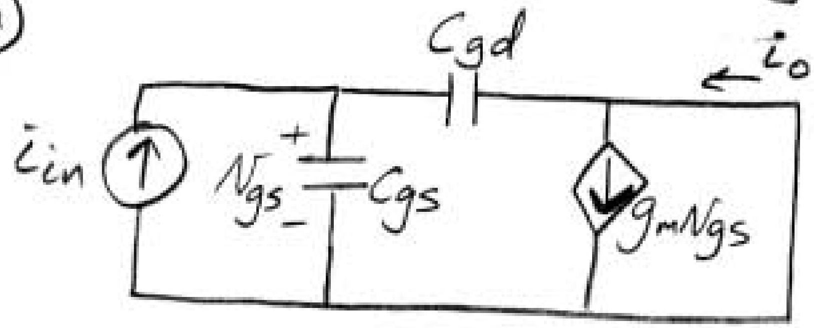
EE105 - Spring 2001 - Homework 7 solution
by William Holtz

1a)



Assuming Saturation, even though it's in triode

$$i_{in} = v_{gs} j\omega C_{gs} + v_{gs} j\omega C_{gd} = v_{gs} j\omega (C_{gs}+C_{gd})$$

$$i_o = g_m v_{gs} - v_{gs} j\omega C_{gd} = v_{gs}(g_m + j\omega C_{gd})$$

solve both of these for v_{gs} and set equal

$$\frac{i_{in}}{j\omega(C_{gs}+C_{gd})} = \frac{i_o}{g_m + j\omega C_{gd}} \Rightarrow \boxed{\frac{g_m + j\omega C_{gd}}{j\omega(C_{gs}+C_{gd})} = \frac{i_o}{i_{in}}}$$

1b) $$\left|\frac{i_o}{i_{in}}\right| = \left|\frac{g_m + j\omega C_{gd}}{j\omega(C_{gs}+C_{gd})}\right| = \frac{\sqrt{g_m^2 + C_{gd}^2\omega^2}}{\omega(C_{gs}+C_{gd})} = 1$$

$$\sqrt{g_m^2 + C_{gd}^2\omega^2} = \omega(C_{gs}+C_{gd})$$

$$g_m^2 + C_{gd}^2\omega^2 = \omega^2[C_{gs}^2 + 2C_{gs}C_{gd} + C_{gd}^2]$$

$$\omega_T = \frac{g_m}{\sqrt{C_{gs}^2 + 2C_{gs}C_{gd}}} \; ; \; C_{gs} = \frac{2}{3}WLC_{ox} + WC_{ov} ; \; C_{gd} = WC_{ov}$$

$$\boxed{\omega_T = \frac{\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_T)(1+\lambda V_{DS})}{\sqrt{\left(\frac{2}{3}WLC_{ox}+WC_{ov}\right)^2 + 2\left(\frac{2}{3}WLC_{ox}+WC_{ov}\right)(WC_{ov})}}}$$

from Fig 2.8 $\mu_n = 775\,cm^2/Vs$; $\frac{\mu_n C_{ox}}{\mu_n} = \frac{50\times10^{-6}}{775} = 64.5\times10^{-9}\ F/cm^2$

$= 0.645\,fF/\mu m^2$

1c)

$$\omega_T = \frac{10\times50\times10^{-6}(5-1)(1+0.1\times2.5)}{\sqrt{\left(\frac{2}{3}\times10\times1\times0.645\times10^{-15}+10\times0.2\times10^{-15}\right)^2+2\left(\frac{2}{3}\times10\times1\times0.645\times10^{-15}\right)\left(10\times0.2\times10^{-15}\right)}}$$

$\boxed{\omega_T = 331\ Grad/sec}$ $\boxed{f_T = 53\,GHz}$

1d)

$$\omega_T = \frac{10\times50\times10^{-6}(3-1)(1+\frac{0.1}{0.6}\times1.5)}{\sqrt{\left(\frac{2}{3}\times6\times0.6\times0.645\times10^{-15}+6\times0.2\times10^{-15}\right)^2+2\left(\frac{2}{3}\times6\times0.6\times0.645\times10^{-15}\right)\left(6\times0.2\times10^{-15}\right)}}$$

$\omega_T = 358\ Grad/sec$ $\boxed{f_T = 57\,GHz}$

2a) find $\frac{v_{out}}{v_{in}}$ via voltage dividers

$$v_{mid} = v_{in}\left[\frac{\left(\frac{1}{j\omega C_1}\right)}{R_1+\frac{1}{j\omega C_1}}\right] \; ; \quad v_{out} = 10v_{mid}\left[\frac{R_2}{\frac{1}{j\omega C_2}+R_2}\right]$$

substitute

$$v_{out} = 10\,v_{in}\left[\frac{1}{j\omega R_1C_1+1}\right]\left[\frac{j\omega R_2C_2}{1+j\omega R_2C_2}\right]$$

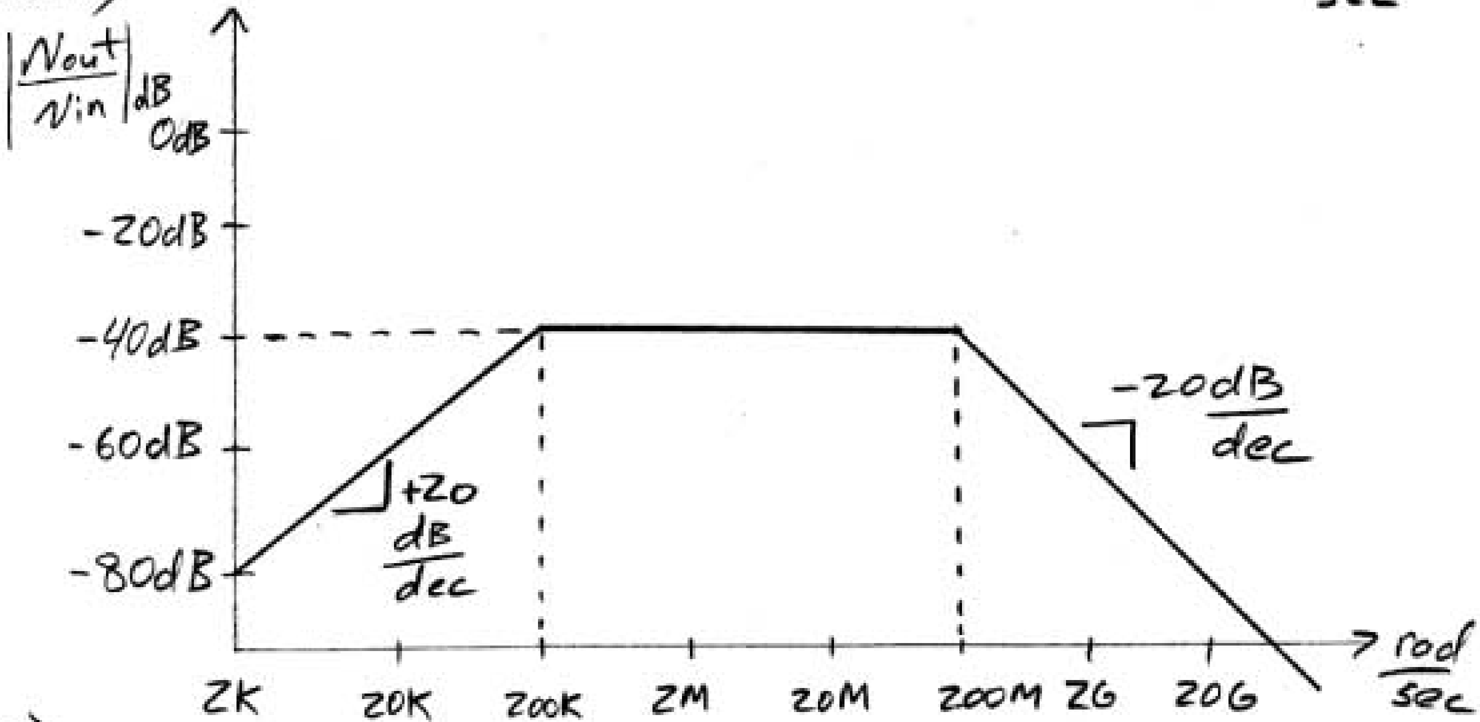
$$\frac{v_{out}}{v_{in}} = \frac{10\times j\omega R_2C_2}{(j\omega R_1C_1+1)(1+j\omega R_2C_2)}$$

zero @ $\omega=0$

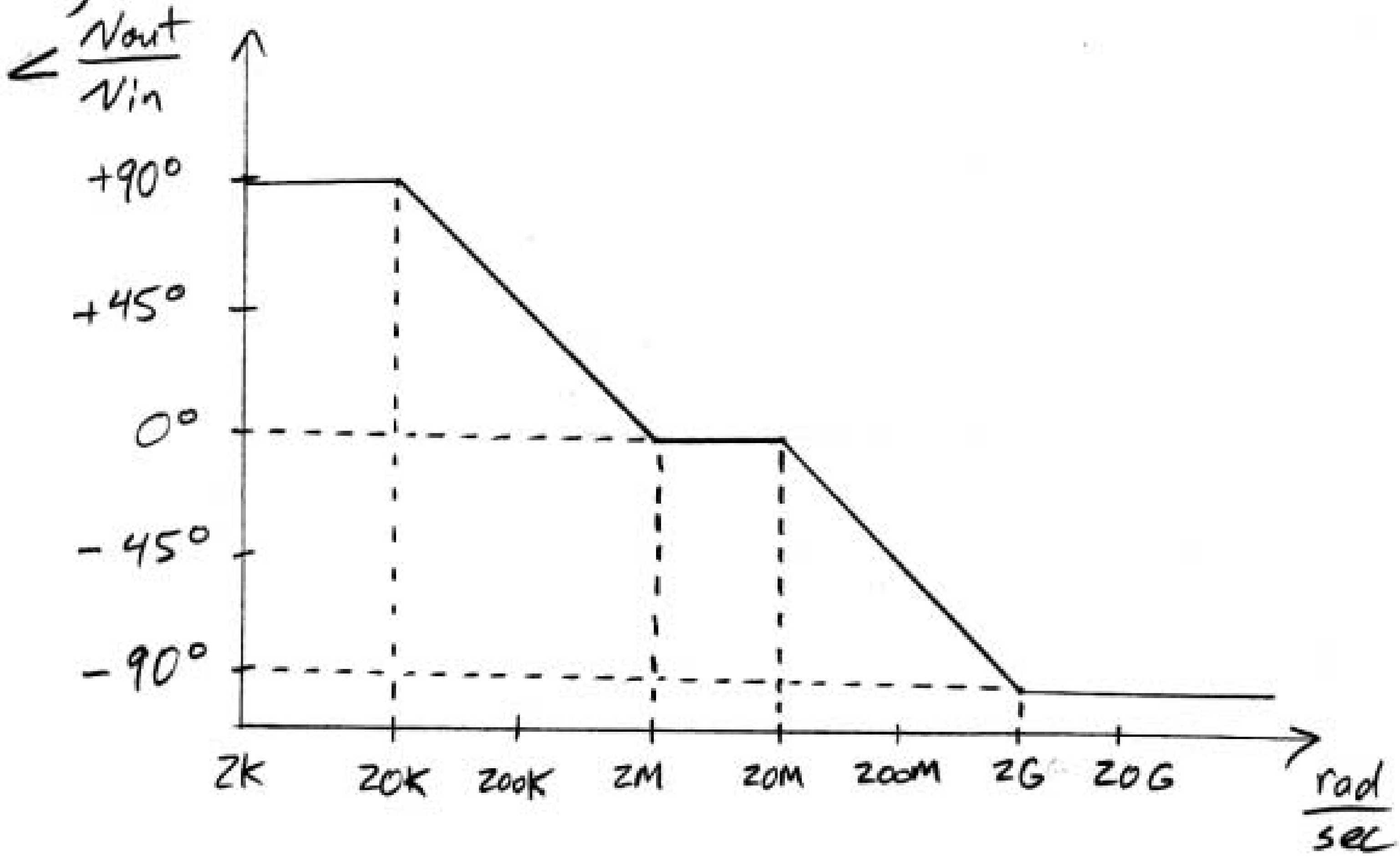
pole @ $\omega = \frac{1}{R_2C_2} = 200\,M\frac{rad}{sec}$

pole @ $\omega = \frac{1}{R_1C_1} = 200\,K\frac{rad}{sec}$

2a) (cont) zero @ $\omega = 0$ => 0dB at $\frac{1}{10 R_2 C_2} = 20M \frac{rad}{sec}$



2b)



3a) $P_{P_0}(-x_p) = N_A = \boxed{10^{18}/cm^3}$; $P_{n_0}(-x_p) = \frac{n_i^2}{N_D} = \frac{10^{20}}{10^{16}} = \boxed{10^4/cm^3}$

$n_{n_0}(x_n) = N_D = \boxed{10^{16}/cm^3}$; $n_{P_0}(x_n) = \frac{n_i^2}{N_A} = \frac{10^{20}}{10^{18}} = \boxed{100/cm^3}$

3b) $P_n(x_n) = P_{n_0} e^{V_D/V_{th}} = 10^4 e^{0.1/0.026} = \boxed{4.68\times10^5/cm^3 = P_n(x_n)}$

$n_p(-x_p) = n_{p_0} e^{V_D/V_{th}} = 100 e^{0.1/0.026} = \boxed{4.68\times10^3/cm^3 = n_p(-x_p)}$

$P_n(x_n) << n_{n_0}$ and $n_p(-x_p) << P_{P_0}$ => low level injection

$\therefore$ $\boxed{P_P(-x_p) = 10^{18}/cm^3}$ $\boxed{n_{n_0}(x_n) = 10^{16}/cm^3}$

4a) $I_0 = q n_i^2 A \left(\frac{D_P}{N_d W_n} + \frac{D_n}{N_a W_p}\right)$ $\mu_p = 460\ cm^2/Vs$ on N side

$\mu_n = 320\ cm^2/Vs$ on P side

$D = \mu V_{th}$; $D_p = 460\times0.026 = 12\ cm^2/s$ on N side

$D_n = 320\times0.026 = 8.32\ cm^2/s$ on P side

$$I_0 = 1.6\times10^{-19} \times 10^{20} \times (2\times10^{-4})^2 \left(\frac{12}{10^{16}\,10^{-4}} + \frac{8.32}{10^{18}\,0.5\times10^{-4}}\right)$$

$\boxed{I_0 = 7.79\times10^{-18} A}$

4b) $I_D = I_0\left(e^{V_D/V_{th}} - 1\right) = 10^{-3} A$

$$\ln\left(\frac{I_D}{I_0} + 1\right) V_{th} = V_D = 0.026 \ln\left(\frac{10^{-3}}{7.79\times10^{-18}} + 1\right)$$

$\boxed{V_D = 0.845\ V}$

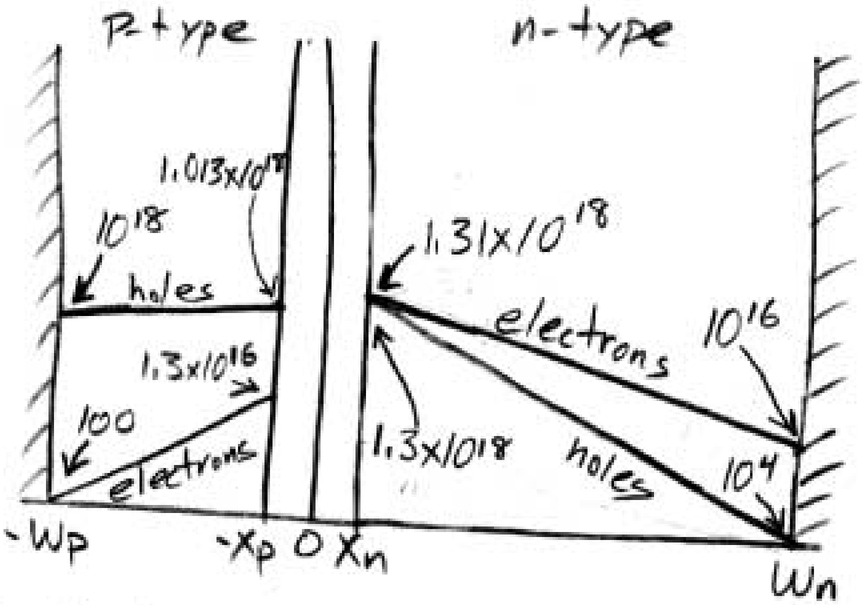
4c) $p_n(x_n) = p_{no} e^{V_D/V_{th}} = 10^4 e^{\frac{0.845}{0.026}} = 1.3\times10^{18}/cm^3$

$n_p(-x_p) = n_{po} e^{V_D/V_{th}} = 100\, e^{\frac{0.845}{0.026}} = 1.3\times10^{16}/cm^3$

low level injection no longer holds

$p_p(-x_p) = p_{po} + n_p(-x_p) = 10^{18} + 1.3\times10^{16} = 1.013\times10^{18}/cm^3$

$n_n(x_n) = n_{no} + p_n(x_n) = 10^{16} + 1.3\times10^{18} = 1.31\times10^{18}/cm^3$



all concentrations in cm^{-3}

4d) find what fraction of current is from holes

$$\frac{\left(\frac{D_p}{N_d W_n}\right)}{\left(\frac{D_p}{N_d W_n} + \frac{D_n}{N_a W_p}\right)} = \frac{\left(\frac{12}{10^{16}\,10^{-4}}\right)}{\left(\frac{12}{10^{16}10^{-4}} + \frac{8.32}{10^{18}\times0.5\times10^{-4}}\right)} = 0.986$$