4.1 EECS 105 SPRING 2001, Solution HW #4

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a). $\Phi_n = (60mV)\cdot\log(\frac{N_d}{10^{10}}) = (60mV)\cdot\log(\frac{10^{15}}{10^{10}}) = 300\ mV$

$\Phi_p = -(60mV)\cdot\log(\frac{N_a}{10^{10}}) = -(60mV)\cdot\log(\frac{10^{17}}{10^{10}}) = -420\ mV$

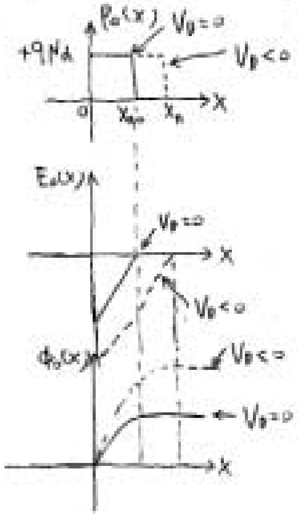
$\Phi_B = \phi_n - \phi_p = 300-(-420) = \boxed{720\ mV}$

b). $X_{do} = X_{no}+X_{po} = \sqrt{(\frac{2\varepsilon_s\phi_B}{q})(\frac{1}{N_a}+\frac{1}{N_d})}$

$= \sqrt{\frac{2(11.7)(8.85\times10^{-14}\ F\cdot cm^{-1})\cdot(0.72V)}{1.6\times10^{-19}C}\cdot(\frac{1}{10^{17}cm^{-3}}+\frac{1}{10^{15}cm^{-3}})}$

$=\boxed{970\ nm}$

c). Assume the PN junction is a one-sided PN junction



$\frac{dE_o}{dx} = \frac{\rho_o(x)}{\varepsilon_s}$

On the n-side of the depletion region

$E_o(x) = E_o(X_n) - \int_x^{X_n} \frac{q\cdot N_d}{\varepsilon_s}\, dx'$

$= 0 - \frac{q\cdot N_d}{\varepsilon_s}(X_n - x)$

$\phi_o(x) = \phi_o(0) + \int_0^x -E_o(x')\,dx'$

$= 0 + \int_0^x \frac{qN_d}{\varepsilon_s}(X_n - x')\,dx'$

$= \frac{qN_d}{\varepsilon_s}X_n\cdot x - \frac{qN_d}{2\varepsilon_s}x^2$

$\phi_o(X_n) = \frac{qN_d}{2\varepsilon_s}X_n^2$

Since $\phi_B - V_D = \phi_o(X_n) - \phi_o(0) = \phi_B(X_n)$ in the one-sided PN junction

$\phi_B - V_D = \frac{qN_d}{2\varepsilon_s}X_n^2$

$\therefore \boxed{X_n = \sqrt{\frac{2\varepsilon_s(\phi_B - V_D)}{qN_d}}}$

4.2

a). $V_{DS} > V_{GS} - V_{Tn}$, $\boxed{\text{Saturation}}$ $V_{Tn} = V_{Ton}$

$I_D = (W/2L)\mu_n C_{ox}\cdot(V_{GS}-V_{Tn})^2(1+\lambda_n V_{DS})$

$= (\frac{100\mu m}{2\cdot2\mu m})\cdot 50\times10^{-6}\frac{A}{V^2}\cdot(4V)^2\cdot[1+(\frac{0.1}{2}V^{-1})\cdot4.5V]$

$=\boxed{24.5\ mA}$

b). $V_{Tn} = V_{Ton} + \gamma_n(\sqrt{-2\phi_p - V_{BS}} - \sqrt{-2\phi_p})$

$= 1V + (0.6\ V^{\frac{1}{2}})\cdot(\sqrt{-2(-0.42V)+2V} - \sqrt{2\cdot0.42})$

$= 1.46\ V$

$V_{DS} > V_{GS} - V_{Tn}$, $\boxed{\text{Saturation}}$

$I_D = (W/2L)\cdot\mu_n C_{ox}\cdot(V_{GS}-V_{Tn})^2(1+\lambda V_{DS})$

$= (\frac{100\mu m}{4\mu m})\cdot(50\times10^{-6}\frac{A}{V^2})\cdot(3-1.46V)^2\cdot[1+(\frac{0.1}{2}V^{-1})\cdot2.5V]$

$=\boxed{3.335\ mA}$

$V_{SD} < V_{SG} + V_{TP}$, $\boxed{\text{Triode}}$ $V_{TP} = V_{T0P}$

$I_D = (W/L)\mu_p C_{ox}[V_{SG}+V_{TP}-(V_{SD}/2)](1+\lambda_p V_{SD})V_{SD}$

$= (\frac{100\mu m}{2\mu m})\cdot 25\times10^{-6}\frac{A}{V^2}[4V-1V-\frac{2.5}{2}V]\cdot[1+(\frac{0.1}{2}V^{-1})\cdot 2.5V]\cdot 2.5V$

$= \boxed{6.152 mA}$

). $V_{TP} = V_{T0P} - \gamma_p(\sqrt{2\phi_n - V_{SB}} - \sqrt{2\phi_n})$

$= -1V - 0.6V^{\frac12}\cdot(\sqrt{2\cdot0.42V+1V} - \sqrt{2\cdot0.42V})$

$= -1.264V$

$V_{SD} < V_{SG} + V_{TP}$ $\boxed{\text{Triode}}$

$-I_D = (W/L)\cdot\mu_p C_{ox}[V_{SG}+V_{TP}-(V_{SD}/2)]\cdot(1+\lambda_p V_{SD})\cdot V_{SD}$

$= (\frac{100\mu m}{2\mu m})\cdot 25\times10^{-6}\frac{A}{V^2}\cdot[3V-1.264V-\frac{1.5}{2}V]\cdot[1+(\frac{0.1}{2}V^{-1})\cdot1.5V]\cdot1.5V$

$= \boxed{1.987 mA}$

3. $kp = \mu_n C_{ox}$, from the graph, $V_{TH} \doteq 0.5V$

a). In triode region

$I_D = (W/L)\cdot kp\cdot[V_{GS}-V_{TH}-\frac{V_{DS}}{2}]V_{DS}$ (neglect $\lambda\cdot V_{DS}$)

$\frac{dI_D}{dV_{DS}} = kp(\frac{W}{L})(V_{GS}-V_{TH}-V_{DS})$

$\frac{dI_D}{dV_{DS}}(V_{DS}=0) = kp(\frac{W}{L})(V_{GS}-V_{TH}) = kp\cdot(\frac{100\mu m}{20\mu m})\cdot(V_{GS}-0.5V)$

when $V_{GS}=5V$, $\frac{dI_D}{dV_{DS}}(V_{DS}=0) \doteq 1\frac{mA}{V}$

$\therefore kp = \frac{dI_D}{dV_{DS}}(V_{DS}=0)\cdot(\frac{20\mu m}{100\mu m})\cdot\frac{1}{V_{GS}-0.5V} = 10^{-3}\frac{A}{V}\cdot\frac{1}{5}\cdot\frac{1}{4.5V} \doteq \boxed{44\mu A/V^2}$

b) In saturation region

$I_D = \frac{kp}{2}\cdot\frac{W}{L}(V_{GS}-V_{TH})^2$

when $V_{GS}=5V$, $V_{DS}=5V$, $I_D = 2.25 mA$

$kp = \frac{2I_D}{\frac{W}{L}\cdot(V_{GS}-V_{TH})^2} = \frac{2\cdot2.25mA}{\frac{100\mu m}{20\mu m}(5V-\frac{1}{2}V)^2} = \boxed{\doteq 44\mu A/V^2}$

4.

a). In triode region,

$\begin{cases} I_D = (W/L)\mu_n C_{ox}[V_{GS}-V_{TH}-(\frac{V_{DS}}{2})]V_{DS} \\ V_{DS} = V_{out} = 5V - I_D\cdot R_L \end{cases}$

$\therefore V_{out} = 5V - (W/L)\cdot\mu_n C_{ox}\cdot[V_{in}-V_{TH}-\frac{V_{out}}{2}]\cdot V_{out}\cdot R_L$

$\therefore (W/L)\mu_n C_{ox}\frac{V_{out}^2}{2}\cdot R_L - [(W/L)\cdot\mu_n C_{ox}\cdot(V_{in}-V_{TH})\cdot R_L+1]\cdot V_{out} + 5V = 0$

$\therefore V_{out} = \frac{[(W/L)\cdot\mu_n C_{ox}(V_{in}-V_{TH})\cdot R_L+1] \pm \sqrt{[(W/L)\mu_n C_{ox}(V_{in}-V_{TH})\cdot R_L+1]^2 - 4\cdot5V\cdot A}}{2\cdot(W/L)\mu_n C_{ox}\cdot R_L\cdot\frac12}$

$A = \frac12(W/L)\mu_n C_{ox}R_L$

$(W/L)\mu_n C_{ox}\cdot R_L = 25\cdot V^{-1}$

$\therefore V_{out} = \frac{[(25V^{-1})\cdot(V_{in}-1V)+1] \pm \sqrt{[(25V^{-1})(V_{in}-1V)+1]^2 - 4\cdot5V\cdot\frac12\cdot25V^{-1}}}{25V^{-1}}$

b). $V_{in}=3V$, the MOSFET is in triode region

$\therefore V_{out} = \frac{[25V^{-1}\cdot(3V-1V)+1] - \sqrt{[(25V^{-1})\cdot(3V-1V)+1]^2 - 2\cdot5V\cdot25V^{-1}}}{25V^{-1}}$

$\doteq \boxed{100mV}$

$I_{out} = \frac{5V-V_{out}}{R_L} = \frac{5V-100mV}{10k\Omega} = \boxed{0.49mA}$

c). $V_{in} = V_{in0} + \Delta V_{in}$, $V_{out} = V_{out0} + \Delta V_{out}$

From a).

$$V_{out0} + \Delta V_{out} = 5V - (W/L)\mu_n C_{ox} \left[(V_{in0} + \Delta V_{in} - V_{Tn}) - \frac{V_{out0} + \Delta V_{out}}{2} \right] \cdot (V_{out0} + \Delta V_{out}) \cdot R_L$$

$$\therefore \Delta V_{out} = -(W/L)\mu_n C_{ox} \cdot \left[\Delta V_{in} - \frac{\Delta V_{out}}{2} \right] \cdot (V_{out0} + \Delta V_{out}) \cdot R_L$$

$$- (W/L)\mu_n C_{ox} \cdot \left[V_{in0} - V_{Tn} - \frac{V_{out0}}{2} \right] \cdot \Delta V_{out} \cdot R_L$$

Neglect higher-order terms and using $(W/L) \cdot \mu_n C_{ox} \cdot R_L = 25\,V^{-1}$

$$\therefore \Delta V_{out} = -25V^{-1} \cdot \left[\Delta V_{in} - \frac{\Delta V_{out}}{2} \right] \cdot V_{out0} - 25V^{-1} \cdot \left[V_{in0} - V_{Tn} - \frac{V_{out0}}{2} \right] \cdot \Delta V_{out}$$

$$\therefore \frac{\Delta V_{out}}{\Delta V_{in}} = \frac{-V_{out0}}{V_{in0} - V_{Tn} - V_{out0} + \frac{1}{25V^{-1}}} = \frac{-100\,mV}{3V - 1V - 0.1V + 0.04V} = \boxed{-\frac{1}{19.4}}$$

$$\doteqdot \boxed{-0.0515}$$

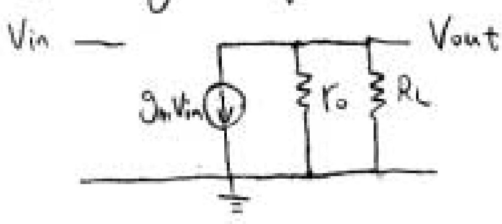
d) $$g_m = \frac{\partial I_D}{\partial V_{GS}} = (W/L) \cdot \mu_n C_{ox} \cdot V_{DS} = \left(\frac{100\,\mu m}{2\,\mu m}\right) \cdot 50\,\mu A/V^2 \cdot V_{DS}$$

$$= 50 \cdot 50\,\mu A/V^2 \cdot 0.1V = \boxed{250\,\mu A/V}$$

$$\frac{1}{r_o} = \left(\frac{\partial I_D}{\partial V_{DS}}\right)$$

$$\therefore r_o = \frac{1}{(W/L)\mu_n C_{ox} [V_{GS} - V_{Tn} - V_{DS}]} = \frac{1}{50 \cdot 50\,\mu A/V^2 \cdot 1.9V} = \boxed{210.5\,\Omega}$$

e). Small signal equivalent circuit



$$\frac{V_{out}}{V_{in}} = -g_m (r_o \| R_L) = -(250\,\mu A/V) \cdot (210.5\,\Omega \,\|\, 10k\Omega)$$

$$= \boxed{-0.0515}$$