EE105 - Spring 2001 - Homework 5 Solution
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5.1a) for border line between triode and saturation

$V_{DS} = V_{GS} - V_{Tn}$. Define I to be drain to source current. From Ohm's law $V_{DS} = 5 - 2IR$

$V_{GS} = 5 - IR$; $V_{BS} = -IR$

source and bulk are at different potentials so

$V_{Tn} = V_{Ton} + \gamma_n\left(\sqrt{-2\phi_p - V_{BS}} - \sqrt{-2\phi_p}\right)$ substituting

$$5 - 2IR = 5 - IR - V_{Ton} - \gamma_n\sqrt{-2\phi_p + IR} + \gamma_n\sqrt{-2\phi_p}$$

$$\frac{V_{Ton} - \gamma_n\sqrt{-2\phi_p} - IR}{-\gamma_n} = \sqrt{-2\phi_p + RI}$$

Let $X \equiv V_{Ton} - \gamma_n\sqrt{-2\phi_p}$

$$\frac{(X - IR)^2}{\gamma_n^2} = -2\phi_p + RI$$

$$X^2 - 2XIR + I^2R^2 = (-2\phi_p + RI)\gamma_n^2$$

$$(X^2 + 2\phi_p\gamma_n^2) + (-2X - \gamma_n^2)RI + R^2I^2 = 0$$

$$aI^2 + bI + c = 0$$

$a = R^2 = 10^8$

$b = \left[-2\left(V_{Ton} - \gamma_n\sqrt{-2\phi_p}\right) - \gamma_n^2\right]R = -12.602 \times 10^3$

$c = \left[V_{Ton} - \gamma_n\sqrt{-2\phi_p}\right]^2 + 2\phi_p\gamma_n^2 = -0.0998$

$I = -7.89\ \mu A$ or $134\ \mu A$

need $I > 0$ so $I = 134\ \mu A$

$V_{BS} = -IR = -134 \times 10^{-6} \times 10^4 = -1.34\ V$

$V_{Tn} = 1 + 0.6\left(\sqrt{+0.84 + 1.34} - \sqrt{+0.84}\right) = 1.335\ V$

$$I = \frac{W}{2L} \mu_n C_{ox} (V_{GS} - V_{Tn})^2 (1 + \lambda_n V_{DS})$$

solve for λ_n

$$\lambda_n = \left(\frac{2I}{\frac{W}{L} \mu_n C_{ox} (5 - IR - V_{Tn})^2} - 1 \right) \frac{1}{5 - 2IR} = 0.4151$$

$$\lambda_n = \frac{0.1}{L}; \; L = \frac{0.1}{\lambda_n} = 0.24 \, \mu m; \; \frac{W}{L} = \frac{1}{2} \Rightarrow \boxed{W = 0.12 \, \mu m}$$

5.1b) as shown above $\boxed{I = 134 \, \mu A}$

5.1c) $\lambda_n = \frac{0.1}{L}$ so increasing $W \Rightarrow$ increasing L

$\Rightarrow$ decreasing λ_n

$I \propto (1 + \lambda_n V_{DS})$

so decreasing $\lambda_n \Rightarrow$ decreasing I and from ohm's law this implies the voltage drop across the resistors will decrease, resulting in more voltage dropped across the transistor, therefore V_{DS} increases and $V_{GS} - V_T < V_{DS} \Rightarrow$ $\boxed{\text{saturation}}$

5.1d) $$I = \frac{W}{2L} \mu_n C_{ox} (5 - IR - V_{Tn})^2 [1 + \lambda_n (5 - 2IR)]$$

$$-\sqrt{\frac{2I}{\frac{W}{L} \mu_n C_{ox} [1 + \lambda_n (5 - 2IR)]}} + 5 - IR = V_{Tn}$$

set equal to V_{Tn} equation

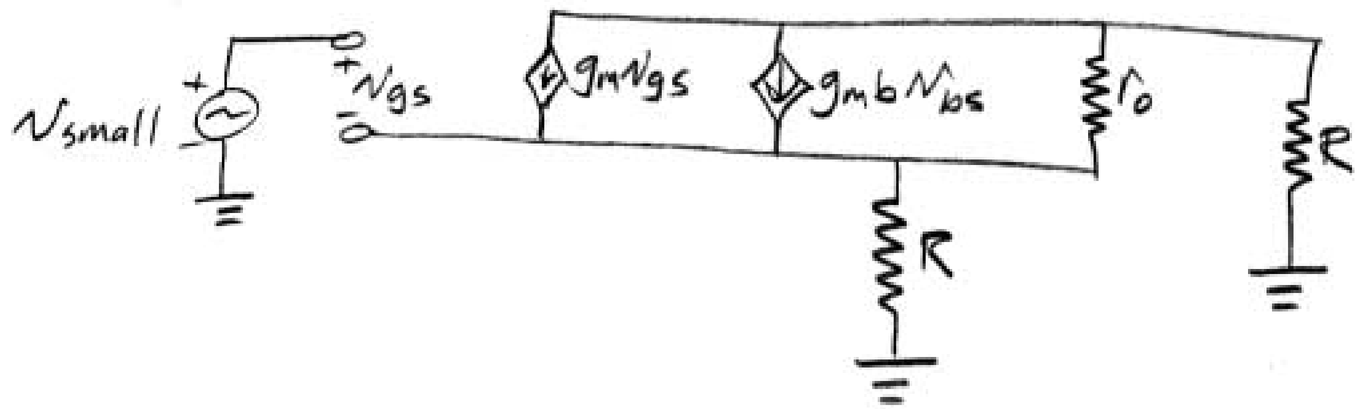
$$5 - IR - \sqrt{\frac{2I}{\frac{W}{L} \mu_n C_{ox} [1 + \lambda_n (5 - 2IR)]}} = V_{T0n} + \gamma_n \left(\sqrt{-2\phi_p + IR} - \sqrt{-2\phi_p} \right)$$

This is nasty, solve for I numerically $\boxed{I = 120 \, \mu A}$

5.1e) $g_m = \sqrt{\frac{2W}{L} \mu_n C_{ox} I} = 77.5\mu A/V$

$$g_{mb} = \frac{\gamma_n g_m}{2\sqrt{-2\phi_p - V_{SB}}} = 5.6\mu A/V$$

$$r_o = \frac{1}{\lambda_n I} = 40.2K\Omega$$



5.2a) $I = \frac{W}{2L} \mu_p C_{ox} (V_{SG} + V_{TP})^2 (1 + \lambda_p V_{SD})$

$V_{SG} = 5 - 2$; $V_{SD} = 5 - V_{out}$

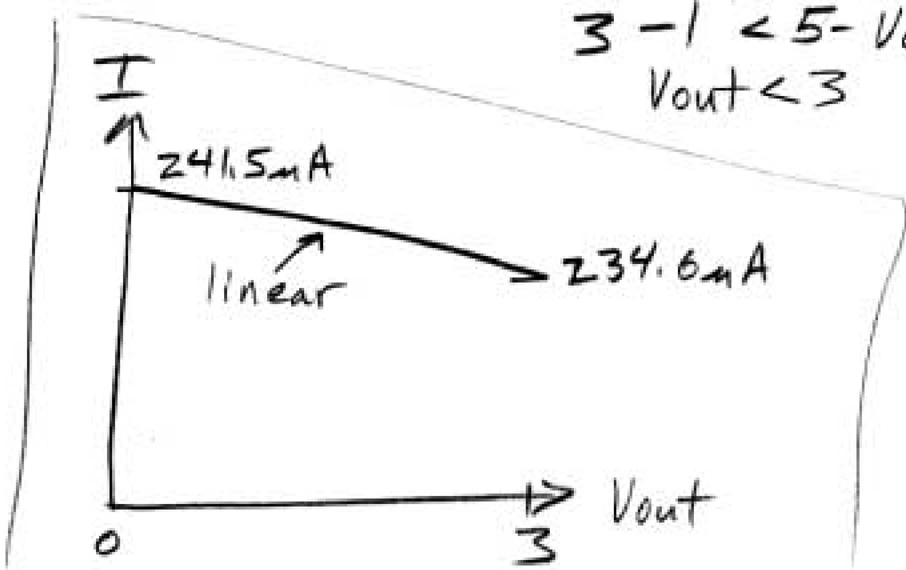
$I = \frac{W}{2L} \mu_p C_{ox} (3-1)^2 [1 + \lambda_p (5 - V_{out})]$

$I = 230 \times 10^{-6} [1 + 0.01(5 - V_{out})]$

for saturation $V_{SG} + V_{TP} < V_{SD}$

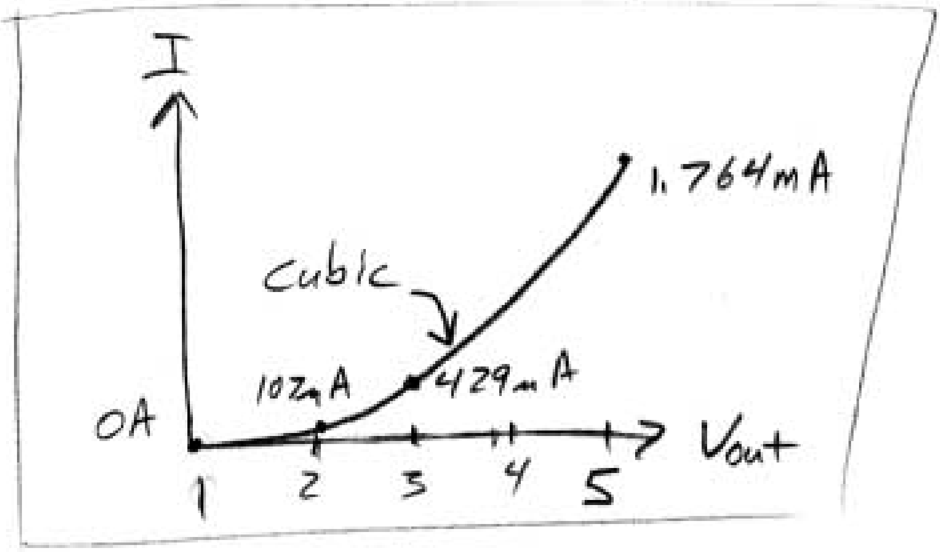
$3 - 1 < 5 - V_{out}$

$V_{out} < 3$



5.2b) $I = \frac{W}{2L} \mu_n C_{ox} (V_{GS} - V_{Tn})^2 (1 + \lambda_n V_{DS})$

$I = \frac{42}{2 \times 10} 50 \times 10^{-6} (V_{out} - 1)^2 (1 + \frac{0.1}{10} V_{out})$



5.2c) I could set the current from 5.2a equal to the current from 5.2b and solve for Vout, however since the range of I for 5.2a is small compared to 5.2b I'll make some approximations to make this easier.

take average I from 5.2a $\frac{241 + 234}{2} = 238 \mu A$

set equal to 5.2b current equation

$238 \times 10^{-6} = \frac{42}{20} 50 \times 10^{-6} (V_{out} - 1)^2 (1 + 0.01 V_{out})$

only weakly dependent on $(1 + 0.01 V_{out})$

so guess a Vout value and stick into that term

say $V_{out} = 2.75$

$238 \times 10^{-6} = 2.1 \times 50 \times 10^{-6} (V_{out} - 1)^2 (1.0275)$

$\sqrt{\frac{238 \times 10^{-6}}{2.1 \times 50 \times 10^{-6} \times 1.0275}} + 1 =$ **$V_{out} = 2.48$ volts**

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5.2d) $g_{m_n} = \sqrt{\frac{2W}{L}\mu_n C_{ox} I} = 316\mu A/V$

$g_{m_p} = \sqrt{\frac{2W}{L}\mu_p C_{ox} I} = 234\mu A/V$

$r_{on} = \frac{1}{\lambda_n I} = 420K\Omega$

$r_{op} = \frac{1}{\lambda_p I} = \frac{1}{\lambda_n I} = 420K\Omega$