0.72}$$

$$V_{Tn} = 1.24\,V$$

Change N_a by 10x

$$V_{Tn} = -0.96 + 0.84 + \left(\frac{50\times10^{-7}\,cm}{3.45\times10^{-13}\,F/cm}\right)\sqrt{2\times1.6\times10^{-19}\,C\times1.035\times10^{-12}\times10^{17}\times0.84}$$

$$V_{Tn} = 2.30\,V$$

$\boxed{\text{doping by 10x}}$ d

3.4) polysilicon/oxide/substrate form a MOS cap, which will be in inversion (V_T can be found to be 3.85V). Capacitance in inversion is just $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$

(a)

$$C_{poly\,unit} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45\times10^{-13}\,F/cm}{10^{-4}\,cm} = \boxed{3.45\,nF/cm^2}$$

$$C_{poly\,total} = 3.45\times10^{-9}\,\frac{F}{cm^2}\times 33\,\mu m^2 = \boxed{1.14\,fF}$$

metal/oxide/substrate also forms a MOS cap, which will be in depletion (V_T can be found to be 7.93V and V_{FB} is $-0.84V$)

$$C_{metal\,unit} = \frac{C_{ox}}{\sqrt{1+\frac{2C_{ox}^2(V_{GB}-V_{FB})}{q\,\varepsilon_s N_a}}}$$

$$C_{metal\,unit} = \frac{\left(\frac{3.45\times10^{-13}\,F/cm}{2\times10^{-4}\,cm}\right)}{\sqrt{1+\frac{2\left(\frac{3.45\times10^{-13}\,F/cm}{2\times10^{-4}\,cm}\right)^2(5+0.84)V}{1.6\times10^{-19}C\times1.035\times10^{-12}\,F/cm\times10^{15}/cm^3}}}$$

$$C_{metal\,unit} = \boxed{1.57\,nF/cm^2}$$

(b)

$$C_{metal\,total} = 1.57\,nF/cm^2\times 18\,\mu m^2 = \boxed{0.28\,fF}$$

3.4 cont) N doped region and substrate form a pn junction which has a capacitance.

$$C_{active\,unit} = \frac{\varepsilon_s}{X_d} \;\therefore\; X_d = \sqrt{\left(\frac{2\varepsilon_s(\phi_B - V_D)}{q}\right)\left(\frac{1}{N_a}+\frac{1}{N_d}\right)}$$

$$X_d = \sqrt{\left(\frac{2\times 1.035\times10^{-12}\,F/cm \times (-0.72V - -5V)}{1.6\times10^{-19}\,C}\right)\left(\frac{1}{10^{15}/cm^3}+\frac{1}{10^{17}/cm^3}\right)}$$

phi_B is a positive number so -0.72 should not have a negative sign.

$X_d = 2.36\,\mu m$ Xd=2.73um

note that since the substrate is doped 100 times lighter than the n-region so 99% of this depletion region is in the substrate, and thus the n-region is not completely depleted.

Carrying down Xd
Cactive=3.79nF/cm^2

$$C_{active\,unit} = \frac{1.035\times10^{-12}\,F/cm}{2.36\times10^{-4}\,cm} = \boxed{4.39\,nF/cm^2}$$

C

$$C_{active\,total} = 62\,\mu m^2 \times 4.39\times10^{-9}\,F/cm^2 = \boxed{2.72\,fF}$$

Carrying down again...
CactiveTotal=2.35fF

$30\,\mu m^2$ for bottom
plus $32\,\mu m^2$ for "side walls"

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