EE105 - Spring 2001 - Homework #9 Solutions
by William Holtz

1a) $V_{CE} = 2.5 > V_{CE_{SAT}} = 0.1 \Rightarrow$ not saturation
by Ohm's law there is a current through R_C
and by KCL this current must come from the
collector ∴ not cutoff. Current is exiting
collector so not reverse active $\Rightarrow$ $\boxed{\text{Forward Active}}$

1b) Let $Z_{eq} = R_C \| C_L \| R_L = \dfrac{1}{\frac{1}{R_C} + j\omega C_L + \frac{1}{R_L}}$

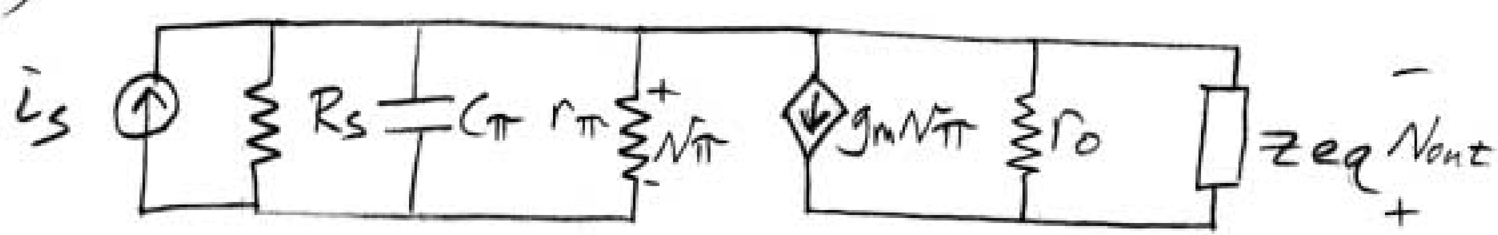
$Z_{eq}(\omega=0) = 833\,\Omega$

$$I_C = \frac{V_{OUT}}{Z_{eq}(\omega=0)} = \frac{2.5}{833} = \boxed{3\text{mA} = I_C}$$

1c)
$$I_B = \frac{I_C}{B} = \frac{5 - V_{BE}}{R_S} + I_{BIAS}\ ;\ I_{BIAS} = \frac{I_C}{B} - \frac{5 - V_{BE}}{R_S} = \frac{3\times10^{-3}}{50} - \frac{4.3}{50K}$$

$$\boxed{I_{BIAS} = -26\,\mu A}$$

1d)



$$g_m = \frac{I_C}{V_{th}} = \frac{3\times10^{-3}}{26\times10^{-3}} = 0.115\,A/V$$

$$r_\pi = \frac{B}{g_m} = \frac{50}{0.115} = 433\,\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{25}{0.115} = 217\,\Omega$$

In the r_0 expression, the value pluged in for I_C is incorrect. Therefore r_0=8333 ohms.

1d cont) $C_\pi = C_b + C_{joE} = g_m \frac{W_B^2}{2D_{nB}} + A\sqrt{\frac{q\varepsilon_s N_{AE} N_{DB}}{2\phi_B (N_{AE}+N_{DB})}}$

$\mu_{nB} = 450\ cm^2/Vs$ $\qquad D_{nB} = \mu_{nB} V_{th} = 11.7\ cm^2/s$

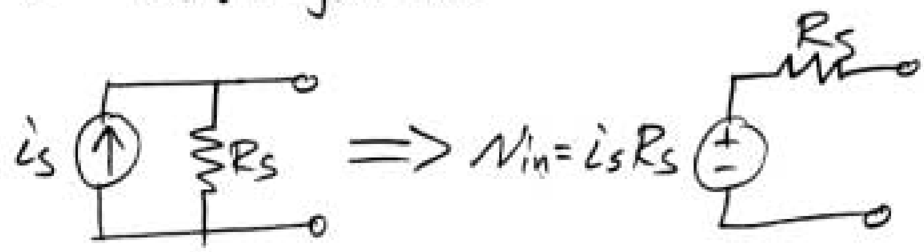
$\phi_B = 60\log\left(\frac{10^{19}}{10^{10}}\right) + 60\log\left(\frac{5\times10^{17}}{10^{10}}\right) = 1.00V$

$$C_\pi = 0.115\,\frac{(10^{-4})^2}{2\times11.7} + (10^{-3})^2\sqrt{\frac{1.6\times10^{-19}\times1.035\times10^{-12}\times10^{19}\times5\times10^{17}}{2\times1\times(10^{19}+5\times10^{17})}}$$

$C_\pi = 49pF$

$Z_{eq} = \frac{1}{1.2\times10^{-3} + j\omega 50\times10^{-12}}\ \Omega$

1e)



Let $Z_\pi = C_\pi || r_\pi$

$v_\pi = i_s\left(\frac{R_s Z_\pi}{R_s + Z_\pi}\right)$; $v_{out} = g_m v_\pi \left(\frac{r_o Z_{eq}}{r_o + Z_{eq}}\right)$

$v_\pi = v_{in}\left(\frac{Z_\pi}{R_s + Z_\pi}\right)$; $v_{out} = g_m v_{in}\left(\frac{Z_\pi}{R_s+Z_\pi}\right)\left(\frac{r_o Z_{eq}}{r_o + Z_{eq}}\right)$

$$\boxed{\frac{v_{out}}{v_{in}} = g_m\left(\frac{Z_\pi}{R_s+Z_\pi}\right)\left(\frac{r_o Z_{eq}}{r_o+Z_{eq}}\right) \quad \text{where } Z_\pi = C_\pi || r_\pi \text{ and } Z_{eq} = R_C || C_L || R_L}$$

1f) expand out equation from part e so that the poles and zeros are easy to see

$$\frac{Z_\pi}{R_S + Z_\pi} = \frac{\left(\frac{r_\pi}{1+j\omega r_\pi C_\pi}\right)}{\left(R_S + \frac{r_\pi}{1+j\omega r_\pi C_\pi}\right)} = \frac{r_\pi}{R_S + r_\pi + j\omega R_S r_\pi C_\pi}$$

$$\frac{r_o Z_{eq}}{r_o + Z_{eq}} = R_C \| R_L \| r_o \| C_L = \frac{1}{\frac{1}{R_C} + \frac{1}{R_L} + \frac{1}{r_o} + j\omega C_L}$$

$$= \frac{R_C R_L r_o}{R_L r_o + R_C r_o + R_C R_L + j\omega R_C R_L r_o C_L}$$

$$\frac{V_{out}}{V_{in}} = \frac{\left[\frac{g_m r_\pi R_C R_L r_o}{(R_S + r_\pi)(R_L r_o + R_C r_o + R_C R_L)}\right]}{\left[1 + \frac{j\omega R_S r_\pi C_\pi}{R_S + r_\pi}\right]\left[1 + \frac{j\omega R_C R_L r_o C_L}{R_L r_o + R_C r_o + R_C R_L}\right]}$$

due to updated r_0 value...
the numerator is 0.748

$$\frac{V_{out}}{V_{in}} = \frac{0.17}{(1 + j\omega\, 21\times10^{-9})(1 + j\omega\, 8.6\times10^{-9})}$$

and the 8.6e-9 gets replaced with
3.79e-8

Poles at $\frac{1}{21\times10^{-9}} = 47.6\ \text{Mrad/s}$

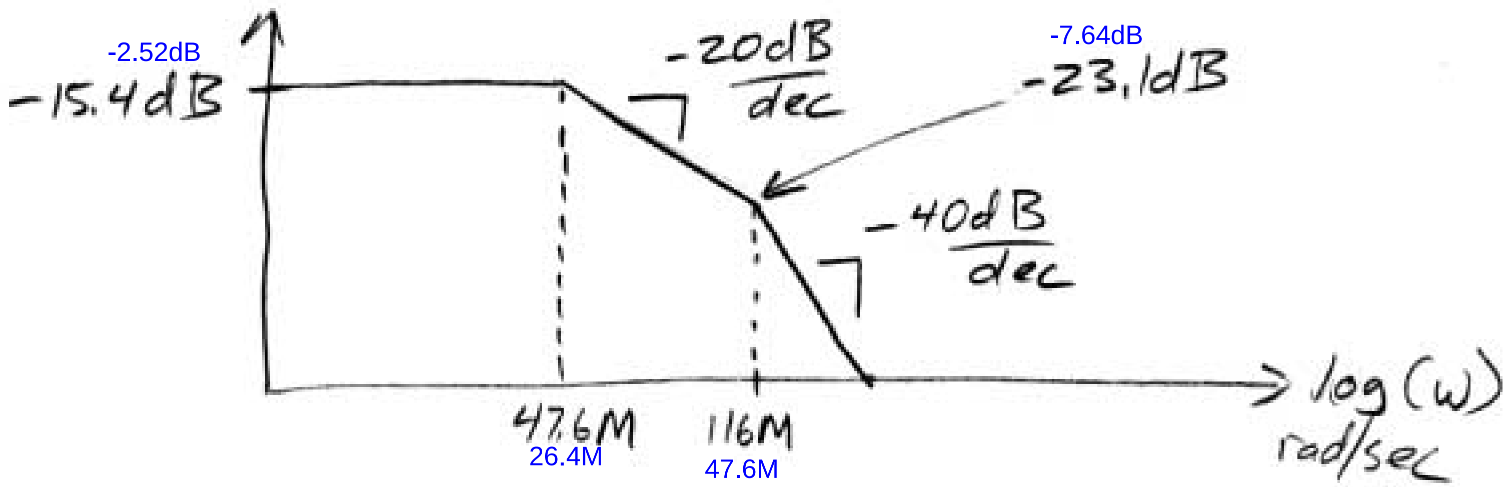
and $\frac{1}{8.6\times10^{-9}} = 116\ \text{Mrad/s}$ 1/3.79e-8=26.4Mrad/s

20log(0.748)=-2.52dB

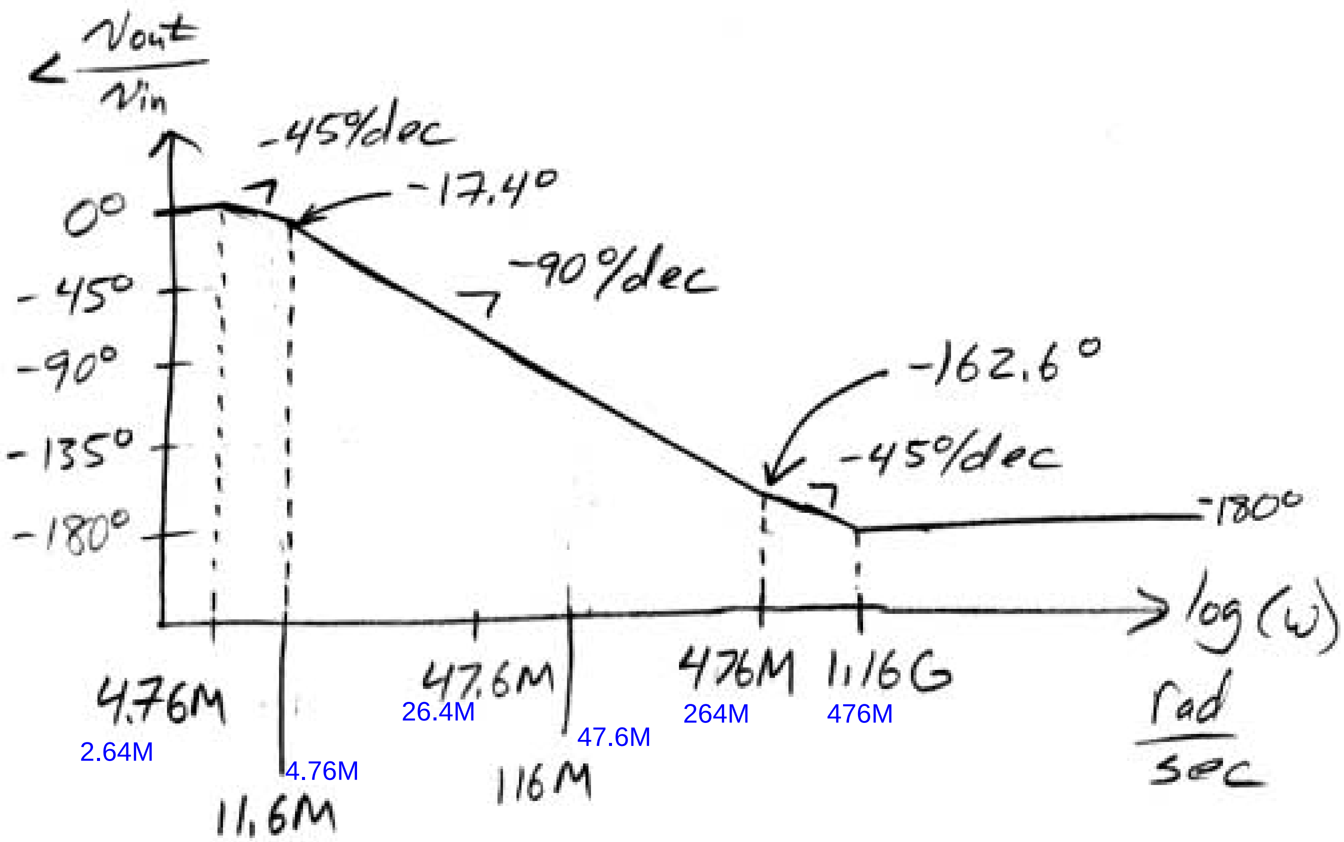
$20\log(0.17) = -15.4\text{dB}$

1f) cont)

$\left|\frac{V_{out}}{V_{in}}\right|_{dB}$



$-15.4 - 20\log\left(\frac{116}{47.6}\right) = -23.1\,dB$ -2.52-20log(47.6/26.4)=-7.64dB



$0 - 45\log\left(\frac{11.6}{4.76}\right) = -17.4°$ -45log(4.76/2.64)=-11.52

$-17.4 - 90\log\left(\frac{476}{11.6}\right) = -162.6°$ -11.52-90log(264/4.76)=-168.5

$-162° - 45\log\left(\frac{1.16}{0.476}\right) = -180°$ -168.5-45log(476/264)=-180

2a) $I_D = q n_i^2 A \left(\frac{D_{nB}}{N_{AB} W_B}\right)\left(e^{V_{BE}/V_{th}} - 1\right)$

D_{nB} is a function of N_{AB}, so solve for $\frac{D_{nB}}{N_{AB}}$ then use trial + error to find a value

$$\frac{I_D W_B}{q n_i^2 A \left(e^{V_{BE}/V_{th}} - 1\right)} = \frac{D_{nB}}{N_{AB}}$$

$$\frac{10^{-3} \times 10^{-4}}{1.6\times10^{-19}\, 10^{20} (10^{-3})^2 \left(e^{0.7/0.026} - 1\right)} = 12.69\times10^{-15} = \frac{D_{nB}}{N_{AB}}$$

Guess $D_{nB} = 10 \Rightarrow N_{AB} \sim 10^{15} \Rightarrow \mu_n = 1375\ cm^2/Vs$

need larger try $N_{AB} = 10^{16} \Rightarrow \mu_n = 1200\ cm^2/Vs$

$$\boxed{N_{AB} \approx 3\times10^{15}/cm^3}$$

2b) $1.269\times10^{-15} = \frac{D_{nB}}{N_{AB}}$ $\boxed{N_{AB} \approx 2.3\times10^{16}/cm^3}$

2c) $1.269\times10^{-16} = \frac{D_{nB}}{N_{AB}}$ $\boxed{N_{AB} \approx 1.7\times10^{17}/cm^3}$

3) saturated $\Rightarrow V_{CE} = V_{CESAT} = 0.1V$

$$I_C = \frac{5-0.1}{40\times10^3} = 123\mu A$$

forward active $\Rightarrow I_B = \frac{I_C}{\beta}$

$$I_B = \frac{123\mu A}{100} = 1.23\mu A$$

$$V_{BIAS} - I_B 10K - V_{BE} = 0$$

$$V_{BIAS} = V_{BE} + I_B 10K = 0.7 + (1.23\times10^{-6})(10^4) = \boxed{0.712V = V_{BIAS}}$$